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Students' Thinking Way: Learning Trajectory of Realistic Mathematics Education

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Abstract – **The research goal is to find the students' thinking ways at each creative thinking process stage at the learning trajectory of RME they undergo. The results show that the learning trajectories of RME they undergo sequentially are problem-posing, the real model, mathematics model, and innovation model. The creative thinking process stages from "problem-posing - real model": orientation & preparation; "real model - mathematics model": incubation & illumination; and "mathematics model - innovation model": verification. The students' thinking ways are: orientation: methodical thinking, redefinition, and exploration; preparation stage: incubation, exploration, methodical thinking, recycling idea, and redefinition; incubation: incubation and serendipity; illumination: synthesis, incubation, serendipity, and lateral thinking; and verification: methodical thinking, redefinition, and lateral thinking.**

Keywords – **Thinking Way, Learning Trajectory, RME, Creative Process**

I. Introduction

Sitorus & Masrayati (2016)have developed a five-stage-creative thinking process that the elementary school students underwent in the realistic mathematics education (RME), namely: orientation, preparation, incubation, illumination dan verification. The most interesting thing to more study of these findings is that the research does neither explore yet how students' ways to thinkofeach of creative thinking process stage nor the learning trajectory of RME.Koestler (1949) has developed a nine-way-thinking, namely: exploration, incubation, lateral thinking, methodical thinking, redefinition, recycling ideas, serendipity, synthesis, and spontaneous, while Cheung (2007) stated that in addition to the nine-way-thinking, he found "thinking out of the box" as an uncommon and unique thinking way done by someone.

In the context of RME, the students themselves construct their thinking ways at each creative thinking process stage in the learning trajectory of RME by the mathematics modeling process, namely: situational, referential, general and formal, (Gravemeijer, 1994, Gravemeijer & Terwel, 2000; Wijaya, 2012).

A. Way of Thinking

Koestler (1949) has developed the bisociation concept as a combination of two different perspectives, namely: convergent and divergent thinking tobe a nine-way-thinking, namely: exploration, incubation, lateral thinking, methodical thinking, redefinition, recycling ideas, serendipity, synthesis, and spontaneous. Cheung (2007) conducted a study of a number of modern American societies to study their thinking ways. The research has found that the modern American society has a ten-way-thinking, namely: exploration, incubation, methodical thinking, recycling idea, redefinition, serendipity, spontaneous, lateral thinking, synthesis, and thinking out of the box.

Exploration refers to someone explores the problem, circumstances, environment, and other aspects to be potentiallydeveloped. The facts, conditions, information, experiments, perspectives, and environment encourage to create the new ideas (Sitorus & Masrayati, 2016, Sawyer, 2008, Sternberg, 2006, Silvia & Beaty, 2012, Simonton, 2010). Exploration is a part of learning the creativity. When someone fully explores the situation, she/heis more prepared to provide the solutions to solve the problems encountered(Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995). This process is usually simultaneous, sustainable sequentiallyfor each other.

Incubation is a process of the brain relaxation from the activity of incubating all information during a certain time. A process occurs in the subconscious mind and finally finds the solution. The subconscious and conscious minds become constant and stable to re-arrange all ideas(Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995, Poincare, 1929). Someone takes a back step to search the information, situation, and problem complexity. According to Wallas, (1926), incubation as a creative process provides the empty space for brain to solve the problem and grant the brain performance based on the problem background(Sternberg, 2006; Sternberg, 2012). Guilford (1967)stated that the brain could potentially work during the incubation period. Transformation, insight, intuition, emotion, mental, environmental, personality, personal experience and motivation have an important role in the incubation process. Information and perspective can be modified and changed. The incubation processis potential to bring out the insight.Intuition is innate someone-self to solve the problem. The sufficient information possessed by creative people can develop anoccurred perspective to facilitate the problem-solving.

Methodical thinking involves the step by step process to produce creative products (Koestler, 1949). The methodical thinking requires the utilization of facts, experiences, and situation analysis. The analysis process is begun with the definition of the problem, experiment, collecting the facts, and understanding the situation, issue environment, and perspective principle. Methodical thinking procedure can be done by differentiating the problem structure and integrating theproblem function. Searching the data fully is potential to generate thecreative ideas and solutions as quick as possible. The methodical thinking process is done by verification, elaboration, consolidation, assimilation, interpretation and clarification the creative ideas. Guilford, (1967)stated that themethodical thinking process is done by 7 steps, namely:

(1) to observe the need or difficulty of information byvisual-figural, symbolic, semantic, and behavioral information;
(2) to formulate the problem by writing, explaining, and revealing the problem. The lack and need of information should be searched and completed in such an away, so it can be solved by creative people;
(3) to survey the available information in the brain;
(4) to formulate a solution by arranging the problem-solving;
(5) torevise the solution critically by examining the information;
(6) to formulate new ideas; and
(7) examiningand acceptance the ideas.

