International Journal of Innovation in Science and Mathematics
Volume 7, Issue 3, ISSN (Online): 2347–9051

On the Nature of N-Dimensional Mathematical Objects, with $n \geq 0$

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Date of publication (dd/mm/yyyy): 16/05/2019

***Abstract* – In the Aristotelian viewpoint a line cannot be made by points. The debate on the nature of the continuum has lasted for centuries till the present time. The prevailing opinion is that lines are generated by the movement of points, planes by the movement of lines and so on. In this paper we will try to clarify the nature of the relation between points and other geometric objects taking into account the actual nature of each individual object. In this way we will show that the generation process of geometric objects is opposite to the prevailing opinion mentioned above.**

We will also introduce a question about the relation between sets and power sets, which in our opinion is similar to the relation among geometric objects with a different number of dimensions, although the generation process of sets and power sets is the opposite of the geometrical objects' one. We sustain that power sets seem greater than sets since, so as they have been defined, they have an additional dimension, here defined Event axis.

In a short appendix it is shown how the set of points of a n-dimensional geometric object (with $n > 1$) may be put in a biunivocal correspondence even with a single point.

***Keywords* – Continuum, Point, Line, N-Dimensional Objects, Set Theory, Power Set, Cardinality.**

I. Introduction

The debate on the nature of the continuum has gone on for centuries until the present.

In the Aristotelian sight due to the fact that points have no size a line can not be composed by points (Bottazzini, 2018). We can find this point of view also in modern times: Hegel (1831), for example, states that a sum of points does not give a line. About the nature of the line, recalling Aristotle, Proclus (Bottazzini, cited) writes that a line is the flux of a point in movement. For Giordano Bruno (Bottazzini, cited) a line is a point which moves. We can find this viewpoint also in Newton (1684-1691). Leibniz (1686), on the contrary, does not consider the movements: he states that a line is a collection of infinitesimals (although he does not state that infinitesimals exist, but that we can reason as if they exist) [12].

The problem of the nature of these different geometric objects suggests us to ask such a question about the nature of sets and power sets. We know that, given a set A with infinite elements, the cardinality of the set P(A) is the cardinality of the continuum and then it is not possible to put in biunivocal correspondence the elements of A with the elements of P(A). However, we state that, as for geometric objects, sets and power sets have not the same number of dimensions and then a comparison among them is problematic.

In what follows we will try to give a new interpretation to these problems. About the nature of points, lines, planes and solids we will advance the thesis that a n-dimensional geometric object derives its existence from objects with a greater number of dimensions, and not the contrary, as it has been sustained until now. In the case of the Set Theory we will introduce a time axis (as in Paolilli, 2018), here named “Event Axis”: in this way also the sets will be classified in accordance to the number of their dimensions.

II. THE CONTINUUM IS NOT CONSTITUTED BY POINTS

A point has no dimension but it is not nothing (in Paolilli, 2018, it is indicated with 1 (0)). It exists as a location identified by one or more coordinates in a n-dimensional geometric object (with $n > 0$). Therefore

2.1 *There are no Points without N-Dimensional Geometric Objects (With n > 0).*

Therefore we assert that

2.2 *No one of Geometric Objects with at Least one Dimension is Generated by Points.*

The nature of all these geometric objects is quite different: a point is a position, a line is a distance, a surface is a two-dimensional boundary. Therefore in the same way, as infinite points do not make a segment or a straight line, infinite straight lines do not make a plane, infinite planes do not make a space [13], and this even if it is possible to put in biunivocal correspondence the points of a straight line, a plane or a volume with the points of a segment as small as you want, so considering equipotent these sets of infinite points. Effectively these sets are equipotent, but they are only sets of points which we can identify into a segment, a straight line and so on, and not segments, straight lines and so on.

When we say that the points of a segment have the same cardinality of a straight line or plane or space we mean that, by means of a biunivocal correspondence, we can build an image of them in a segment, but the nature of the image is not the nature of the represented object [14]. The segment is infinitely shorter then the straight line. In this segment there are infinite points, but if we overlap this segment on the straight line, banally in the straight line there are infinite other points that are not in the segment [15]. Segments and straight lines are not sets of points, even if we can individuate points in them. A line is absolutely different from a point, due to the fact that it has one dimension while the point has no dimension. The same for surfaces, volumes and objects with more than three dimensions [16].

