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Alternative to Reduced Euler Equations in Turbulent Incompressible Flows where Pressure Has Isotropy Assumption: Vorticity Spectrum

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Abstract **– We look at reduced Euler Equations in turbulent incompressible flows where pressure has isotropy assumption where we find that vorticity is relative to principle axes. We solve and find how vorticity gives an energy spectrum integral. This integral is integrated and gives us the dissipative spectrum from the vorticity in a turbulent flow.**

Keywords **– Euler Equations, Pdes, Partial Differential Equations, Turbulence, Incompressible, Flow, Pressure, Isotropy, Vortex, Energy, Spectrum, Dissipative, Dispersion and Vorticity.**

We have the following vorticity functions:

$$\omega(x) = \nabla \times u(x)$$

$$\omega_i(x) = e_{ijk}\partial u_k(x)/\partial x_j$$

Where vorticity function omega is perpendicular or normal to the flow velocity u(x) implies the vorticity correlations at points p, p' from the following:

$$\omega_i(x)\omega_j(x+r) = \omega_i(x)\omega_j(x') = e_{ilm}e_{jpq}(\partial u_m(x)/\partial x_l)(\partial u_q(x)/\partial x'_p) \quad (1)$$

Where we have the following mapping:

$$e_{ilm}e_{jpq} = -\begin{pmatrix} \delta_{ij} & \delta_{lp} & \delta_{iq} \\ \delta l_j & \delta_{lp} & \delta_{lq} \\ \delta_{mj} & \delta_{mp} & \delta_{mq} \end{pmatrix} = -(-1) = 1 \quad (2)$$

x' = x + r in RHS of the above:

$$(\partial u_m(x)/\partial x_l)(\partial u_q(x')/\partial x'_p) = \partial^2 R_{mq}(r)/\partial r_l \partial r_m \quad (3)$$

Therefore, we get the following equation:

$$\omega_i(x)\omega_j(x') = -\delta_{ij}\nabla^2 R_{ll}(r) + \frac{\partial^2 R_{ll}}{\partial r_i \partial r_j} + \nabla^2 R_{ij}(r) \quad (4)$$

where

$$R_{ij,j}(r) = R_{ij,i}(r) = 0 \Rightarrow \omega_i(x)\omega_j(x+r) = -\nabla^2 R_{ii}(r)$$

Fourier transforming the above implies the following:

$$\Omega_{ij}(k) = (\delta_{ij}k^2 - k_i k_j)\phi_{ll}(k) - k^2\phi_{ji}(k)$$

And

$$\Omega_{ii}(k) = k^2\phi_{ii}(k) \quad (5)$$

These are the vorticity spectrum tensors, where Omega of i and I is the wave number k-space density of the contributions of diagonal (eigenvalues) terms to the mean-square total vorticity:

$$\Omega_{ii}(k) = k^2\phi_{ii}(k) \quad (5b)$$

This is equal to the spectrum of viscous dissipation of Kinetic Energy (KE), phi. So, for ordinary isotropy:

$$\bar{\epsilon} = \frac{d}{dt}(\frac{1}{2}u_i\bar{u}_i) = \frac{d}{dt}\int_0^\infty E(k,t)dk$$
$$= \int\int\int \frac{1}{2}\phi_{ii}(k,t)d^3k$$
$$= \int\int\int k^2\phi_{ii}(k,t)d^3k$$
$$= \int\int\int k^2\phi_{ii}(k)d^3k$$
$$= \int\int\int \Omega_{ii}(k)d^3k$$

This implies the following:

$$\Omega \sim \phi \Rightarrow \omega \sim \epsilon \Rightarrow \omega\alpha\epsilon \Rightarrow allignment.$$

Remark:

$$= 2\nu\int_0^\infty E(k)dk$$
$$= 2\nu\int_0^\infty 4\pi k^2 A(k)dk$$
$$= 8\pi\nu\int_0^\infty k^2 A(k,t)dk.$$

Compliance with Ethical Standards

The author declares that they have no conflict of interest.

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Steve Anglin, Sc.M, Ph.D. (h.c.) is an applied mathematician, a member in The Society of Industrial and Applied Mathematics, and a former visiting lecturer at Case Western Reserve University and Saint Leo University. He received his Master of Science in Applied Mathematics from Brown University of The Ivy League and Hon. Doctorate from Trinity College.