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# Struggling to Understand the Fundamental Role Played by the Golden Mean Number System in Science and Art: Speculation on Penrose's Fractional Tiling in Art, High Energy Physics and Quantum Cosmology

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*Abstract* – The aim of the present paper is to explain the deep connection between El Naschie's E-Infinity Cantorian spacetime theory and the work of Sir R. Penrose's fractal tiling universe, Prof. A. Connes' non-commutative geometry, Von Neumann's pointless continuous geometry, L. Hardy's quantum entanglement, Nobel Laureate D. Gross' Heterotic string theory and Nobel Laureate Gerard t'Hooft's renormalon. Thus we present here a short but nevertheless comprehensive attempt to comprehend the maze of interrelations between art, science and philosophy wrapped into the mathematical language of the golden mean number system which was developed and flourished in Alexandria-Egypt due to the dedicated work of the luminous stars of the Greco-Egyptian civilization notably Socrates, Pythagoras, Aristotle and Plato.

*Keywords* – Golden Mean Number System, Unification of Science and Art, Penrose Fractal Tiling, L. Hardy's Quantum Entanglement, A. Connes' Non-commutative Geometry, Von Neumann Pointless Fractal Geometry, Golden Mean Scaling, Non-super Symmetric Quantum Gravity Coupling, Heterotic Superstring Theory of D. Gross et al.

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## I. INTRODUCTION - BACKGROUND INFORMATION

The main scientific motivation of the present work was initially induced by the theoretical physics and quantum cosmology connected to cosmic dark energy and Penrose's golden mean fractal tiling universe [1-17]. However, the scope and horizon of our frame of mind grew spontaneously and our questions became more general and more philosophical in a natural autonomous way [1-13]. Thus, we start here in the first place by inquiring about the location of any demarcation lines between reason and emotion that is if there are such lines? [1-5]. Of course, most of us think that our action and behavior is guided mainly by rational reasoning particularly if we are engineers and or scientists as is the case with the present author [1-5], [13]. For all of that and maybe more, let me start with a disclaimer and an honest admission that the author is not at all sure that on a deep level, reality or what we consider real is not an artificial self-imposed mirage [1.13] and undue simplification to an extent difficult to square with what we should aspire to achieve, namely absolute honesty [1, 13].

The present work starts from the above premises and will not pretend that at the end things would be clearer. Just the opposite. There is a considerable probability that at the end things will be even more unclear.

However, if this is true then we are better off with less clarity than with an illusion of a falsity [1-17]. Let us momentarily return to the more secure ground of science and say that the present work is motivated by the research results of L. Hardy's seminal work on quantum entanglement and its relation to A. Connes non-

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commutative geometry as well as E-Infinity Cantorian space time developed in the last 25 years by the present Author and his many associates [2-4].

In a nutshell, Hardy's result states that the probability for quantum entanglement of two quantum particles is given by [11], [14-17]:

$$P(\text{Hardy}) = \phi^5 \tag{1}$$

From this remarkably simple result which was confirmed by many sophisticated actual experiments and gained by some miraculous and simple mathematical reasoning, it was possible to reason that the ordinary energy density of the cosmos is given by half the value  $\phi^5$  given by equation (1) namely [11,12], [14-17].

$$\gamma(\text{ordinary}) = \left(\frac{1}{2}\right)(\phi^5) \tag{2}$$

Consequently, and if the above is true and it is true, it follows that the missing energy density of the cosmos, i.e. what we sensibly call dark energy must amount to [8-17]:

$$\gamma(\text{ordinary}) = 1 - (\phi^5 / 2) = 5\phi^2 / 2 \tag{3}$$

which is almost 95.5% of the total energy of the cosmos given by Einstein's famous maximal energy  $E = mc^2$  [8-12].

It seems we are already in deep waters even though we have not started yet in earnest. The rest of the paper will be an attempt to clarify or destroy some of the deepest contradictions that modern science, philosophy and art have presented us with in connection with Einstein's theory of relativity and Bohr-Schrodinger-Dirac-Feynman quantum mechanics [1-13].

### **III. STRUGGLING FOR UNDERSTANDING IN TEN STEPS**

In what follows we will attempt to summarize the global scientific situation in ten steps as a pun on the ten dimensions of superstring theory.

#### *2.1. Step One*

We know that there is no fully understood connection between Hardy's quantum entanglement, ordinary energy [14-17] and the golden mean [14-16]. In the first instance, this is because Hardy's quantum entanglement [14-17] is given by the golden mean  $(\sqrt{5}-1)/2$  to the fifth power, i.e. the dimension of Kaluza-Klein spacetime [11-17].

Consequently, dark energy would be given by the maximal energy density, i.e. one minus  $\phi^5/2$ . Simple golden mean arithmetic shows this to be equal to  $5\phi^2$  divided by 2. It is remarkable how  $D = 5$  of Kaluza-Klein theory enters this calculation for the ordinary energy as a power of  $\phi$  where  $\phi$  is representative of the zero set pre quantum particle and for dark energy as a fivefold of  $\phi^2$  which is representative of the pre- quantum wave empty set  $\phi^2$  giving us  $5\phi^2$ . The next steps will shed more light on this first step and its implications [11-17].

#### *2.2. Step Two*

In addition to what we mentioned in step 1, it is possible to determine the dark energy density in a completely different way namely as the ratio of renormalization spacetime dimension of 'tHooft- Weltman-Wilson  $D = 4-k$  and  $D = 4$  where  $k = 2\phi^5 = \phi^3(1-\phi^3) = 0.18033989$  is 'tHooft's renormalon [1-17]. Here  $\phi^3 = \phi^2\phi = 1/(4+\phi^3)$  is the intersection of a Connes'-El Naschie pre-quantum wave empty set  $\phi^2$  with the same pre-quantum particles  $\phi$  or the inverse of Einstein's scale invariant spacetime dimension  $4+\phi^3$  discovered by M. Selvam in India and independently by M.S. El Naschie in Egypt and U.K [1-17].