Recycling ideas can be done by analogizing the prior issueto look for the similarities (Indriastuti, 2009). Koestler (1949) stated that the cross-reference, hidden analogy, metaphor, and associative context are as mediums to resolve one of reference frames into the new things(Gilhooly, 1982). The analogy process can be used effectively when the concept is well understood to parallelize the prior issue into the new situation. The well-understood concept can help to identify some problem structures that someone will possibly obtain the information about the problem. An analogy can also be used to identify the new solutions. Recycling the old ideas into a new context, frame, applications, andother disciplines is a creativity source.

Redefinition is a thinking wayby changing the problem into the real word to well understand it. When the problem seems intractable, the creative people can change the resolution parameters.Guilford & Hoepfner, (1971)stated that someone changes the problem nature by utilizing the real objects and adaptingthem into the new things. The redefinition aspect does not change the purpose, however,to redefine the items involved in the problem. The problem sensitivity is an awareness need to change the strategy or methods of problem-solving and awareness of the information lack. Someone can identify something reviewed, the lack and need of information completed, and problem to be solved.If someone is ongoingto solve and sensitive to the problem, possibly find the new policy for solvingit. Likewise, the creative people reimagine the problem by redefining what need to be resolved.

Serendipity refers to someone who gets the lucky factor to identify the new ideas. Serendipity is a thinking way that can be constructed and developed by him/herself by seeing what not thought to be seen, considering each piece of information to potentially expand, and findingthe similarity of ideas or other totally different things(Mednick, 1962). The serendipity thinking involves asetting stage.

Spontaneous thinking is a thinking way by out of the clogged mind to solve the problems with the creative and innovative solutions. Someone uses the opportunity of the mind's ability to spontaneously generate ideas. According to Wallas (1926),every action on spontaneous thinking is the result of an intellectual process when someone first thought of what she/he want to solve,and then calculate the medium used to achieve the end result. The action that the person acted is not deliberate results to search the final ones. The spontaneous thinking does not involve a setting stage.

Lateral thinking refers to a thinking way to obtain the idea feedbacksas much as possibly, even though the crazy ideas, and select and rank of the ideas to earn the best solution. To search the best solution, someone explores the thinking power both logically and imaginatively. The lateral thinking process includes:

(1) selecting and defining the focus. At this stage, problems, challenges, and opportunities are identified. The problem focus is redefined carefully and thoroughly;
(2) generatingthe ideas. There are several tools togeneratethe ideas, namely: concept extraction, challenge, random entry, as well as provocation and movement;
(3) abstractingthe ideas. At this stage, someone will search the connection among ideas and choose some best and unique ideasto identify the concept. Those ideas are later ranked by using several criteria to be well implemented; and
(4) selectingthe best ideas. Ofthe treatment and assessment, someone will get truly remarkable ideas to be applied.

Synthesis is an integration two or more objects or concepts to generate the new and innovative ideas (Giangreco, Cloninger, Dennis, & Edelman, 1994). The object and concept are other aspects of synthesis. Synthesizing the objects means to combine the simple objects into new and conceptual objects. Synthesizing theconcept means to integratetwo or more ideasinto one new idea. Koestler(1949) stated thatthe synthesis process is done by different fertilization and associative contexts. Koestler describes that someone needs to explore the components of ideas, context, reference term, or concept to find the solution before synthesizingthem. Instead, Guilford and Hoepfner explore various types of synthesis resultant. In other words, Koestler does not directly consider various synthesis used. According to Wallas, (1926), during the thinking process, the brain continuouslycalls other relevant and related ideas each other. The interesting aspect of Wallas is that the synthesis poweris in the connecting or comparing ideas created by him/herself. A lot of knowledge has been generated through a proper comparison. Wallas noted that the idea is obtained, compared, and concluded from 2 different things in the same condition. Synthesis as a creative process uses an association to combinethe different concepts into new one.

Thinking out of the box refers to an unusualthinking technique, unorthodox thinking methods, and different completion approach(Robinson, 2009, De Barros, Primi, Miguel, Almeida, & Oliveira, 2010). Someone thinks to solve the problem by brainstorming rationally, accept and no restrict him/herself from any ideasin the brain including the crazy ideas, keep all ideas including theideasever rejected becausethe rejected ideas possibly become intoone great dan big idea to solve the problem.

B. Creative Thinking Process Stage

Sitorus &Masrayati (2016)have developed a five-stage-creative thinking process, namely:orientation,preparation, incubation, illumination dan verification.

