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Mathematical and Electronic Perception of Electromagnetism

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***Abstract* – The article begins with a reminder of Maxwell's equations and their usefulness in the modeling of the cerebral electromagnetic field, as explained in article [2]. The manipulation of these equations results in a couple of wave equations. A solution of this equation is explained for plane waves. This one-dimensional model can be applied to plane light waves. Finally, the article presents electronic components that can process these signals. Some are passive, others are active. We then arrive at an open question: Can the electronic components process the brain signals, picked up by electrodes or SQUID, in order to extract other useful forms, other than plots? In the article, [1], several applications of brain signal processing are mentioned: Since emotions are electrochemical reactions generated in the brain, can we filter them in brain waves, to improve daily human life such as renovating emotional education in schools?**

***Keywords* – Maxwell Equation, Wave Equation, Electromagnetic Fields, Plane Wave, Electronic Components, Emotional Education.**

I. Modeling of the Electromagnetic Field

Electromagnetic fields were studied physically during the study of light in the seventeenth century. The phenomenon of electromagnetic radiation is a form of linear energy transfer. The electromagnetic wave is one of the representations of the phenomenon. Photons are another representation. In addition, the wave theory of light was mainly developed by Christian Huygens in the 1670s. Then Augustin Fresnel developed the notions of interference and wavelength. In 1860, the great theoretical advance was the synthesis of the laws of electromagnetism by James Clerk Maxwell. His equations predicted the existence of electromagnetic waves, and their speed, allowing the hypothesis that light is an electromagnetic wave. Maxwell's equations constitute the basic postulates of electromagnetism, with the expression of the Lorentz electromagnetic force. These formulas show that in stationary conditions, the electric and magnetic fields are independent of each other. Whereas they are not when they vary over time. J.C. Maxwell succeeds in writing them in the form of integral equations.

In the article [2], the mathematical model is applied to the electromagnetic fields (E, B) generated by the neurons of the brain. It is composed of a system of equations of linear operators. The neural brain sources function as a dipole. A dipole is an entity formed by two equal electric charges of opposite signs, located at a short distance. There are two kinds of dipoles: the generators of the electric current and the receivers of the current. The EEG and MEG sensors are defined in a three-dimensional coordinate system. Assuming that the medium is homogeneous and isotropic, the researchers obtain simple formulas for electromagnetic behavior of the brain. In this conductive medium, isolated in the air, the electric field E and the magnetic field B verify Maxwell's equations (1). The studies carried out in electrophysiology, in particular thanks to the electroencephalogram, use the quasi-static model due to the low frequency of the signal and the low electrical capacity of the tissues of the head. In fact, the frequency of EEG signals most often varies between 0.5 Hz and 100 Hz. On the other hand, the tissues of the head appear as passive conductors, so the researchers consider that electric currents and magnetic fields behave in a stationary way at all times, see thesis [4]. However, in this work we consider them as transient. Consequently, the brain electromagnetic fields (E, B) verify the following Maxwell's equations:

$$\begin{cases} \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla . E &= \frac{\rho}{\varepsilon_0} \\ \nabla \times B &= \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} \\ \nabla . B &= 0 \end{cases} \tag{1}$$

With

1. J is the current density.
2. ρ is the charge density.
3. ε_0 is the permittivity of the vaccum. $\varepsilon_0 = 8.85\ 10^{-12}\ \ Fm^{-1}$
4. μ_0 is the permeability of the vaccum. $\mu_0 = 4\pi\ 10^{-7}\ \ Hm^{-1}$

The first equation, known as the Maxwell-Faraday equation, gives the relation between the circulation of the electric field along the solid circuit and the temporal change magnetic field flux through a surface that rests on this circuit. The second equation, known as the Maxwell-Gauss equation, expresses the fact that the flow of an electric field through a closed surface is related to the electric charge contained inside this surface. The third equation, known as the Maxwell-Ampère equation, expresses the relation between the circulation of the magnetic field along the solid circuit and the leakage current through a surface resting on this circuit. Finally, the fourth equation, called the Maxwell-Flow equation, expresses that the magnetic field flow through any closed surface is zero. In the article [2], the author shows that the system of equations (1) can be transformed into a couple of waves equations. Therefore, the problem of identifying the electric field E is reduced to solving the following d'Alembert equation:

$$\Delta E - \mu_0\, \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \mu_0 J \tag{2}$$

