

A Mean Flow Equation and Solution Problem

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Abstract – Picture a function $U(2h) = 0$. $Y = h$ and $U(0) = 0$. Symmetric flow implies the following: The only non-zero term is $-\rho u_1 \bar{u}_2$ which depends on $x_2 = y$. Also, Where P of Omega is mean pressure at walls, $u_2 = 0$ by no-slip depends on streamwise coordinates $x_1 = x_i = x$. We have the following: Where Tau at the other wall, $x_2 = 2D$, should be $-\tau$ of omega. Therefore, from above, we have the following: Then, it follows that the above reads as follows where $\tau = 0$ on channel center plane and x of 2 or $y = D$. Therefore, we get the following: This is for Reynolds shear stress as a function of x of 2, otherwise y .

Keywords – Mean Flow Equations, Flow, Symmetric, Streamwise, Reynolds, Fluid Dynamics.

Picture a diagram here with the following flow and parameters:

$$U(2h) = 0$$

$$Y = h$$

$$U(0) = 0$$

$$\begin{aligned} (\bar{U}_i)_t + \bar{U}_j (\bar{U}_i)_y &= -\frac{1}{\rho} \bar{P}_x + \nu (\bar{U}_i)_{yy} - (u_i \bar{u}_j)_y \\ &= \frac{1}{\rho} [-\bar{P} \delta_{ij} + \mu ((\bar{U}_i)_y + (\bar{U}_j)_x) - \rho u_i \bar{u}_j] \end{aligned}$$

Symmetric flow implies the following:

$$\Rightarrow x_3 = 0 \Rightarrow u_1 \bar{u}_3 = u_2 \bar{u}_3 = 0 \Rightarrow$$

The only non-zero term is $-\rho u_1 \bar{u}_2$ which depends on $x_2 = y$. Also,

$$\bar{u}_2 = \bar{u}_3 = 0$$

$$\Rightarrow \mu (\bar{U}_1)_{yy} = \bar{P}_x + \rho (u_1 \bar{u}_2)_y$$

$$\bar{P}_y + \rho (u_2^2)_y = 0$$

$$(\bar{P})_z = 0$$

$$\Rightarrow \bar{P} = P_\omega(x_1) - \rho u_2^2(x_2)$$

Where P of Omega is mean pressure at walls, $u_2 = 0$ by no-slip depends on streamwise coordinates $x_1 = x_i = x$.

$$\Rightarrow (P_\omega)_x = (\tau)_y$$

Where

$$\tau(x_2) = \mu (\bar{U}_1)_y - \rho u_1 \bar{u}_2$$

We have the following:

$$\tau = x_2 (P_\omega)_x + \tau_\omega$$

$$\tau_\omega = \mu (\bar{U}_1)_y |_{y=0}$$

Where Tau at the other wall, $x_2 = 2D$, should be $-\tau$ of omega. Therefore, from above, we have the following:

$$\tau_\omega = -D (P_\omega)_x$$

Then, it follows that the above reads as follows where $\tau = 0$ on channel center plane and x of 2 or $y = D$.

$$\tau / \tau_\omega = 1 - (x_2 / D) = 1 - (y / D)$$

Therefore, we get the following:

$$\Rightarrow \tau_\omega (1 - (x_2 / D)) + \rho u_1 \bar{u}_2 = \mu (\bar{U}_1)_y$$

$$\Rightarrow -\rho u_1 \bar{u}_2 = \tau_\omega (1 - (x_2 / D))$$

This is for Reynolds shear stress as a function of x of 2, otherwise y .

COMPLIANCE WITH ETHICAL STANDARDS

The author declares that they have no conflict of interest.

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