At the orientation stage, the students read, understand, and search information from the contextual problems. Osborn (1953)stated that people orient and recognize the problem. Osborn stated that someone will not be able to solve the problem when the issue is not understood. At the preparation stage, students collect the data and information, representand manipulatethe contextual problems into mathematics objects, and formulate the problem-solving model and strategies. According to Hill, Johson, Pryor, & Rahim(2011), someone gets information from the students' internal and external factors . Students have some prior knowledge obtained fromthe previous learning as their internal factors, and spontaneous experiences as their external factors.At the incubation stage, students undertake two thinking phenomena, namely: self-appeasement or reflection and deadlock thinking. Inspiration comessuddenly after the incubation period. The students read and try to understand the preparation process again, recall the prior knowledge and learning experiences, imagine the connection of each mathematics object, and bring outthe rudimentary mathematicsideas. In the context of mathematics learning, the students get new thinking energy as advantages from the incubation period to solve the mathematics problem after the brain takesarest to think by diverting the problem to the other more relaxed thinkingactivities(Sternberg, 2006; Sternberg, 2012).At the illumination stage, students analyze the pieces of mathematics ideas and synthesize them, find the main mathematics ideas, connect the mathematics ideas to other ones, and solve the problems. Someone's experience fromthe preparation until incubation process is accumulated into a set of knowledge that she/he usesin the illumination stage to generate the new method or strategy to solve the problem.At the verification stage, students verify the mathematicsideas, revisethe errormathematicsideas, and find the creative and innovative mathematics solutions. In the learning context, students need to communicate their creative and innovative mathematics solutions to the others, such as teachers or colleagues who have higher knowledge or competence than the students themselvesto consider the novelty and advantage as their responsibilities to what they have got(Christensen, 2005).

C. *Learning Trajectory of Realistic Mathematics Education*

The students construct the learning trajectory of RME by modeling process to find the creative mathematics solutions. Blum stated that the students begin the modeling process from the real-world (Maaß, 2006; Gravenmeijer, 1994; Zulkardi, 2010).The students simplify, structure, and idealize the contextual problem as their thinking process towards the real model to go in the direction of the mathematics model then.In this mathematics model, students have worked on the mathematical framework to obtain the mathematics solution. Furthermore, this solution is first interpreted and validated then. If it is error evidently, the students apply the modeling process repeatedly.

According to Galbraith &Stillman(Stillman, 2015; Ee & Widjaja, 2013), the students' learning trajectories when they have gotthe issues or problems are:

(1) understand and structure the issues or problems
(2) simplifyand interpret the context;
(3) assume and formulate the problem, and perform the mathematical process. At this trajectory, someone has to work mathematically and derive the mathematical output;
(4) verify the results by comparing, critiquing, validating, communicating(Rahayu, 2015), justify and report in writing; and
(5) revise the error results based on the verification results obtained.

Voskoglou, (2012) stated that the students'mathematics learning trajectories are:

(1) analyze and understand the problem and search the relevant additional information;
(2) construct the mathematics model and do the mathematics process from the real situation to the mathematics model;
(3) find a suitable mathematics modelby manipulating the mathematics objects;
(4) validate and introduce the model; and
(5) understand the mathematics results and its implementation in the real system to provide the mathematics solution.

According to Gravemaijer, the learning trajectories of RME that the students construct by mathematics modeling process are situational level, referential level, general level, and formal level (Gravemeijer, 1994; Gravemeijer & Terwel, 2000; Wijaya, 2012). The situational level is the most basic level of mathematicsmodeling as a starting point ofthe learning trajectory that the students traverse. The knowledge and modeling process at this level is still in the context of the problem situation they use. The students who are in this level still try to understand and identify the problem andsearch some whatever information to know from the problem. The students represent the problem intothe real-world.At the referential level, the models and strategies that the students develop are not in the real situation but refer to the context. Students create the models to describe the real situation called "model of". The students representthe problem into problem-solving models. At this level, the students possibly design manydifferent problem-solving strategies and models for each other. The students representthe real situation into "model of". Atthe general level, the modelsthat the students already develop have set in the solution mathematically called "model for". At this level, students focus on math, no the real situation again. The students use the concepts of subtraction, addition, division, and so on as the basis for solving the mathematics problems. At the formal level, students already usethe symbols and mathematical representation. The formal level is the formulation and confirmation levels of the mathematics concepts that the students have developed. Students begin to develop the algorithms or procedures. The teacher's role is very crucial in concluding the mathematics concept fromthe students' mathematics activities.

Based on the explanation above,the students have thinking waysat each creative thinking process stage at the learning trajectory of RMEthat the students undergo. The research questions are:

Q1: How is the learning trajectory of RME the students undergo to find the innovative mathematics solutions?

