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Adomian-Like Decomposition Method in Solving Navier-Stokes Equations

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Abstract **— The aim of this paper is to construct a scheme for the accurate simulation of the solutions of the non stationary Navier-stokes equations. This is achieved through the so called Adomian-Like Decomposition Method that we developed.**

The nonlinear Problem is decomposed into an infinite set of linearized Navier-Stokes equations for which the existence and numerical schemes are constructed in part I In Part I a product formula intended to simulate numerically the solution of linearized problem.

In the second part the idea of Adomian decomposition Method is used to define the splitting of thenonlinear problem into an infinite set of linearized problems.

The convergence proof very technical shall be presented in a fortcoming paper.

Keywords **— Navier-Stokes, Equations-Product, Formula-Linearized, Equations-Adomian-Like, Decomposition Method, Convergence.**

I. INTRODUCTION

We consider the movement of an incomprehensible viscous fluid in a bounded smooth domain Q which is governed by the Navier-Stokes equations:

$$\begin{cases} (1.1)\ \dfrac{\partial u}{\partial t} - v\nabla u + u\nabla u + \nabla p = f in\Omega \times (0,T) \\ (1.2) \operatorname{div} u = 0 \quad in\Omega_T \\ (1.3) u|_{\partial\Omega} = 0 \quad \text{for all } t \in (0,T) \\ (1.4) \quad u(0,x) = u_0(x), x \in \Omega \end{cases}$$

Where $f(x,t)$ is the density of the external forces, v is the kinematic density of the fluid. $P = p(t,x)$is the pressure $u=(u_1,u_2,u_3)$ is the velocity field.

Due to the lack of the analytical solutions of the Navier stokes equations and even the global existence theorem forthe strong solutions of the Navier-stokes equations and event the global existence theorem for the strong solutions of the (*NSE*) is not known for arbitrary f, v and u_0...., the development of the efficient numerical schemes and algorithms intended to approximate the solutions of these equations are of the great importance.

A number of numerical algorithms have been proposed algorithm of Chorin [13] is given without any proof of convergence.

In Giovani [18], a convergent product formula was proposed but without any explicit rate of convergence. But even here the product formula is very complicated and therefore for less use in computational point of view.

In Rautman [17], a product formula for the following linearized Navier-Stokes

Have been proposed:

$$\begin{cases} (1.1)\ \dfrac{\partial u}{\partial t} - v\nabla u + v * \nabla u + \nabla p = f \quad in\Omega \times (0,T) \\ (1.2) \operatorname{div} u = 0 \quad in\Omega \times (0,T) \\ (1.3) u|_{\partial\Omega} = 0 \quad \text{for all } t \in (0,T) \\ (1.4) \quad u(0,x) = u_0(x), x \in \Omega \end{cases}$$

Where f is the density of the external forces, v is a constant and describe the viscosity of the fluid

$P = p(t,x)$ is the pressure

$u= (u_1, u_2, u_3)$ is the velocity of particle.

Due to the lack of analytical solutions of Navior-Stokes equations and even the problem of the solution in the large (globe solution).

Numerical schemes and algorithms intended to approximate solutions of these equations are of great importance both for theorical and computational point of view.

In Marsden [11], a product formula intended to formulizea numerical of A. Chorin was given without proof of convergence. [14] a convergent product formula was proposed but without any explicit rate of convergence.

But even here the product formula is very complicated and therefore for less used in computational point of view.

In Rautmann [16] a product formula for the following linearized equations was given:

$$\begin{cases} (1.5)\ \dfrac{\partial u}{\partial t} - v\nabla u + b(x)\cdot\nabla u + \nabla p = 0 \quad in(0,T) \times \Omega \\ (1.6) \nabla \cdot u = 0 \quad in(0,T) \times \Omega \\ (1.7) u|_{\Gamma} = 0 \quad \text{for } t \in (0,T) \\ (1.8) \quad u(0,x) = u_0(x), x \in \Omega \\ b(x) \textit{ is independant of time} \end{cases}$$

It is said that the product formula converges but no proof is given there.

