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Effect of Rotation on Convective Diffusive Mass Transfer in a Magnetically Conducting Fluid

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***Abstract* – An analytical study on the effect of rotational flow on convective-diffusive mass transfer in a magnetically conducting two phase fluid is carried out. Gill and Sankarasubramanian's generalized dispersion model is modified to obtain concentration profile of injected drug in a rotating fluid. The effects of parameters arising out of rotation of ferromagnetic fluid on convective diffusive mass transfer of solute are analyze and results are graphically depicted.**

***Keywords* – Dispersion model, Ferro-fluid, Pulsatile flow, Rotating fluid.**

I. Introduction

In physiological fluid dynamics, flow generated by pulsatile motion of the boundary finds importance. If magnetic field is also present, it gives rise to a rotating motion in presence of angular velocity. Mass transfer in this case is affected by the rotation of fluid motion and is interesting to analyze.

Ghosh [1] has studied motion of an incompressible viscous fluid in a channel bounded by two rigid coaxial cylinders with inner cylinder set to motion by longitudinal pulses. Chakra borty and Ray [2] studied the unsteady magneto hydrodynamic coquette flow between two parallel plates when one of the plates is subjected to random pulses. Makar [3] presented the solution of magneto hydrodynamic flow between two parallel plates when the velocity tooth pulse is imposed on the upper plate and the induced magnetic field is neglected. Ghosh and Sarkar [4] have analyzed the hydro magnetic channel flow of a dusty fluid induced by velocity tooth pulses and arrived at the solution by using Fourier analysis. Ghosh and Debnath [5] have used Laplace transforms for solving the same problem. Ghosh and Ghosh [6] have solved the hydro dynamic flow of two phase fluid near a pulsating plate. Yang H T et.al [7] have studied on Stokes problems by considering incompressible, viscous, conducting fluid in the presence of magnetic field.

H P Greenspan [8] has explained various features of rotating fluids and its applications in various fields. P G Saffman [9] has explained on stability characteristics of a dusty gas in the case of a plane parallel flow. D H Michael et.al [10] have solved problems on the motion of a dusty gas using Saffman's formula and Laplace transform.

Haleh Alimohamadi and Mohsen Imani [11] have explained the effect of external magnetic field on blood flow patterns in a stenosis artery. By considering the geometrical and biological aspects of the area where stenosis has occurred, they have solved fluid dynamic equations in coupled porous and various factors related to the occurrence of stenosis are investigated. Shigang Wang et.al [12] has discussed on the method of magnetically targeted drug delivery where the drug containing the nano particles are targeted towards affected tissue. They have developed a model presenting the hydrodynamics of ferro fluids flowing in a blood vessel under the application of magnetic field.

A 3D flow is also analyzed in this case to understand clinical application of magnetic targeting drug delivery. Results obtained in the analysis present further scope for improving delivery in favour of the clinical approach.

Zhong Tan and Yanjin Wang [13] have discussed on the differential system to describe the motion of an incompressible ferro fluid under the influence of external magnetic field. They have provided the solution by considering the Navier-stokes equations, the magnetization equations and the magneto static equations. Various other hydro dynamical aspects have been discussed by solving a Cauchy problem and establishing a blow-up criterion for strong solutions.

Haleh Alimohamadi and Mohsen Imani [14] have done investigations and obtained numerical solutions of blood flow patterns in aneurysm artery under the effect of magnetic field. They have considered the governing equations of these phenomena and analyzed that there is a large variation between unsteady pulsatile inlet velocity and human heart beating frequency. They have shown that wall porosity of the blood vessel has great importance in the performance of the magnetically targeted drug delivery.

In the present study the effect of rotational flow on convective-diffusive mass transfer in a magnetically conducting two phase fluid is considered. Modified generalized dispersion model of Gill and Sankarasubramanian is applied here to obtain concentration profile of injected drug in a rotating fluid.