Q2: How is the students' thinking way at each creative thinking process stage at the learning trajectory of RME the students undergo?

For addressing the two questions above, the research goal is to findthe students' thinking ways at each creative thinking process stage at the learning trajectory of RME that they undergo.

II. Research Method

This research is qualitative. Operationally, the research was conducted by several steps, namely: research initiation; data collection; and data analysis (Sitorus & Masrayati, 2016).

In the early research step (research initiation), the researcher did some preparations(Sitorus& Masrayati, 2016), namely: design the instructional tools, preparethe research instruments, and then implement RME. The researcher designed the instructional tools based on the principles and characteristics of RME integratedwith the creativity in education, such as lesson plan, guidance handbook for teacher and students; and students' activity sheet.The research instruments are mathematics thinking test, and interview and observation guides. The test is a set of open-ended questions based on the mathematics material"two-dimensional figure" in class VI ofelementary school. The test is used to measure the students' creative thinking level. Three of all students who took a test are keyinformants in this research. The three students as key informantsare classified into "very creative", "creative", and "quite creative", 1 student from each other.

The creative thinking indicators are:

(1) "very creative", students are able to solve a problem with one or more alternative answers or different andnovel solving-problemways;
(2) "creative", students are able to solve a problem with one or more alternative answers or different andflexible solving-problemways;
(3) "quite creative", students are able to solve a problem with one or more alternative answers or different andfluent solving-problemways, and
(4) "uncreative", students are not able to solve the problem.

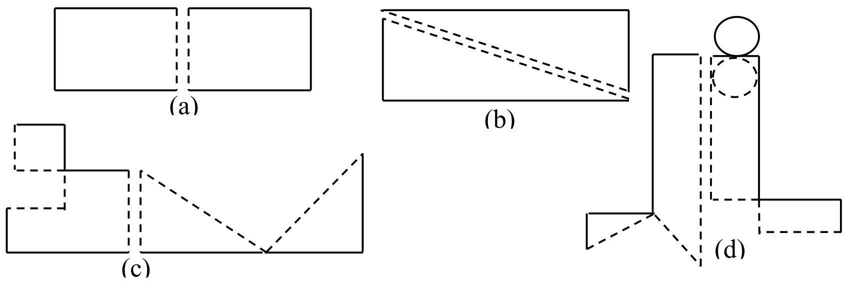
The research was conducted in June 2016 at HikmatulFadillahElementary School in Medan, North Sumatra Province-Indonesia.

The data was collected in three ways, namely: test, observation, and in-depth interview. The researcherinterviewed the three students as key informants. The researcher used the three ways to collect data to complete them each other by triangulationprocess.Data was analyzed qualitatively in 3 steps, namely: data reduction, data presentation, and conclusion or verification.

III. Results

The RME implementation was conducted in several steps: problem-posing, discussing, representing, and concluding. The RME implementation process isrepresented in the following matrix Table 1.

Table1.RME Implementation Processon the Mathematics Material "Two-Dimensional Figure"

Teacher's activities	Students' activities
Step 1. Contextual problem-posing	
Demonstrate a piece of rectangular paper to the students and ask them to divide it into two same area pieces	Divide it into two same area pieceswith the same or different two-dimensional figureforms by using a piece of scissors or cutter as an instructional tool. The followings are variety of answersgrouped into 4 various ones 
Step 2. Discussing	
Ask the students to discuss their mathematics solutions together in team and write them on the student's activity sheet	Discuss their various answers and exchange informationforeach other. Some students debate and defend them for each other Write the answers on thestudent's activity sheet
Step 3. Presenting	

Ask the students individually or vicariously from team to present their mathematics solution on the board in front of the class	Write their mathematics solutions on board, and others look Correct the error answersonbeingdirected by teacher
Step 4. Concluding	
Direct the students to make the conclusion derived from the facts/reality, concept until procedure as students' answers strengthening	Solvethe problem by manipulating the mathematics objects by usingthe paper asan instructionalmediato form the rectangles, circles, triangles, and other shapes to more understand the concept oftwo-dimensional figure

Based on the answer variety above, the student who has "answers (d)" as a mathematics solution is called "very creative" because he is ableto combine various forms of thetwo-dimensional figure, such as circle, triangular, trapezoidal, and rectangular, presented in a unique and interesting form, hereinafter refers to students (S1). The student who has "answers (c)" as a mathematics solution is called "creative" because she is ableto combine two different forms of two-dimensional figure, such as triangles and rectangles, hereinafter refers to students (S2). The student who has "answers (b)" as mathematics solution is called "quite creative" because she is ableto presenttwo same forms of two-dimensional figure, namely triangle, hereinafter refers to students (S3), and it is different with "answers (a)" which only present two same area and form of rectangular and the form is same with the original paper.