In all those works the product formula is of the form:

$$u_n(.,) = \left(E_{t|n} o H_{t|n}\right)^n (u_0)$$

Or

$$u_n(.,) = \left(H_{t|n} o \Phi_{t|n} E_{t|n}\right)^n (u_0)$$

where E_t is an approximation procedure for the solution of the Euler equations, $\Phi_t$is some extension operator H_t the exact operator solution of the Stokes problem with the same initial condition.

Our idea although very close to the works cited above are of interest since:

i) the convergence proof is given
ii) it is based on linearized equations easy to tackle.

We consider the following Navier-Stokes equations in a regular bounded cylinder.

$$Q_T = (0,T) \times \Omega \text{ of } \mathbb{R}^4$$

$$\begin{cases} (1.9)\ \frac{\partial u}{\partial t} - \nu \nabla u + b(t,x)\cdot \nabla u + \nabla p = 0 \quad \text{in} Q_T \\ (1.10) \nabla \cdot u = 0 \quad \text{in Q} \\ (1.11) u|_{\partial\Omega} = 0 \quad \text{for } t > 0 \\ (1.12) \quad u(0,x) = u_0(x), x \in \Omega \end{cases}$$

We study the existence and regularity properties of (P_T). The corresponding Adomian decomposition factors to solve are in this form.

The theorems established in this form enable the proof of the well poseness of the method developed in the second part of this work. Under some suitable assumptions on b (t,x).

We state of corresponding compatibility conditions necessary and sufficient for higher regular solutions.

A product formula for approximating solutions of (P_T) Is then proposed.

The proof of the convergence is strongly based on lemma 4.3

In the second part of this work we apply this product formula to study an approximation of the non linear Navier - Stokes equations.

II. Preliminaries

2.1 Functional spaces and notations

In all what follows Ω is a bounded domain of IR^2 that for simplicity we suppose simply connected and of class C^2.

$L^2(\Omega)$ denotes the space of measurable square integrable functions on Ω equipped with the scalar product:

$$(u,v)_{L^2} = \int_\Omega u\bar{v}dx$$

and the norm $\|u\| = [(u,u)_{L^2}]^{1/2}$

$$H^{m,2}(\Omega) = \{u \in L^2(\Omega), D^a u \in L^2(\Omega), |\alpha| < m\}$$

$$\alpha = (\alpha_1, \alpha_2, \alpha_3) \text{ and} |\alpha| = \alpha_1, +\alpha_2 + \alpha_3$$

With the scalar product

$$(u,v)_{H^M} = \sum_{|\alpha|<m} (D^a u, D^a v)_{L_2}$$

Let $\mho = \{v \in C_0^\infty(\Omega), \nabla . v = 0\}$ be the space of regular solenoidal functions with compact support in Ω.

Let H and V denote respectively the adherence of $\mho$ in $L^2(\Omega)$, $H^1(\Omega)$ respectively.

Let us recall the following result, see for example [46]

$L^2(\Omega) = H \oplus H^\perp$ where

$$H = \{u \in L^2(Q), \nabla . u = 0, \Upsilon_n u = u.n|\Gamma = 0\}$$

$$H^\perp(\Omega) = \{\nabla\varphi, \varphi, \in H^1_{loc}(\Omega)\}$$

$$\Upsilon_n \in L(E(\Omega); H^{-1/2}(\Gamma)), \Gamma = \partial\Omega$$

$$E(\Omega) = \{u \,\hat{}\, u \in L^2(\Omega), V.u \in L^2(\Omega) \text{in the distribution sense} y\}$$

These properties holds for domain at least Lipshitz continuous P denotes the orthogonal projectional of $L^2(\Omega)$ into H which is continuous with $\|P\|_{L^2(\Omega)H} < 1$, and maps, $H^m(\Omega)$ into $H^m(\Omega) \cap \boldsymbol{H}$

Definition 2.1

The closure of the operator $-\mathbf{P}\Delta$ in $L^2(\Omega)$ is denoted A and is the Stokes operator .