II. Mathematical Formulation

A flow of incompressible viscous fluid through a channel is considered. Due to suspended particles an angular velocity arises in the fluid which is normal to the axis. The lower boundary is moving with a velocity u_o. This along with the magnetic field causes motion of the fluid. The governing equations are

$$\frac{\partial u}{\partial t} + (2i\Omega)u = \nu\frac{\partial^2 u}{\partial y^2} + \frac{K}{\tau}(v-u) - nu \qquad (1)$$

$$m\frac{\partial v}{\partial t} + (2i\Omega)v = \frac{1}{\tau}(u-v) \qquad (2)$$

Where u and v are velocities of the fluid, ν is the kinematic viscosity given by $\nu = \frac{\mu}{\rho}$, μ is the viscosity of the fluid, ρ is the density, t is the time, $K = \frac{mN_0}{\rho}$, N_0 the couple stress parameter, mN_0 is the mass of the particle, $\frac{\sigma_0 B_0^2}{\rho}$ is the external magnetic field applied on the flow, σ_0 is the stress tensor, B_0 is the magnetic effect and Ω is the rotation parameter.

To solve (1) and (2), we define the boundary conditions as

$u = 0 \; at \; t = 0$ (3)

$u = u_0 e^{-nt} \; at \; y = 0$ (4)

$u = 0 \; at \; y = h$ (5)

Above equations have been non dimensionalised using

$$(u^*, v^*) = \frac{(u, v)}{u_0}, y^* = \frac{y}{\sqrt{\nu\tau}}, t^* = \frac{t}{\tau}$$

Using these variables equations (1) and (2) takes the form

$\frac{\partial u}{\partial t} + (Ei)u = \frac{\partial^2 u}{\partial y^2} + K(v - u) - \alpha u$ (6)

$\frac{\partial v}{\partial t} + (Ei)v = (u - v)$ (7)

Where, $E = 2\pi\tau$ is a non dimensional flow parameter and $\alpha = n\tau$ is a magnetic parameter.

The boundary conditions in non-dimensional form are

$u = 0 \; at \; t = 0$ (8a)

$u = u_0 \; at \; y = 0$ (8b)

$u = 0 \; at \; y = h$ (8c)

Let us assume that $u = Ue^{-wt} \; and \; v = Ve^{-wt}$ then eliminating v from (6) and (7), we get

$\frac{\partial^2 U}{\partial y^2} + \lambda^2 U = -P$ (9)

Where $\lambda^2 = \left[\frac{k(w-Ei)}{1+Ei-w} + w - Ei - \alpha\right]$ (9a)

Solving (9) we get the velocity profile as

$U = u_0(cos\lambda y + cot\lambda h sin\lambda y]$ (10)

The average velocity is given by

$\bar{U} = \frac{u_0}{\lambda}\left(\frac{1-cos\lambda h}{sin\lambda h}\right)$ (11)

The particle velocity V can be obtained using the assumption $V = \frac{U}{1+Ei-w}$ (12)

III. Dispersion Model

A slug input with initial concentration co of length Xs is introduced into the flow. As the particles are magnetically conducting and due to the flow the solute stars dispersing. The governing species equation is given by

$\frac{\partial c}{\partial t} + u(y,t)\frac{\partial c}{\partial x} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right)$ (13)

With boundary conditions

$$c(0,x,y) = \begin{cases} c_0, \mid x \mid \leq \frac{1}{2}X_s \\ 0, \; otherwise \end{cases}$$

$-D\frac{\partial c}{\partial y} = k_s c \; at \; y = h$ (14a)

$D\frac{\partial c}{\partial y} = k_s c \; at \; y = -h$ (14b)

$C(t,\infty,y) = \frac{\partial c}{\partial x}(t,\infty,y) = 0$ (14c)

Non dimensionalising (13) to (14c) using the following

$\xi = \frac{y}{h}, \tau = \frac{Dt}{h^2}, \theta = \frac{C}{C_0}, X = \frac{Dx}{h^2\bar{u}}, U = \frac{u}{\bar{u}}, P_e = \frac{h\bar{u}}{D}, \beta = \frac{k_s h}{D}$

we get

$\frac{\partial\theta}{\partial\tau} + U\frac{\partial\theta}{\partial X} = \frac{1}{P_e^2}\frac{\partial^2 c}{\partial X^2} + \frac{\partial^2\theta}{\partial\xi^2}$ (15a)

$\theta(0,X,\xi) = \psi(X)Y(\xi),$ (15b)

$\frac{\partial\theta}{\partial\xi} = -\beta\theta \; at \; \xi = h$ (15c)