The three students as key informants were interviewed in-depth, and the interview material is about their thinking ways at each creative thinking process stage at the learning trajectory of RME that they underwent.

The students' learning trajectory began from "problem-posing" designed with the imaginable context for them or contextual problem designed in such a way close to the students'daily lives. Based on interview results, the three students did some thinking activities, namely: try to recognize and understand what information known and asked from the contextual problem, what requirements to solve it, and restate the original problem more operationally. Based on observation results, student (S1) just read the problem, while students (S2) and students (S3) more read references, asked their friends and teacher. The two students asked what additional information to know and search. The three students have not done the writing activities yet, and seem in a deep thought about something to do. All process they have done is still going on their heads without writing anything. Then, the students prepared to collect the data and information, planned some problem-solving strategies, and preparethe problem-solving planning (interview and observation results), namely:(1) wrote all information known and asked from the contextual problem and the requirements to problem-solving, simplified the contextual problem into the real situation, and manipulated the problem into the mathematics objects. Based on observation results, the three students used the paper as an instructional media to draw various forms of the two-dimensional figure. Furthermore, they stated that they manipulated the problem to more understand it; and (2) tried to search or remember the same problem they have ever solved which has similarities with the problem to be being solved. According to them, their teacher has ever giventhe similar problems to the problems they weresolving.From the previous problem, they analogized the known and asked information as well as copiedthe strategies or methods to solve the problems they were doing. Based on interview results, the three students experienced the incubation phenomenon when they prepared the data and information. Student(S1) and Student (S2) experienced the thinking fatigue. Based on observation results, they distractedout of the classroom to relax the brain for a few minutes (interview results). Students (S3) experienced the thinking deadlock. He said that he could not analogize the previous problem completion with the problem completion he was doing, especially to construct various forms of the two-dimensional figure. This is because he has lower creative thinking level than both of his friends.

The next students' learning trajectory is to use the mathematics concepts and procedures to generate the creative mathematics ideas. At this trajectory, the students experienced the incubation phenomenon as atrajectory transition they have traversed previously, but the starting point and required time of the incubation process are different. Based on interview results, Student (S1) and Student (S2) experienced more early and ahead for incubation process than Student (S3). Student (S1) and Student (S2) had high emotion to solve the problem immediately as soon as possible, which triggered and stimulated their brains to work more quickly. They just needed to relax their brains from the burden some thinking for the unpredictable time till they got inspiration or insight to solve the problem. Student (S3) needed the scaffolding from his teacher as soon as possible to solve the problem. Based on interview results, the three students solve the problem based on the strategy and method they had planned in the preparation stage. Based on the answer sheets, the students performed a series of completion process with the different mathematics representations and used the symbols and language of formal mathematics process. Students (S1) combined 4 different forms of the two-dimensional figure, such as circle, triangular, trapezoidal, and rectangular, and drewthemin a unique and attractive form. Students (S2) combined two different forms of the two-dimensional figure, such as: triangle and rectangle. Student (S3) only drew two trianglesbut has modified the rectangle paper into two same area triangles. Based on interview results, Student (S3) found the difficulty to modify the original paper form variously and uniquely.

The last students' learning trajectory is to verify the students' answers towards the innovative solutions. Based on interview results, the three students verified

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theprocedure correctness ofthe problem completion as well as the answer novelty value. The three students' ways to verify their mathematics solutions are to do the metacognition by asking them selves for some questions, for example: "is the answer I have made novel? is the two-dimensional figure I have drawn various?". Based on the answer sheets, Student(S1) and Student (S2) have found the novel mathematics solutions, but no for Student (S3) yet. Because of the innovation concept does not only refer to the novelty of the way, method, strategy, and product the researcher asked the students the advantage and connection of their innovative mathematics solutions they have generated to the other disciplines. Student (S1) and Student (S2) stated that at least, they have got training and had the knowledge to design something various, unique and attractive. Further more, they also said that this lesson is similar and integrated with the architect's and designer's works.