A similar reasoning is applied to the magnetic field *B*. So the problem boils down to solving the equation:

$$\Delta B - \mu_0\, \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 J \tag{3}$$

We note

$$\mu_0\, \varepsilon_0 = \frac{1}{c^2} \tag{4}$$

The equalities 2 and 3 become:

$$\begin{cases} \frac{\partial^2 E}{\partial t^2} = c^2 \Delta E - \frac{1}{\varepsilon_0} J \\ \frac{\partial^2 B}{\partial t^2} = c^2 \Delta B + \frac{1}{\varepsilon_0} J \end{cases} \tag{5}$$

the parameter c is homogeneous at a speed : it is the speed of propagation of the wave. Indeed, in SI base units, we have:

$F.H.m^{-2} = A.s.V^{-1}.V.A^{-1}.s.m^{-2}$

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$= s^2.m^{-2}$ (6)

with V the volt and A the ampere, s the second and m the meter. In any medium, it does not depend on the vacuum measurements μ_0 and ε_0.

II. Plane Wave Modeling

Maxwell's equations result in a couple of hyperbolic equations. A study of the wave equation exists in [3]. In dimension 1, the equation of the waves is called the equation of the vibrating string. Indeed, it describes the vibratory movement of a finite or infinite stretched string. Moreover, it was formulated in the 17th century by scientists studying the movement of the string of a violin. This equation models not only the oscillations of a string but also the propagation of a plane electromagnetic wave. That is to say a wave whose wave fronts are infinite planes, all perpendicular to the same direction of propagation designated by the vector $\vec{n}$. The best known example is the plane light wave. Taking for example $\vec{n}$ in the x-axis direction, then this wave does not depend on the coordinates y and z:

$$u(x, y, z, t) = u(x, t) \tag{7}$$

Thus, the measured quantity depends only on the time t and on a single space variable in Cartesian coordinates.

A. *Solution of the Homogeneous Equation in one Dimension*

The uniqueness of the solution necessitates imposing initial temporal conditions and conditions at the edges of the spatial domain. In this section, we will present the general form of the solutions of this equation, as in the book [3].

1. *Theorem*

We have, $u : \mathbb{R}^2 \to \mathbb{R}$ is a solution of,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{8}$$

If and only if there are two twice derivable functions f and g such that,

$$u(x,t) = f\,(x - ct) + g(x + ct) \tag{9}$$

2. *Demonstration (1 Theorem)*

We will break down the demonstration into two parts:

First Sense:

Let's show that : if u is written in the form 9 then u is a solution of 8. On the one hand,

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= \frac{\partial}{\partial t}(-c\, f\,'(x - ct) + cg'(x + ct)) \\ &= c^2\,(f\,''(x - ct) + g''(x + ct)) \end{aligned} \tag{10}$$

On the other hand,

$$\frac{\partial^2 E}{\partial x^2} = c^2\,(f\,''(x - ct) + g''(x + ct)) \tag{11}$$

Hence the results.

Second Sense:

Let's show that : if u is a solution of 8 then u is written in the form 9. In the equation 8, we make the following change of variables.

$$r = x - ct \,, s = x + ct \tag{12}$$

and we pose,

$$u(x,t) = v(r,s) \tag{13}$$

We have,

$$\begin{aligned} \frac{\partial u}{\partial t} &= \frac{\partial v}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial v}{\partial s}\frac{\partial s}{\partial t} \\ &= -c\frac{\partial v}{\partial r} + c\,\frac{\partial v}{\partial s} \end{aligned} \tag{14}$$