$\frac{\partial\theta}{\partial\xi} = \beta\theta \; at \; \xi = 0$ (15d)

$\theta(\tau,\infty,\xi) = \frac{\partial\theta}{\partial x}(\tau,\infty,\xi) = 0$ (15e)

By using the generalized dispersion model proposed by Gill and Sankarasubramanian (1970), the solution of (15a) using (15b) to (15e), we get

$\theta = \sum_{k=0}^{\infty} f_k(\tau,\xi)\frac{\partial^k\theta_m}{\partial X^k}$ (16)

where $\theta_m = \int_0^h \theta \, d\xi$ (16a)

is the dimensionless average concentration.

Integrating (15a) w.r.t ξ between 0 $and \; h$ we get

$\frac{\partial\theta_m}{\partial\tau} = \frac{1}{P_e^2}\frac{\partial^2\theta_m}{\partial X^2} + \left|\frac{\partial\theta_m}{\partial\xi}\right|_0^h - \frac{1}{2}\frac{\partial}{\partial X}\int_0^h U\theta \, d\xi$ (17)

Where θ_m is introduced into (17)

Let us introduce the dispersion coefficient

$\frac{\partial\theta_m}{\partial\tau} = \sum_{i=0}^{\infty} K_i(\tau)\frac{\partial^i\theta_m}{\partial X^i}$ (18)

In the dispersion model proposed by Gill and Sankarasubramanaian (1970) if the process of distributing θ is diffusive in nature.

The values of $K_i(\tau)$ can be obtained from the following relation:

$K_k = \frac{\delta_{i,2}}{Pe^2} - 2\frac{\partial f_i}{\partial\xi}(\tau,1) - \int_0^h f_{i-1}Ud\xi \;\; i = 0,1,2,$ (19)

Where $f_{-1} = f_{-2} = 0, \delta_{i,2}$ is the kronecker delta defined by

$\delta_{ij} = \begin{cases} 1, for \; i = j \\ 0, for \; i \neq j \end{cases}$ (20)

Equation (18) reduces to the following form due to truncation

$\frac{\partial\theta_m}{\partial\tau} = K_0(\tau)\theta_m + K_1(\tau)\frac{\partial\theta_m}{\partial X} + K_2(\tau)\frac{\partial^2\theta_m}{\partial X^2}$ (21)

To solve (6.4.0) we need the values of $K_i(\tau), for \; i = 0,1,2$ and $f_k, for \; k = 1,2,3,..$ Hence we substitute (16) into (15a) and use (17) to get the set of partial differential equations given below:

$$\frac{\partial f_k}{\partial\tau} = \frac{\partial^2 f_k}{\partial\xi^2} - Uf_{k-1} + \frac{1}{Pe^2}f_{k-2} + \sum_{i=0}^{k} f_{n-i}K_i,$$

$k = 0,1,2, ...$ (22)

Subjected to

$\theta_m(0,X) = \int_0^h \theta \, d\xi$ (22a)

$f_k(0,\xi) = 0 \; and \; f_k(\tau,0) = finite \; for \; k = 1,2,3,..$ (22b)

$\frac{\partial f_k}{\partial r}(\tau,1) = -\beta f_k(\tau,1)$ (22c)

$\theta_m(\tau,\infty) = \frac{\partial\theta_m}{\partial X}(\tau,\infty) = 0$ (22d)

We have $\int_0^h f_k d\xi = \delta_{k,0}$ (23)

The functions f_0 and exchange coefficient K_0 are obtained from the following:

$K_0(\tau) = \left|\frac{\partial f_0}{\partial\xi}\right|_0^h$ (24) $\frac{\partial f_0}{\partial\tau} = \frac{\partial^2 f_0}{\partial\xi^2} - K_0 f_0$ (25)

Introducing $f_0(\tau,\xi) = e^{-\int_0^\tau K_0(\eta)d\eta} g_0(\tau,\xi)$ (26)

the solution is obtained as

$g_0(\tau,\xi) = \sum_{n=0}^{\infty} A_n e^{-\mu_n^2\tau} cos\mu_n\xi$ (27a)