Based on the explanation results above, the researcher conclude that the learning trajectories of RME the students traversedsequentiallybeginfrom the problem-posing, simplifying the problem to the real model, using the mathematics concept and procedure towards a mathematics model, and verifying the mathematics solutions as well as connecting them to other disciplines towards an innovation model. The students do the thinking activities at the learning trajectory of RME they traversewith some creative thinking process stages, namely (1) the creative thinking process stages from the problem-posing toward the real model are: orientation and preparation. At the preparation stage, the students experienced the incubation phenomenon. This incubation process occurs due to two factors, namely: thinking fatigue, experienced by students who have creative thinking level "very creative" and "creative" as well as thinking deadlock, experienced by students who have creative thinking level "quite creative"; (2) the creative thinking process stages from the real model toward the mathematics model are incubation and illumination. The incubation process occurs as a trajectory transition they have traversed. The students who have creative thinking level "very creative and" creative" are more early and ahead for incubation processthan the students who have creative thinking level "quite creative"; and (3) the creative thinking process stage from the mathematics model toward the innovation model are verification.

Thestudents' thinking ways at each creative thinking process stage are: (1) orientation stage:methodical thinking, redefinition, and exploration; (2) preparation stage: incubation, exploration, methodical thinking, recycling idea, and redefinition; (3) incubation stage: incubation and serendipity; (4) illumination stage: synthesis, incubation, serendipity, and lateral thinking; and (5) verification: methodical thinking, redefinition, and lateral thinking.

For more understanding, the following can befigured the elementary school students' thinking way at each creative thinking process stage at the learning trajectory of RME they traverse as showninFigure 1

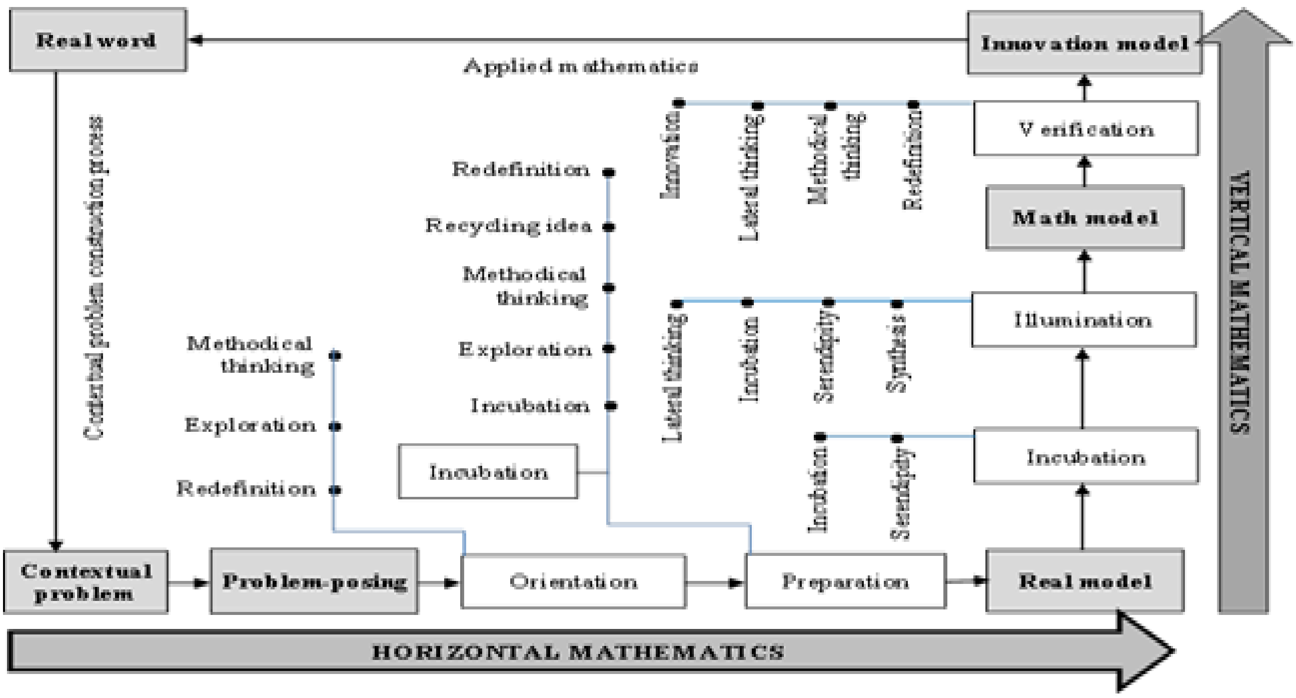

Figure 1. Students' Thinking Way in Each Creative Thinking Process Stage at Learning Trajectory of RME