III. THE SET A IS NOT CORRECTLY COMPARABLE WITH P (A)

A quite similar observation can be done on sets.

In the Set Theory is:

$$P(A) > A \tag{1}$$

In Figure 1 we show a set A (with 3 elements: {x, y, z}) compared with the power set P(A). Here we assert:

$$Pi(A) \leq A \tag{2}$$

Note that we have added to P a subscript i. So for the reason that the two sets, A and P(A), as for geometric objects with different number of dimensions, are not correctly comparable if another dimension (here the Event Axis) is not considered.

In fact A is a set of elements which exist in a given moment. If the elements of A are infinite, then we have an actual infinity. On the contrary the power set P(A) contains all the possible subsets (all the possible combinations of the elements) of A, and then, so as it is defined, can not exist in a single moment. In a single moment every element is a part of a unique combination and then the combinations in every moment are less or equal to the number of elements, never more numerous. If the elements of A are infinite, the elements of P(A) are then a potential infinity. In other words, P(A) is bigger than A only if we observe all its possible elements without taking into account that they can not all exist simultaneously. On the contrary if we add another dimension, the time [17], here named Events, the sight is quite different. There is a series of events i in which P(A) contains only the combinations for which each element appears only once. It could be objected that an element can be a member of more than one subset in a single moment: for example an element (individual) of A who is an element (member) of more than one subset (club) of P(A). This is true, but it is also true that no one can have more than one interaction if not dividing himself (his attention), and therefore not in all his entirety. In mathematical terms we could have in the subsets, instead of elements of A, fractional elements of A.

In Figure 2 it is exemplified the case of the finite set A compared with P(A), already shown in Figure 1, but adding the Event axis [18] and then saying it Pi(A): in all the events the number of elements of Pi(A) is never greater than the number of the elements of A.

What we said about P(A) is valid for P(P(A)), P(P(P(A))) and so on: other dimensions are added from time to time, so as for geometrical objects. Note, however, that the process of the generation is the inverse with respect to the geometrical objects: there dimensions are subtracted, here dimensions are added.

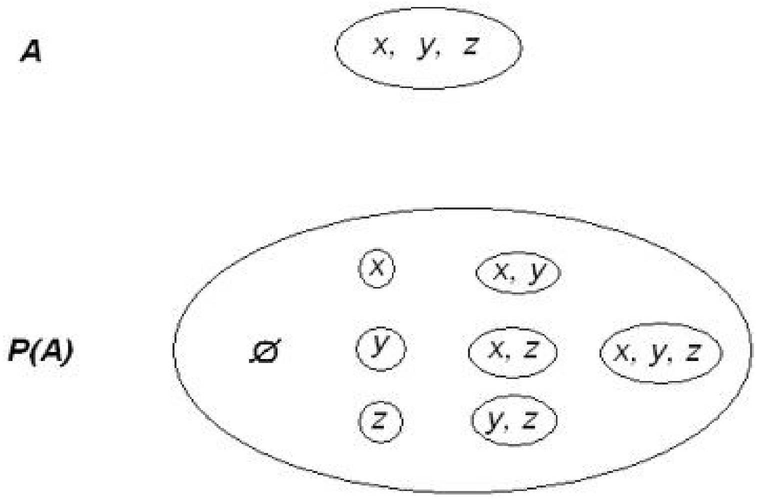

Fig. 1. The set *A*, with *3* elements, is compared with the power set *P(A)*, with 2^3 elements.