Where $A_n = \frac{\int_0^h Y(\xi)cos\mu_n\xi d\xi}{\left[1+\frac{sin2\mu_n h}{2\mu_n}\right]\int_0^h Y(\xi)d\xi}$ (27b)

And $\mu_n's$ are Eigen values satisfying the following equation:

$\mu_n sin\mu_n = \beta cos\mu_n \; for \; n = 0,1,2, ...$ (27c)

$f_0(\tau,\xi) = \frac{\sum_{n=0}^{\infty} A_n e^{-\mu_n^2\tau} cos\mu_n\xi}{\sum_{n=0}^{\infty}\frac{A_n}{\mu_n}e^{-\mu_n^2\tau} sin\mu_n\xi}$ (28)

And $K_0(\tau) = -\frac{\sum_{n=0}^{\infty}(A_n\mu_n)e^{-\mu_n{}^2\tau}sin\mu_n\xi}{\sum_{n=0}^{\infty}\frac{A_n}{\mu_n}e^{-\mu_n{}^2\tau}sin\mu_n\xi}$ (29)

As τ→∞, the asymptotic representations for f_0 and K_0 are given by

$$f_0(\infty,\xi) = \frac{\mu_0}{sin\mu_0 h} \quad (30)$$

$$K_0(\infty) = -\mu_0{}^2, \quad (31)$$

After the injection of the solute, for longer time $f_K(\xi)$ will satisfy the equation

$$\frac{d^2 f_k}{d\xi^2} + \mu_0{}^2 f_k = \frac{u}{\bar{u}} f_{k-1} - \frac{1}{P_e^2} f_{k-2} + \sum_{i=1}^{k} K_i\, f_{k-i} \; for\; k = 1,2,3,\ldots \quad (32a)$$

With boundary conditions

$$f_k(0) = finite, \frac{\partial f_k}{\partial\xi}(1) = -\beta f_k(1), for\; k = 1,2,3,.. \quad (32b)$$

$$\int_0^h f_k d\xi = 0, for\; k = 1,2,3,\ldots. \quad (32c)$$

Following Gill and Sankarasubramanian approach using (19) we have

$$K_k = \frac{1}{\int_0^h f_0 cos\mu_0\xi d\xi}\int_0^h \left[\frac{u}{\bar{u}} f_{k-1} + \frac{1}{P_e^2} f_{k-2} - \sum_{i=1}^{k} K_i\, f_{k-i}\right] cos\mu_0\xi d\xi \quad (33)$$

Introducing $f_k = \sum_{j=0}^{\infty} B_{j,k}\, cos\mu_j\xi \; for\; k = 1,2,.$ (34)

Where $B_{j,k} = \frac{1}{\mu_j{}^2-\mu_0{}^2}\left[\frac{1}{P_e^2} B_{j,k-1} - \sum_{j=1}^{\infty} B_{j,k-1} - \left[1 + \frac{sin 2\mu_j h}{2\mu_j}\right]^{-1} \sum_{l=0}^{\infty} B_{l,k-l} I(j,l)\right]$ and $For\; j \neq l \quad I(j,l) = \int_0^h \frac{u}{\bar{u}} cos\mu_j\xi cos\mu_l\xi d\xi$ (35)

$For\; j = l$

$$I(j,l) = \int_0^h \frac{u}{\bar{u}} cos\mu_j\xi cos\mu_j\xi d\xi \quad (36)$$

$$K_1(\tau) = \frac{I(0,0)}{\left[1+\frac{\sin\mu_0 h}{2\mu_0}\right]} \quad (37)$$

$$K_2(\tau) = \frac{1}{P_e^2} - \frac{sin\mu_0 h}{\mu_0\left(1+\frac{\sin 2\mu_0 h}{2\mu_0}\right)}\sum_{j=0}^{\infty} B_{j,1}\, I(j,0) \quad (38)$$

The solution is obtained in one half of the channel and A_n mentioned in (27b) is independent of

$\psi(x)$ *and* $\phi(\xi)$ *given by* $\phi(\xi) = \begin{cases} 1, 0 \leq \xi \leq \xi_s \\ 0, \xi_s \leq \xi \leq 1 \end{cases}$ (39)