Note:
Learning trajectory
Creative thinking process stage
Thinking way

IV. Discussion

The implementation of RME in this research isin line with the opinion of Freudenthal that: (1) RME begins from the contextual problem-posing related to the students' daily lives; (2) using the problem-solving model constructed by the students themselves with the teacher's guidance; (3) generating the various ways, strategies, and solutions as thestudents' contributions; (4) maximizing the interaction process between the student-student, student-teacher and student-learning resources; and (5) connecting the mathematics material to the other mathematics topics (intertwinement)(Gravemeijer, 1994).The teachermust construct an interesting learning process to make the students to think creatively, more emphasize and provide the opportunitiesthe students to search and try their abilities and independence to construct their own knowledge by themselves. The teacher implements a learning system that activates the discussion in the classroom(Gardner, 2004; Smith, 2008). The students discuss dan debate actively. The communicative dialogue between students and teacher takes place byasking, probing, searching, and disputing something to arouse their curiosities. N.A.C.C.C.E, (1999) stated that both of teaching creatively and teaching for creativity 'usetheimaginative teaching approach to design the more interesting and effective learning process. The teaching approach is mentioned to develop the students' thought and behavior. Both of these terms have a close connection to affirmthat teaching for creativity involves teaching creatively. The students'creativity ability is just most likely developed by the creativeteachers.

The contextual problem-posing as the first step in this research is to develop the students' creative thinking. This is in line with the opinion of Stiff, Curcio & Frances, (1999) stated that the problem-posing is a medium to constructthe students' creative thinking. Leung et al. (2009)stated that the problem-solving and problem-posing can improve the creativity abilitywith the indicators: fluency, flexibility, and novelty. Leung explained that the creativity and problem-posing havethe same characterin its multiplicity. Sitorus & Masrayati, (2016) stated that "fluency"refers to the students' ability to solvethe problem by providing the variousand true responses; "flexibility"refers to the students' ability to solvethe problem by providing the variousand flexible responses; and "novelty"refers to the students' ability to solvethe problem by providing the variousand novel or unique responses.

The students' mathematics answers obtained by thecreative mathematics thinking test are very various, including the patterns, ways, as well as strategies of the problem-solving. Saefudin, (2012), Ruseffendi, (2006), Sitorus & Masrayati, (2016), Usdiyana, Purniati, Yulianti, & Harningsih, (2013) found that the implementation of RME can develop the creative thinking abilitybecauseRME has principles and characteristics they applied in their researches. "Reinvention" as one of RME principlesenables the students themselves to construct the mathematics concept. "Self-modeling as one of RMEcharacteristics enables the students themselves to develop their creative thinking abilities. Creativity is not strict on the existingpatterns and ways but skilled at finding the new routes.

The problem-posing as the starting point of the learning trajectory on this research finding is in line with the opinions of Hadi & Fauzan, (2003) stated that the learning trajectory sequentiallystarts from the real problem, informal solution from the real problems (model, illustration, sketch, pattern), and formal solution by using the formula, concept, and algorithms mathematically. The mathematics activities to solve the problem are more given inthe early learning, but it can also be added in the middle or at the end of the lesson.

The students' creative thinking ways at the first learning trajectory as these research findings are in line with the opinions of Sitorus & Masrayati, (2016)stated that the creativethinking process stage of RME for the elementary school students begins with the problem-orienting. The students read and try to understand the problem (Mace & Ward, 2002); wish to get mathematics ideas; searchthe whatever information asked and known from the problem; and search the pieces of information from the contextual problem (eg: the size and formula to calculate the geometry volume).All the problems are transformed into possible positions to solve the mathematics problems and can be seen alternately and complexly. All process is still going on the mind without writing them. To solve the problems, the students dothe preparation process as the second stage, namely: (1) collect all the mathematics material, including the books and students' answer sheet; (2) represent/manipulate the contextual problem into the mathematics objects (situational level) to more understand; and (3) formulate the model/strategy used to solve the contextual problem. The students communicate the received information with their own languages, for example: write whatever informationwas known and asked from the contextual problem. Russefendy stated that this strategy is used to manage, analyze, and synthesize the data or other ones encountered in the problem.

Osborn as Juntune'sexplanationstated thatsomeone does an effort to solve the problem continuously with some steps, namely: problem-identifying, fact-finding, problem-finding, idea-finding, solution-finding, and finding-accepting. Problem-identifying is afirst stepto solving the problem creatively. The students perceivethe problem, and without perceiving the problem, no solution can be provided; "fact-finding" refers to the students enroll all the facts, questions, data and information to solve the problem; and organize the facts; "problem-finding"refers to the students clarify the problem by focusing the problem very reallyto be solved; "idea-finding" refers to the students find the strategies/ideas to solve the problems; "solution-finding" refers to the students selectthe appropriatestrategies/ideas to solve the problem; and "finding-accepting"refers to the students plan the required action to implement the such solutions.

Riedesel et al. (1996)stated that mathematics is problem-posing and problem-solving. The students basically experience 2 mathematics activities, namely: what problems should be maybe submitted from a number of facts and how to solve the problem. The students get the opportunity to develop their ability to identify the factsandissues from the problem-posing action. Meanwhile, they are able to develop their ability to solve the non-routine problems from the problem-solving action.