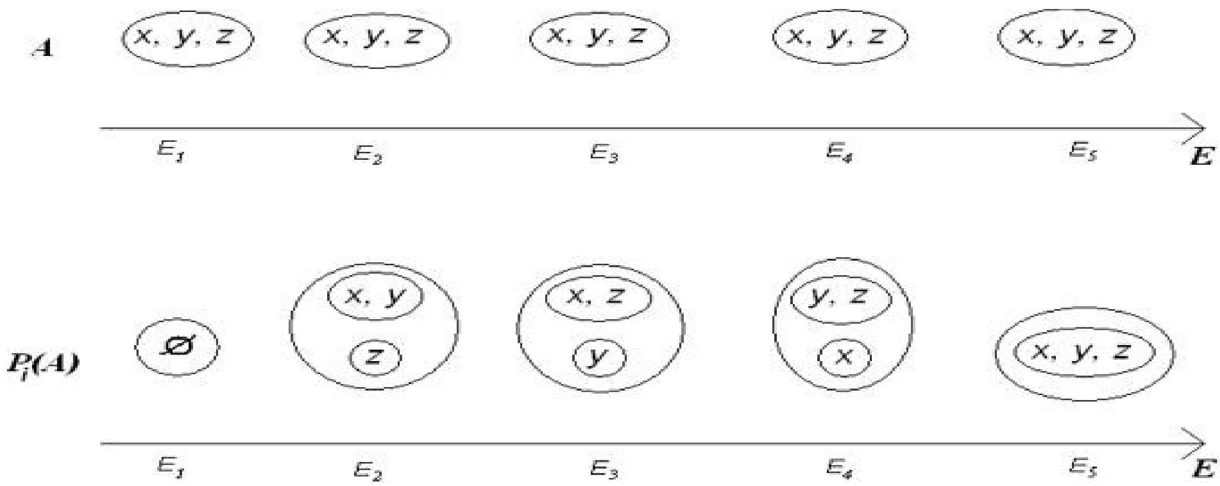

Fig. 2. In the upper section of the figure it is shown the *A* set in different moments. In the lower section it is shown the power set *P(A)* in the same moments. The number of elements of *P(A)* is greater than the number of elements of *A* only if we ignore the Event axis, that is if we consider the set of possible combinations as present all in a single event. The figure, for the sake of simplicity, does not show the events in which only one or two elements of *A* appear in $P_i(A)$.

IV. Conclusion

The debate on the nature of points, lines and other geometric objects has lasted for centuries till present time. We propose an alternative approach to the nature of these objects, defining them by means of a classification which starts from a n-dimensional object and continues subtracting dimensions instead of adding them. In this way the procedure of putting in biunivocal correspondence the points of different geometric objects takes a new aspect. In fact this procedure establishes a comparison among sets of points belonging to geometric objects with different numbers of dimensions but these sets are not the geometric objects to which they belong. These objects have different natures and are not comparable and even their sets of points are comparable.

We underline that the number of dimensions is critical also to understand the nature of other mathematical objects as power sets, which show to have a time dimension. If derived by a set *A* with infinite elements, the cardinality of *P(A)* is aleph 1, but it is a potential and not an actual infinity.

V. Appendix Biunivocal Correspondence between a Point and a Set of Points

We have stated that a point can exist only into a n-dimensional object (with $n > 0$). If the object has at least two dimensions the point can be traversed by infinite lines, each of them with a direction and two possible verses. According to the definition of the line given by Proclus, Bruno and others (a point which moves), we can associate to a point even a speed. Therefore we can put the infinite potential verses and the infinite potential speeds associable to that point in biunivocal correspondence with the points of any geometric object.

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[12] Later Euler (1748) claims that the differentials (and therefore the infinitesimals) are null, being important, for the purposes of the calculus, only the ratio between the differentials (which is not null).
[13] Forgive me for the rough example, but in the material world it is not the organs that generate organisms, but the latter that differentiate themselves giving rise to organs.
[14] In Appendix we show how even a single point could be put in biunivocal correspondence with the points of a n-dimensional geometric object (with $n > 0$).
[15] About the difference among infinities see Smullyan (1992, pp. 234-237).
[16] About the objects with more than three dimensions, see Riemann (1857, posthumous edition). The Riemann's geometry is at the basis of the Einstein's theory of relativity (1905, 1916).
[17] See Paolilli, 2018.
[18] In the lower section of Figure 2, for Pi(A), an improper subset, coincident with the set A, appears in the Event 5. This is not in contrast with what asserted in Paolilli (2017) on the non existence of improper subsets: here the subset seems equal to P(A) but it is not so, being different its nature (a set of three elements in a given moment instead of the power set which changes its conformation in the time: actually P(A) in E5 is a possible combination of the three elements of A, and then it is one of the possible configurations of P(A)taking into account the Event axis.

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