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Boundary Layer in Jet using Euler Equations

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Abstract **– We look at the Boundary Layer Problem involving a jet using the Euler Equations. Integrating, we then get an integral of the velocity of flow that is boundless. Seeking a similarity solution and substituting some conditions, we get some ordinary differential equations with boundary conditions. Then, we look at a particular solution f for a given u. Where at given x, u trends towards infinity as eta changes by zero. Then, there exists a delta = 0 such that there exists a similarity expression that happens only for a thin jet that is bounded and greater than one.**

Keywords **– Euler Equations, BL, Boundary Layer, Jet, Flow, Velocity, Similarity.**

We look at the Boundary Layer Problem involving a jet using the Euler Equations.

$$uu_x + vu_y = \nu u_{yy} \quad (**)$$

$$\bigtriangledown p = 0, u \to 0, u_y = 0, y = 0$$

Integrating (**) above, we get the following:

$$\rho \int_{-\infty}^{\infty} u^2 dy = M = const.$$

Seeking a similarity solution, define the following:

$$\psi = F(x) f(\eta), \eta = y/g(x)$$

$$F(x) = (\frac{3M}{2\rho})^{1/2}(g(x))^{1/2}, if \int_{-\infty}^{\infty} f(\eta)^2 d\eta = \frac{2}{3}.$$

$$-\frac{1}{2} g^{1/2} g'(f'^2 + ff'') = \nu(\frac{2\rho}{3M})^{1/2} f'''$$

And, substituting the following into (**)

$$g(x) = (3x\nu)^{2/3}(\frac{2\rho}{3M})^{1/3}$$

We get the following:

$$\Rightarrow g(x)\alpha(x^{2/3}) \Rightarrow Const.$$

$$\Rightarrow f''' + (ff')' = 0, f(0) = f''(0) = 0, f' \to \infty \Rightarrow f' = 0$$

$$\Rightarrow f' + \frac{1}{2} f^2 = const.$$

If the following f happens for the following given u:

$$f = (2A)tanh(A_n), u = \frac{1}{2}(\frac{3M^2}{4\nu\rho^2 x})^{1/3} sech^2(\frac{1}{2}\eta)$$

Where at given x, u trends towards infinity as eta changes by zero. Then, there exists a delta = 0 such that the following similarity expression:

$$\delta \sim (\frac{\rho\nu^2}{M})^{1/3} \times \frac{2}{3}, \delta << x$$

Happens only for a then jet such that:

$$(\frac{Mx}{\rho\nu^2})^{1/3} >> 1.$$

Compliance with Ethical Standards
The author declares that they have no conflict of interest.

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Steve Anglin, Sc.M, Ph.D. (h.c.) is an applied mathematician, a member in The Society of Industrial and Applied Mathematics, and a former visiting lecturer at Case Western Reserve University and Saint Leo University. He received his Master of Science in Applied Mathematics from Brown University of The Ivy League and Hon. Doctorate from Trinity College.