According to Stiff et al., (1999), the problem-posing is a students' analogy reasoning to make a modeling processfor the existing problemsbased on the previous problemssuch as the research findings in this study, especially at the preparation stage. Another finding is that the students experience the incubation phenomenon at the preparation stage because of the lack of their knowledge and experience. This means that the incubation phenomenon is not only a stage of the creative thinking process but can also occur in the stage of the creative thinking process. According to Petty that someone can undergo the random and repeated stages of the creative thinking processfor a moment.In his/her research, Petty stated that the incubation phenomenon precisely occurred at the end of the creative thinking process stage.

The problem-solving process from the real model towards the mathematics model as students' learning trajectories after the problem-posing in this research isin line with the opinion of Zulkardi (2010). He formulates the mathematics process into two types namely: horizontal and vertical mathematics. The students use the concepts, principles, algorithms of mathematics to help them to organize and solve the problems as a horizontal mathematics process. They connect some concepts and strategies and try to apply them in solving the problem as a vertical mathematics process. The horizontal mathematics is a process from the real world tothe symbol world, while the vertical mathematicsis a transition process from the symbol itself. According to Freudenthal, both of the mathematics processes cannot stand-alone, but an equally important unity in the mathematics learning process (Zulkardi, 2010; Zulkardi, 2000). At thistrajectory, the students experienced the incubation phenomenon as a trajectory transition from the "problem-posing – real model"towards the "real model – mathematics model"as a research finding in this study. This means that the incubation phenomenon does not only occur once. This is in line with the opinion of Rossman (1931) stated that if a problem is very complex, the incubation period is longer and repeated continuously. The incubation process duration for someone to generate the creative ideas is various for each other, and also for the same individual for in other time. The difference is caused by a number of factors, including to someone's situation to stimulate the creative ideas; the problem complexity level to provide the solutions; someone's emotional reaction intensity and way of life, which all factors give the effect to the incubation period. Segal (2004) stated that the incubation functionates to improve the brain performance after relaxing it for a moment or unlimited time. Sitorus & Masrayati, (2016) stated that the students do 3 thinking activities at the illumination stage, namely: analyzethe pieces of mathematics ideas to find the main mathematics ideas; connect the mathematics ideas to the other disciplines, and solve the contextual problems.

The last students' learning trajectory from the "mathematics model" towards "innovation model" as these research findings are in line with the opinion of Riedesel et al. (1996) stated that mathematics does not only comprisethe mathematics substance but has contributed and evolvedin various aspects of human life dynamically. According to Kasper, (2008), "innovation"comprises 5 main steps of the innovative thinking, namely: set the desired conditions to support the innovation; identify the problems or opportunities to innovate; generate the ideas to solve the problems or seize the opportunities; experiment and guidethe ideas to examine how well the ideas in practice; and share the innovations with others. At this trajectory, the students communicate theirmathematics results with other disciplines and connection forthe daily life as these research findings which are in line with the opinion of Coxford that the mathematics connection capabilities comprises (Berlin & White, 1999): connect the conceptual and procedural knowledge by using the mathematics on other topics;implement themathematics for the life activity; consider the mathematics as an integrated unity;implementthe mathematics thinking ability and create a model to solve the problem in other subjects such as: music, art, psychology, science, and business; find a connection of the other mathematics topics; and recognize the various representations for the same concept. NCTM (2000) stated that the students who are able to connect the mathematics ideas to other discipline understand the mathematics well.Cuoco et al. (1996)stated that the mathematics beauty sites in the mathematics connectionitself to bring out the new techniques to solve the problems. At thistrajectory, the students did a metacognition at the verification stage as this research finding which is in line with the opinion of Schoenfeld, (1992) stated that the three-way metacognition in mathematics learning, namely: belief and intuition, having knowledge about the thinking process, and self-awareness. Belief and intuition concern to what mathematics ideas to prepare to solve the problems and how these ideas to solve the problem.Having knowledge about the thinking processconcerns to how accuratesomeone to prepare his/her thinking process. Self-awareness concernsto the accuracy to maintain and organize what to do to solve the problem, and use the observation inputto direct the activities to solve the problem. According to Sitorus & Masrayati, (2016)stated that the students verify the mathematics solutions by (1) verifying the creative mathematics ideas and (2) finding the innovative mathematics solutions. The thinking activities at the verification stage are done bydiscussingtheir creative mathematics ideas with the teacher or peers for the correction process;exploring the data and information for the evaluation process of the mathematics ideas;rechecking the creative mathematics ideas,and revising the error mathematics ideas.

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