The solution of (28) using (39) is

$$\theta_m = \frac{1}{2P_e\sqrt{\pi K_2(\infty)\tau}} exp\left[K_2(\infty)\tau - \frac{\{K_1(\infty)\tau + X\}^2}{K_2(\infty)\tau}\right]$$

(40)

IV. Figures

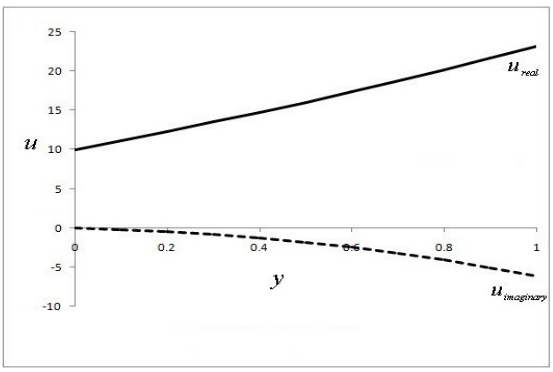

Fig. 1. Velocity Profile

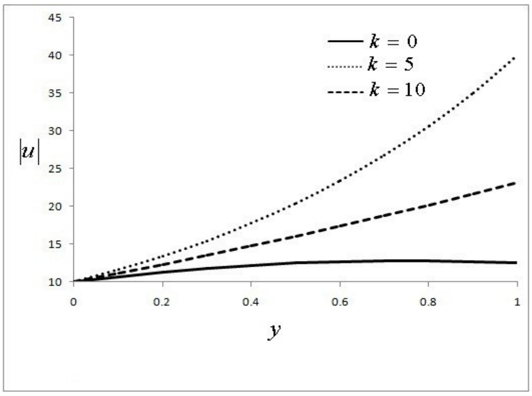

Fig. 2. Velocity profile for different values of drag parameter

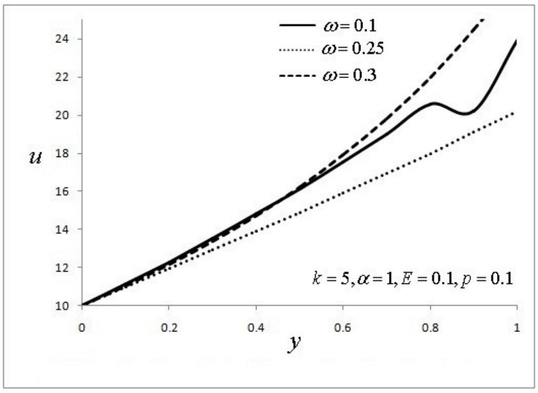

Fig. 3. Velocity profile for different values of angular velocity

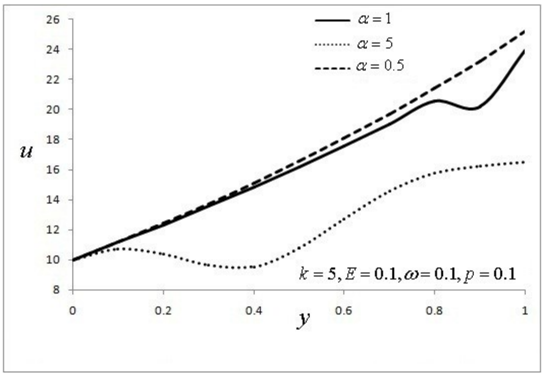

Fig. 4. Velocity profile for different values of hydro magnetic parameter

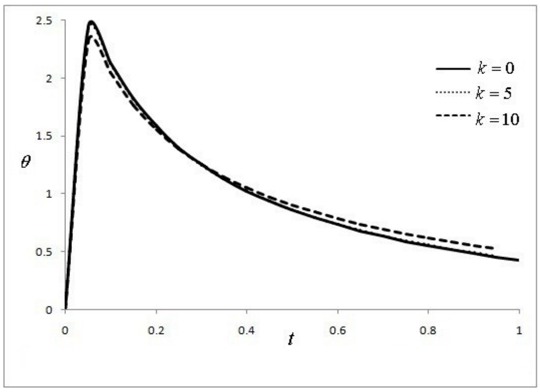

Fig. 5. Concentration profile for different values of drag parameter

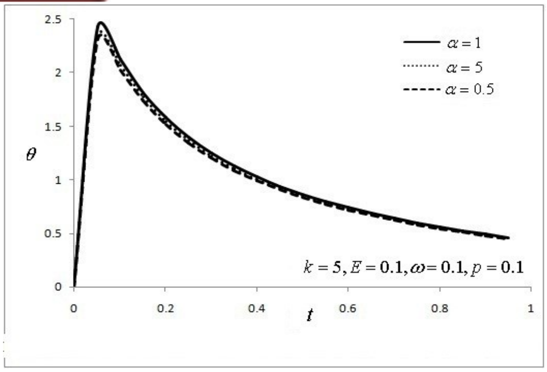

Fig. 6. Concentration profile for different values of hydro magnetic parameter.

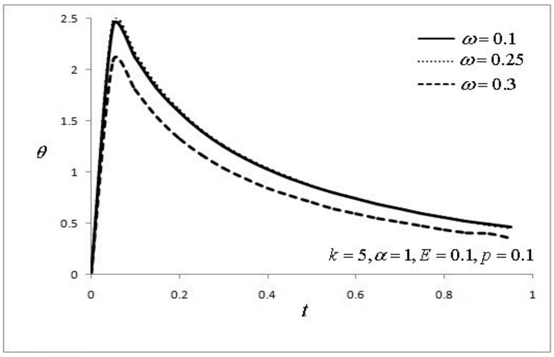

Fig. 7. Concentration profile for different values of angular velocity

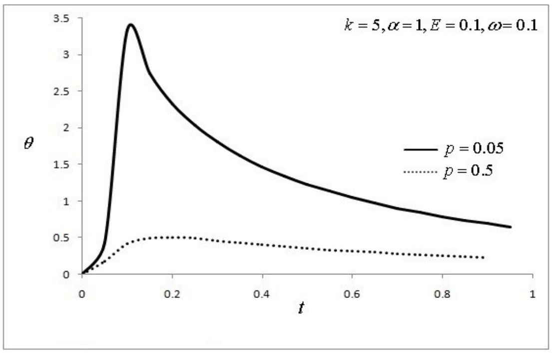

Fig. 8. Concentration profile for different values of Peclet number

V. Results and Discussion

Fig. 1 – 4 represents velocity profile along Y direction plotted for different parameters.

Fig. 1 shows real and imaginary velocity where real part is increasing and dominant imaginary part is initially zero and becomes slightly negative. More contribution is from real part. **Figures 2, 3** and **4** show magnitude of velocity along Y direction.

The fluid consists of suspended nano particles which are magnetically active. The effect of these particles on the flow is measured by drag parameter as k increases from 0 to 5 there is significant increase in velocity but from 5 to 10 again there is a decrease. This is due to the fact that moderate value of drag parameter increases velocity due to the effect of magnetic field but further increase cause resistance to flow.