That way  $\gamma(\text{Dark})$  can be written as  $(4 - k)/4$  where  $D = 4$  is the dimension of the smooth spacetime of relativity.

### 2.3. Step Three

Similar to step 2, the same dark energy density may be found as the ratio of the smooth Kaluza-Klein dimension and the fractal one  $D = 5 + \phi^3$  i.e.  $\gamma(\text{Dark})$  is equal to  $5/(5 + \phi^3)$  where  $D(5 + \phi^3) = D(4 + \phi^3) + 1$  as explained in many previous publications [11-17]:

### 2.4. Step Four

The remarkable fact then is that the vital  $\phi^3$  and the equally vital  $k = \phi^2(1-\phi^3)$  gives us the indispensable parameter which brings the results of loop quantum gravity in line with the results of super string theory i.e. the Barbero-Immirzi parameter as the difference between the two:

$$\gamma(\text{Barbero - Immerzi}) = \phi^3 - k = \phi^3 - (\phi^3(1-\phi^3)) = \phi^6 \tag{4}$$

### 2.5. Step Five

The series of remarkable and inspiring relation does not stop with  $\phi^6$  but goes on and on and one finds that the geometrical mean of the pre- quantum wave  $\phi^2$  and the pre-quantum particle  $\phi$  [17] i.e.

$$\sqrt[2]{\phi^2\phi} = \sqrt[2]{\phi^3} = 0.48586872721 = \langle d \rangle_G \tag{5}$$

will give us naturally  $4 + \phi^3$  when inverted and squared which is not surprising at all but the corresponding intersection [1-17].

$$(1/\langle d \rangle)^5 = (4 + \phi^3)^5 = 1364.000732 \tag{6}$$

is more than remarkable and surprising because the integer part 1364 is quite something when looked at more carefully in the next elementary analysis.

### 2.6. Step Six

An elementary analysis: Noting that.

$$1364/2 = 682 \tag{7}$$

One finds:

$$682/2 = 341 \tag{8}$$

and  $341/8 \approx 42.62$

The above result is very close indeed to the inverse non-symmetric quantum gravity coupling [1-17]

$$\bar{\alpha}_{\text{QG}} = 42 + k \tag{9}$$

### 2.7. Step Seven

A golden mean scaling of the inverse Sommerfeld electromagnetic constant  $\bar{\alpha}_0 / 2$ : we start from the fundamental golden mean scaling of  $\bar{\alpha}_0 / 2$  that is well known from our E-Infinity theory and find that:

$$(\bar{\alpha}_0 / 2)(\phi)^n \xrightarrow{n=1} \left(\frac{137 + k_o}{2}\right)(\phi) \tag{10}$$

$$= 6854101966 (\phi)$$

$$= 42 + 2k$$

$$\xrightarrow{n=2} 26 + k$$

$$\xrightarrow{n=3} 16 + k$$

$$\xrightarrow{n=4} 10$$

$$\xrightarrow{n=5} 6 + k$$

$$\xrightarrow{n=6} 4 - k$$

where  $k = \phi^3(1 - \phi^3)$  and  $k_o = \phi^5(1 - \phi^5)$

The perplexing fact we see from the above dimensional hierarchy is that this scaling is identical to the famous hierarchy of the transfinite heterotic theory of D. Gross and his Princeton quartet [17].

### 2.6. Step Eight

The connection to D. Gross et al Heterotic string theory: Following this route we find indispensable transfinite correction method of M. S. El Naschie without which we would not have obtained the original Heterotic string theory series namely,

$$26, 16, 10, 6 \text{ and } 4 \tag{11}$$

and we have to say that the 0.625% of the equation is a Fibonacci approximation [1-17]

$$1 \tag{12}$$

$$1. \quad 1$$

$$2. \quad 1/2 = 0.5.$$

$$3. \quad 2/3 = 0.666.....$$

$$4. \quad 3/5 = 0.6.$$

$$5. \quad 5/8 = 0.625.$$

$$6. \quad 8/13 = 0.6152.....$$

$$7. \quad 13/21 = 0.619.....$$

8.  $21/34 = 0.617\dots\dots$

### 2.9. Step Nine

Taking the limit to  $n = \infty$  this step gives us the limit  $n = \infty$  for the golden mean  $0.68033989 = (\sqrt{5}-1)/2$  and from all the above we clearly see how everything is connected to everything else via the golden mean number system Alexandrian symphony [1-17].

### 2.10. Step Ten

Realizing that the unity of science, art and philosophy is the Alexandria dream of the Greek-Egyptian civilization [1-7], [13].

## IV. CONCLUSION

For almost a quarter of a century researchers have experimented with the role of the number system in impacting understanding and computational efficiency in science. In the same context, art and philosophy as well as science have been viewed as one coherent entity and we could conclude the following: There is nothing more obvious and simple and at the same time nothing is more complex to comprehend than the role of numbers in nature. This result is in fact classic and dates back to the work undertaken in the Egyptian-Greek-Roman, Islamic cosmopolitan city of Alexandria in Egypt. The Penrose golden mean fractal tiling is the absolute best illustration of the preceding discussion and deep insight.

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### AUTHOR'S PROFILE



**Professor M.S. El Naschie** was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree in Engineering plus being a professional chartered engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt. Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work in cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor and a fuelless interstellar spaceship. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems. Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 83 and his i-10 index is 798 and total citations are 37400 according to Google Scholar Citation.