The following classical properties of the Stokes operator can be found in [14] :

i) A is a positive definite self-adjoint operator , densely defined with: $D_A = H^2 \cap V$
ii) $(Au,v)_{L^2} = (\nabla u, \nabla v)_{L^2}$, $u \in D_A, v \in V$
iii) A has a complete continuous inverse.
iv) $\|Vu\|^2 = \|A^{\frac{1}{2}} u\|$, $u \in V$

2.2 Abstract formulation

Applying the orthogonal projection to the equation (1.9) and assuming that

$u_0\ g$ H

$$(P'T)\begin{cases}(2.3)\ \frac{du}{dt} + \nu Au + pb\nabla u = pf, t \in]0,T] \\ (2.4)\ u(0) = u_0\end{cases}$$

Where u(t) and pf(t) are seen as function from [0. T] into H

Let $A_1(t) = \nu A + pb(t)\nabla$

With domain D_A

In view of establishing existence theorem for the problem (P'_T) some assumption are made on the data b:

Assumption 2.3

$$1 - b(t) \in L^\infty \cap V \text{ for some } p \geq 3 \text{ and } 0 \leq t \leq T, a.e$$

Remark 2.1

It is well known by Sobolev imbedding theorem

$H^1(\Omega) \to L^6(\Omega)$ continuously and also that

$L^6(\Omega) \,\hat{}\!\to L^3(\Omega)$ since Ω is bounded.

It is also to note that in bounded domain assumption 1) implies assumption 2).

Lemma 2.1

If b(t) satisfies the assumption 2.3 , then there exist constants c_1, c_2, $c_3$independantof t. Such that:

a) $\|Au\|_0 \leq 2\|A_1(t)u\| + c_1\|u\|$, $u \in D_A$
b) $\|u\|_2 \leq c_2\|A1(t)u\| + c_3\|u\|$, $u \in D_A$

Proof:

a) Let $u \in D_A$, *we have*

$$Re\langle A_1(t)u, u)\rangle = \langle (Au,u)\rangle > c\|u\|^2$$

Using the Poincare inequality.

It follows that $A1$ is accretive, therefore closable.

$$\|Au\| = \|A_1(t)u - pb(t)\nabla u\| \leq \|A_1(t)u\| + c\|A^{1/2}u\| \ (b \in L^\infty * (\Omega), a.e\ t)$$

$$\leq \|A_1(t)u\| + c_1\|u\|^{1/2}.\|Au\|^{1/2} \textit{(momentinequality)}$$

$$\leq \|A_1(t)u\| + c_1\|u\| + \tfrac{1}{2}\|Au\| \quad \textit{(cauchy - Schwartz inequality)}$$

which yields the result.

b) follows from a) using the Cattabriga estimate for the stokes problem (we suppose that Ω is of class C^2) that is $\|u\|_{2.2} \leq c\|Au\|, u \in D_A$

Remark 2.2

a) It is follows by lemma 2.1 part a) and by the closure of A that $A_1(t)$

b) Is closed for almost all t. ($D_{A1} = D_A$)

c) one can easily check that

$$\|A_1(t)u\| \leq \|Au\| + c(t)\left\|A^{\frac{1}{2}}u\right\|, u \in D_A$$

and c(t) can be taken independant if $\|b(t)\|_\infty \prec \infty$, ***for ae all t Є [0,T]***

Proposition 2.1

If b(t) satisfies assumption 2.3

Then for almost all ***t* Є *[0, T], A₁(t)is m* - sec *torial***

Proof:

A₁(t) is closed by remark **2.2**

Using the fact that

$$\int_\Omega b(t)\nabla u.udx = 0, u\, e\, V, u\ real$$

$(Au, u) = \|\nabla u\|^2 \geq c_0\|u\|^2$

Poincare's inequality and that

$$Re\langle A_1(t)u, u)\rangle = \langle (Au, u)\rangle > c\|u\|^2$$

We deduce that $A_1(t)$ is maximal accretive (Theorem 2.1.4 p21 [16])

This follows by the application of Lax-Milgram's and closed range theorems.

$\text{for all } t \in [0, T], \{\lambda : Re\lambda \succ -c_0\} \subset \rho(-A_1(t))$

(2.1.1) $\|(A_1(t) + \lambda)^{-1}\| \leq \frac{1}{Re\lambda + c_0}$

We have:

$$Im(A_1)(t)u, u) = \int_\Omega b(t)Vudx, u \in D(A)$$

by assumption 2.3, Holder's inequality and Sobolev embedding theorem

$$|Im(A_1)(t)u, u)| \leq \|b(t)\|_{L^3}.\|\nabla u\|_{L^2}.\|u\|_{L^6}$$
$$\leq c\|b(t)\|_{L^3}.\|\nabla u\|^2 = c\|b(t)\|_{L^3}.Re\,(A_1(t)u, u))$$

Hence A_1 is sectorial and by the previous result m-accretive, therefore m-sectorial.