Figure 3 shows influence of angular velocity on velocity. The velocity shows significant change only near the boundary, where the effect of magnetic field is more. For higher value of angular velocity makes the velocity profile linear.

Figure 4 shows velocity profile for magnetic parameter. For higher value of magnetic field the velocity profile tends to become pulsatile.

Figures 5, 6, 7, 8, shows concentration profile for difficult physical parameters. Concentration versus time at axial position $X = 0.1$ is plotted. The concentration initially increases suddenly then decreases and becomes steady after large time. Effect of drag parameter is to initially show a decrease in concentration as more solute particles are dragged away due to magnetic field but stabilizes at a higher value. This is shown in figure 4.

Effect of hydro dynamic parameter is studied in figure 6. Effect of increase in magnetic field is to decrease concentration slightly due to convection of solute particles.

Effect of angular velocity is studied in figure 7. Higher angular velocity facilitates more convection of solute and hence decreases in concentration.

Effect of Peclet number is seen in figure 8. Concentration is inversely proportional to Peclet number. There is a significant decrease in concentration with increase in Peclet number. This is due to decrease of diffusion which results in dominance in convention and loss of solute.

VI. Conclusion

This paper gives an analysis on effect of rotation on mass transfer. Magnetic field results in a rotational flow about the axis. This effect is analyzed using analytical approach. The effect of magnetic field is to create a rotating motion. The angular velocity enhances convection which affects in dispersing the solute away from targeted area.

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