So,

$$\begin{aligned} \frac{\partial^2 u}{\partial t^2} &= -c\,\frac{\partial}{\partial t}\left(\frac{\partial v}{\partial r} - \frac{\partial v}{\partial s}\right) \\ &= -c\left(-c\frac{\partial^2 v}{\partial r^2} + c\,\frac{\partial^2 v}{\partial r\,\partial s} - \left(c\frac{\partial^2 v}{\partial s^2} + c\,\frac{\partial^2 v}{\partial r\,\partial s}\right)\right) \\ &= c^2\left(\frac{\partial^2 v}{\partial r^2} - 2\,\frac{\partial^2 v}{\partial r\,\partial s} + \frac{\partial^2 v}{\partial s^2}\right) \end{aligned} \tag{15}$$

And on the other hand,

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial v}{\partial s}\frac{\partial s}{\partial x} \\ &= -c\frac{\partial v}{\partial r} + c\,\frac{\partial v}{\partial s} \end{aligned} \tag{16}$$

As a result,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial r^2} + 2\,\frac{\partial^2 v}{\partial r\,\partial s} + \frac{\partial^2 v}{\partial s^2} \tag{17}$$

By substituting in the equation 8, we obtain

$$\frac{\partial^2 v}{\partial r^2} + 2\,\frac{\partial^2 v}{\partial r\,\partial s} + \frac{\partial^2 v}{\partial s^2} = c^2\left(\frac{\partial^2 v}{\partial r^2} - 2\,\frac{\partial^2 v}{\partial r\,\partial s} + \frac{\partial^2 v}{\partial s^2}\right) \tag{18}$$

As $c^2 > 0$ we get

$$\frac{\partial^2 v}{\partial r\,\partial s} = 0 \tag{19}$$

To solve 19, we notice that

$$\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial s}\right) = 0 \tag{20}$$

Implies that $\frac{\partial v}{\partial s}$ is constant with respect to r i.e.

$$\frac{\partial v}{\partial s} = F(s) \tag{21}$$

Then

$$v(r,s) = \int F(s)\ ds$$
$$= f(s) + constant\ term\ in\ s \quad (22)$$

Likewise,

$$\frac{\partial}{\partial s}\left(\frac{\partial v}{\partial r}\right) = 0 \quad (23)$$

By integrating, we find

$$v(r,s) = \int G(r)\ dr$$
$$= f(r) + constant\ term\ in\ r \quad (24)$$

It is assumed that the constant term in s is g(r) and that the constant term in r is f(s). Therefore,

$$v(r,s) \ = \ f(s) + g(r) \quad (25)$$

Finally, according to 12, 13 and 25, we obtain

$$u(x,t) = v(r,s)$$
$$= v(x-ct, x+ct) \quad (26)$$
$$= f(x+ct) + g(x-ct)$$

B. *The Non-Homogeneous Equation in one Dimension*

Consider the following equation,

$$\frac{\partial^2 u}{\partial t^2} = \ c^2 \frac{\partial^2 u}{\partial x^2} + \frac{1}{\varepsilon_0} J \quad (27)$$

The term added to the second member $\frac{1}{\varepsilon_0} J$ is called the electromagnetic energy source term. This equation is nonlinear.

III. Electronic Components for Processing the Electromagnetic Signal

An electromagnetic field appears as soon as electric charges are in motion. For example, any conductive wire in the presence of current generates a magnetic field. This paragraph is concerned with electrical components transforming current into another form of energy, such as heat or light; as well as electronic components controlling the movement of electrons to perform operations. A magnetic field can be detected via an antenna. It is transformed into electric current thanks to transformers such as coils. The electric current is processed via electrical and electronic components. Some are passive, others are active. Among the passive components, there are resistors, capacitors and coil. These components are passive because, they do not generate modifications in the electrical characteristics (current, voltage). In other words, they can only store or supply energy but cannot generate it. For example, they oppose the passage of current by presenting an inductive reaction. Some components, such as the resistor, dissipate the current in heat. These components consume only reactive energy. Among the non-passive electrical components, there are the diode and the transistor. These are components that make it possible to increase the power of a signal: voltage, current or both.

A. *Passive Components: the Coil and the Resistor*

A coil is an energy storage dipole. An air coil consists of a winding of conductive wire, such as copper, see figure 1. It generates a magnetic field. In a coil, the voltage lags behind the current. The inductance of a coil is its ability to store magnetic energy created by the flow of current. It can then restore this energy to the rest of the circuit. The coils are used in image reconstructions; see the reference [5].