Furthermore, if $\|b(t)\|_{L^3} = c_0 \prec \infty \text{for all } t \in [0, T]$ then the numerical range of the operator (A1 (t) for all this contained in the fixed sector $\sum_\theta$

$$\left\{0, \theta = Artg\left(\frac{c_0}{v}\right) < \frac{\pi}{2}\right\}$$
$$\sum_\theta \{z \in C, |Artgz| \leq \theta\}$$

Corallary2.1

If b(t) satisfies the assumption 2.3 and $\|b(t)\|_{L^3} \leq c_1 \prec \infty$ *For all* **t** *Є [0, T].*

C_1 independant oft, then $\|(1 + |\lambda|)(A_1(t) + \lambda)^{-1}\|$

Is uniformly bounded with respect to t and λ for Re λ> 0

i.e there exists a constant M such that:

$$\|(A_1(t) + \lambda)^{-1}\| \leq \frac{M}{1 + |\lambda|}, Re\lambda \succ 0$$

Proof:

It follows by the relation (2.1.1) that $\|(A_1(t) + \lambda)^{-1}\|$is uniformly bounded since$\lambda \in \rho(-A_1(t)) \subset \sum_\theta$.

we have

$\lambda\frac{|\lambda|}{Re\lambda}, \in Re\lambda \succ 0$ is uniformly bounded.

It then follows that:

$$\|(1 + |\lambda|)(A_1(t) + \lambda)^{-1}\|$$

is uniformly bounded.

2.4 Existence theorem for the problem (P_T)

To state the existence theorem for the problem (P_T)

We need more assumptions on the data b(t)

Assumption 2.4

i) $b(t) \in C^1([0, T]; L^3(\Omega))$

ii) $b(t) \in C^a([0, T]; L^3(\Omega))\ for\ some\ 0 \prec a \leq 1$

Proposition 2.2

If b(t) satisfies the assumptions 2.3 and 2.4 ii). then

a) ***A₁ (t)*** is continuous D_A

b) $A_1(t).A_1^{-1}(s)$ is uniformly bounded relatively to t

c) $A_1(t).A_1^{-1}(s)$ is Holder continuous in t, uniformly in s.

d) if furthermore b (t),satisfies 2.4 (i) then $A_1(t)$. $A_1^{-1}(s)$ are continuously differentiable respectively on D_A and H.

Proof:

It is sufficient to prove that ***A₁(t) is*** continuously differentiable respectively on D_A And apply lemma 1.7 P 179 of [16].

Let

$u \in D_A, b(t) \in C_1([0, T]); L^3\ then$

$$\left\|\frac{A_1(t)u - A_1(t_0)u}{t - t_0}\right\| \leq \left\|\left[\frac{b(t) - b(t_0)}{t - t_0} - b'(t_0)\right]\nabla u\right\|$$
$$\leq \left\|\frac{b(t) - b(t_0)}{t - t_0} - b'(t_0)\right\|_{L_3}.\|\nabla u\|_{L_6}\quad (\textit{Holder inequality})$$

Hence $A_1(t)$ is differentiable on D_A and $A_1'(t)u = pb'(t)\nabla u$, for all u Є D_A and $t \in [0, T]$

From the relation

$$\|(b'(t) - b'(t_0))\nabla u\| \leq \left\|\left(b'^{(t)} - b'(t_0)\right)\right\|_{L_3}.\|\nabla u\|_{L_6}.$$

Therefore $A'_1(t)u$ is continuous and $A_1(t)$is continuous and $A_1(t)$is continously differentiable on D_A

we then apply lemma I.5 [16] to deduce that $A_1(t)A_1^{-1}(t)$ is Lipschitz continuous in both variable t and s uniformly.

Let us recall some results on the existence and regularity of the solutions of evolution equations due to Tanabe [17] and Krein [15]

In the two assumptions below B(t) is a family of linear operator $t \in [0,T]$ with a constant domain D_B

Assumption 2.5

B(t) is closed operator with constant domain D_B densely defined in the Banach space X for all $t \in [0,T]$

The resolvent of B(t) contains the sector Re $\lambda \leq 0$ and $(1+|\lambda|)||(\lambda) + B(t)^{-1}||$ is uniformly bounded relatively to t and λ.

Assumption 2.6

For all r Є [0, 7] B(t).B^{-1} (r)

Is uniformly continuous in r i.e. there exist some positive real constant L independent of r t and s such that:

$$||B(t).B(t)^{-1}(r) - B(s).B^{-1}(r)|| \leq L|t-s|^a, 0 < a \leq 1$$

Theorem 2.1

If B(t) satisfies the assumption 2.5 and 2.6

Then there exists a bounded operator on X

U(t, s) called fundamental solution of the parabolic equation:

$$\frac{du}{dt} + B(t)u = f(**)$$

such that for u_0 Є X

$f \in C^a([0,T];X), 0 < a \leq 1$

$$u(t) = U(t,s)u_o + \int_s^t U(t,\tau)f(\tau)d\tau$$

is the unique solution of the problem (**) satisfying u(s)=u(0) satisfying:

$u \in C([0,T];D_B) \cap C^1(([0,T];H)$

furthermore , if B(t) is continuously differentiable on D_B with

$$u_0 \in D_B \text{ and } ||(1+\lambda)(B(t)+\lambda)^{-1}|| \leq 1, \lambda \geq 0, \text{then}$$
$$u \in C([0,T];D_B) \cap C^1(([0,T];H)$$

Remark 2.1

The theorem 2.1 is standard and the second part follows from theorem 3.11 P20 of [15]

Theorem 2.2

If data b(t) satisfies assumption 2.3 and 2.4 then the problem P(T) has a unique solution satisfying:

i) $u \in C([0,T];H^2(\Omega) \cap V) \cap C^1(([0,T];H)$

ii) $\|u(t)\|^2 + 2v\int_0^t \|\nabla u\|^2 d\tau = \|u_0\|^2$

Proof:

We take for simplicity f=0

From corollary 2.1 and proposition 2.2 we deduce that the operator $A_1(t)$

Satisfies the assumptions 2.5 and 2.6

Hence the fundamental solution U(t,s) of (P_T) exists and $u(t) = U(t,0)u_0$ is the unique solution and we have the estimate:

$\|u(t)\| \leq \|u_0\|$, for all $t \in [0,T]$

We also have by theorem 2.1

$u \in C([0,T];X) \cap C^1(([0,T];X)$

It follows that. $A_1(t)u(t) \in D_A$

From the lemma 2.1

We have $\|Au(t)\| \leq 2\|A_1(t)u(t)\| + c\|u(t)\|$

hence $u \in C([0,T];D_A)$

The fact that $u \in C([0,T];H^2(\Omega) \cap V)$ follows from cattabriga's estimate. The energy aquality follows from the orthogonality properties:

$$\int_\Omega b(t)\nabla u(t).u(t)dx = 0, \textit{for all } u \in V$$

the equality $(u(t),u'(t)) = \frac{1}{2}\frac{d}{dt}||u(t)||^2$ The result follows by integration.

Remark 2.3

for $f \in C^a([0,T];H)$ and representation

$$U(t) = U(t,0)u_0 + \int_0^t U(t,\tau)f(\tau)d\tau$$

We obtain similar results.

III. Regularity of the Solution

It is well known that higher regularity at t=0 for evolutions requires some compatibility conditions on the data which for the case of Navier-stokes equations are of global nature and "virtually uncheckable" for m> 5/2 ([17] and [16]) we state here some necessary and sufficient conditions to ensure smooth solutions at t=0

Theorem 3. 1

$$u_0 \in H^m(\Omega) \cap V, b(.) \in C([0,T];H^m(\Omega) \cap V), m \geq 2$$

if $f \in C^1(([0,T];H^{m-2k}(\Omega) \cap V)$ then

The following conditions are Equivalent :

i) The solution u of the problem (P_T) belongs *to* $C([0,T];H^m(\Omega) \cap V)$

ii) $\frac{d^k u}{dt^k} \in C(([0,T];H^{m-2k}(\Omega) \cap V)$

Proof

We prove it for m=3.