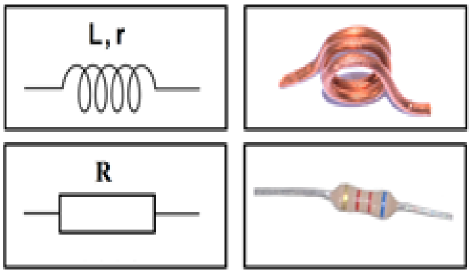

Fig. 1. Passive components: coil and resistance.

An electrical resistor is a receiving dipole that transforms electrical energy into heat. This heat dissipation is considered as an opposition to the passage of an electric current. More generally, the term resistance is associated not only with an electrical component but also with the physical phenomenon and its measurement. Thus, resistance also designates the ability of a conductive material to oppose the passage of an electric current under a given electrical voltage. In addition, its measurement, also called resistance R, can be calculated by the classical expression : $U = R \times I$; with U the voltage at the terminals of the resistor, measured in volt V, I the intensity of the current that passes through the resistor, measured in ampere A and R the value of the resistance, measured in Ohm Ω.

In conclusion, the coil is a component that creates energy thanks to the electromagnetic field but the resistor loses electrical energy in the form of heat.

B. *Active Components : the Transistor and the Zener Diode*

The transistor is a semiconductor electronic component making it possible to control or amplify electrical oltages and currents. It consists of three terminals : the emitter, the base and the collector; as explained in the figure 2. It can be used to regulate the flow of electric current in which a small amount of current in the base controls a larger current between the collector and the emitter. It is possible to use transistors to amplify a weak signal, such as an oscillator or as a switch. It is found inside electronic circuits, whether in high or low voltage.

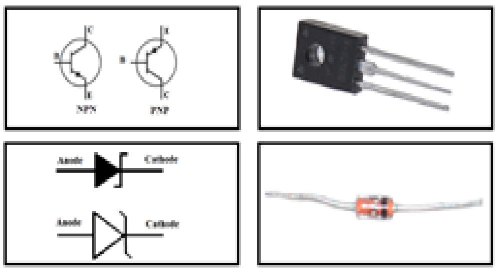

Fig. 2. Active components : transistor and zener diode.

Recall that a diode is a semiconductor device that acts mainly as a one-way current switch. It allows the current to flow easily in one direction, but strongly restricts the current to flow in the opposite direction. A zener diode is an assembly of two semiconductors. They are used to regulate the voltage in a circuit. More precisely, they maintain a known reference voltage at the terminals of the diode when they have a reverse bias.

In conclusion, a transistor makes it possible to amplify an electrical signal, voltage and current. On the other hand, a zener diode makes it possible to obtain a fixed voltage at the out-turn. There are other voltage regulators. For example, a thyristor makes it possible to lower an electrical signal.

IV. Application: Brain Wave Treatment

The brain waves are detected by detectors called electrodes for the EEG and SQUID for the MEG. They are always processed via electronic components and computer software to result in plots. But can they be treated differently?

Brain wave treatment can mean a change in voltage or current or both. In the previous paragraph, we have described electronic components for processing electromagnetic signals. Some are passive, others are active. A treatment of electric waves detected by an EEG or of magnetic waves detected by a MEG may consist in the amplification of the waves via components such as transistors or the modulation of the frequencies via electronic components such as the zener diode.

V. Conclusion

Mathematical modeling always reflects a real physical phenomenon. In this article, the one-dimensional wave equation illustrates the motion of a plane electromagnetic wave. It is important to process electrical and magnetic signals. These attitudes have been generalized for a long time. Indeed, the transformation of radio waves into sound is the most well-known application. This article presents certain electronic components that can intervene in this transformation, such as the transistor. As emotions are electrochemical reactions generated in the brain. Can we extract emotions from brain electromagnetic waves?

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Author's Profile

Yosra Annabi, is a free researcher in applied mathematics. Author of a collection of ten books: The introductions, from European University Editions; as well as two books for the preparation for the aggregation of mathematics. Her research focuses on the mathematical modeling of phenomena in hydrogeology, hemodynamics and brain waves. The mathematical problems studied are direct or inverse.