Suppose $u_0 \in C[0,T]H^3(\Omega) \cap V$,

We have from the eΩuation and the existence theorem 2.2 that

$$\frac{du}{dt} \in C(([0,T];H$$

and

$$\frac{du}{dt} = A_1(t)u(t) + f(t) = vAu(t) - pb(t)\nabla u + f(t)$$

belongs to $H^1(\Omega)u(t) \in H^3(\Omega) \cap V)$ b(t) Є H^3 this implies

$Au(t) - pb(t)\nabla u \in U^1(\Omega)$ for all $t \in [0,T]$

from the estimates

$\|pu(t)\nabla u\|_s \leq c\|u\|_s\|v\|_{s+1}$

Kato and Lai [ ] we then deduce that :

$$\|A_1(t)u(t_1) - A_1u(t_2)\| \leq c_1\|b(t_1) - b(t_2)\|_{H^3} + \|u(t_1) - u(t_2)\|_{H^3}$$

c1 and c2 depending only on Ω and

$Sup_{0\leq t\leq T}\|b(t)\|_{H^3}, \|u(t)\|_{H^3}),$

It then follows that $\frac{du}{dt} \in \mathbf{C}([0,T];\ \mathbf{H}^1(\Omega))$

Since V is closed in $H^1(\Omega)$ and the operator $\frac{d}{dt}$ is closed we deduced that $\frac{du(t)}{dt} \in V$ *for all t* $\in$ *[0, T]*

The converse is immediate from Cattabriga's estimates and the fact that

$$\frac{du(t)}{dt} = A_1(t)u(t) + f(t) \in C([0,T];\ H^1(\Omega))$$

Corollary 3.1

If the conditions of theorem 3.1 are satisfied then

$A_1(0)u(0) + f(0))|_\Gamma = 0$

Proof

By the equivalence of theorem 3.1

$$\frac{du(t)}{dt} \in C([0,T];\ H^1(\Omega)) \cap V$$

Therefore $\frac{du(0)}{dt} = A_1(0)u(0) + f(0)$

This implies $\frac{du(0)}{dt}|\Gamma = 0$

Which is the well known non local compatibility conditions for the Navier-Stokes equations.

Remark 3. 1

a) One can construct solutions of the problems (P_T) by Galerkin approximations for $u_0 \in H^m(\Omega \cap V)$ $u(t) \in C([0,T]; H^m(\Omega)) \cap V) \cap C^1([0,T]; H^{m-2l}(\Omega)) \cap V)$ $l < m/2$

The condition $u(t) \in C([0,T]; H^m(\Omega)) \cap V)$ can also be obtained for sufficiently regular data and the corresponding compatibility conditions for the initial data.

b) The corresponding pressure is obtained by the projection theorem.

3.2 Some remarks on the singularity at t=0

Let

$u_0 \in H^3(\Omega) \cap V)$,

$f(t) \in C([0,T];\ \mathbf{H}^1(\Omega)) \cap \mathbf{V})$

$b(t,x) \in C([0,T]; H^3(\Omega) \cap V)$,

Consider the following problems

$$g(x) = -v\nabla u_0 + u_0\nabla u_0 - f(0)$$

$$(P_1)\begin{cases} \frac{\partial u_1}{\partial t} - b(t)\nabla u_1 + \nabla p_1 = f + g \ in\ (0,T]\ x\ \Omega \\ u_1|_\Gamma = 0 \\ \nabla u_1 = 0 \\ u_1(x,0) = u_0 \end{cases}$$

$$(P_2)\begin{cases} \frac{\partial u_2}{\partial t} - v\nabla u_2 + b(t)\nabla u_2 + \nabla p_2 = f + g \ in\ (0,T]\ x\ \Omega \\ u_2|_\Gamma = 0 \\ \nabla u_{12} = 0 \\ u_2(x,0) = u_0 \end{cases}$$

Note that in (P_2) the external force is time independent and that if u_1 and u_2 Are solution of (P_1) and (P_2) respectively.

Then $u = u_1 + u_2$ and the corresponding pressure

$P = P_1 + P_2$ is a solution of (P_T)

It is evident by the definition of g that the compatibility conditions for (P_1) are fulfilled, hence

$u_1 \in C([0,T]; H^3(\Omega) \cap V)$ therefore, the singularity of (P_T) is fully characterized by u_2 For numerical purpose see for example [17] The order of accuracy of the problem (P_T) is given by that of u_2 which seems easier to tackle than that of (P_T) itself (at least at for finite element approximation).

The abstract formulation of (P_T) is :

$$\frac{du}{dt} + A_1(t)u = g \ in\ [0,T]$$

u(0)=0

g is time independent .

Remark 3.2

A solution of (P_1) has been constructed by the means of product formula in[12]. In the Adomian-like method that follows we use this product formula to construct the adomian coefficients. They are obtained approximately.

IV. Adomian Decompostion Method for Navier - Stokes EΩuations

4.1 Preliminaries

Adomian Decomposition Method

Our equation can be written in the form

$Lu - N(u) = f$ (3)

Where L is a linear operator and N a nonlinear operator .

The linear operator is assumed invertible.

The Adomian method consist of approximating the solutions of (1) - (2) as

an infinite series $u = \sum_{n=0}^{\infty} u_n$ (4)

and expressing the nonlinear operator in the form $N(y) = \sum_{n=0}^{\infty} A_n$ where A_n are the adomian polynomials of y_0, y_i, y_n... as defined by (Adomian 1994) where

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty} \lambda^i u_u\right)\right]$ i=0,1,2 (5)

applying the inverse of the linear operator L^{-1} to the members of (3)after substituting (4) and (5) we obtain the following relations

$\sum_{n=0}^{\infty} u_n - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right) = L^{-1}(f)$ (6)

we can deduce the following iterative relations:

$u_0 = L^{-1}f$

$U_{n+1} = L^{-1}(A_n)$ (7)

The coefficients A_n can all be estimated recursively.

The general formula that we can deduce from the formula established by Abbaoui and Cherruault [18]

$A_n = \sum_{k=1} N^{(k)}(y_0)\frac{\left[\sum_{p1+p2\ldots pn=n} y_{p1}y_{p2}\cdots y_{pn}\right]}{p_1!p_2!\ldots p_n!},\quad n \geq 1$ (9)

We can then deduce the value of

$Y_n = \int_0^t \int_0^\tau A_n d\tau dt$ (10)

y0 = h

Definition of the Adomian polynomial in a Banach space

We shall note in the application of Adomian Method so far it is assumed that the operator

N(u) analytic.

We are going to extend these notions on N to an arbitrary Banach algebra X

N is an operator from XxX→ X

$(x, y) \to N(x, y)$

We assume that N is analytic as function of two variables

Le B be a closed densely defined operator in X Defined the operator

M : X→ X

$u \rightarrow M(u) = N(u, Bu)$

Let us consider the problem (PT) defined in the first part by

$$(P_T)\begin{cases}(1.9)\dfrac{\partial u_1}{\partial t} - v\nabla u + u + \nabla p = f + \; in\; Q_T \\ (1.10)\nabla u = 0 \; in\; Q_T \\ (1.11) u|_{\partial\Omega} = 0, t > 0 \\ (1.12) \quad u(x,0) = u_0, x \in \Omega\end{cases}$$

Here the X=L^2(0, T; H)

Let us define the associated Stokes equations

$$(SP_T)\begin{cases}\dfrac{\partial u_1}{\partial t} - v\nabla u + u + \nabla p = f + \; in\; Q_T \\ (1.10)\nabla u = 0 \; in\; Q_T \\ (1.11) u|_{\partial\Omega} = 0, t > 0 \\ (1.12) \quad u(x,0) = u_0, x \in \Omega\end{cases}$$

The problem (SP_T) has a unique solution for u_o Є V for any fЄL^2(0, T; H)

We are going to choose for simplicity f independent of t. (***here*** f=0)

Le A be the stokes operator as defined in the first part the unique solution of (SP_T)

is defined through the analytic semi-group e^{-At} by

$$u(t) = e^{-At}u_0 + \int_0^t e^{-(t-s)A} f(s)ds = L^1 f \qquad (4.1)$$

Applying the Weyl's projection on the the two member of equation (1.9)

We have

$$Lu + Pu\nabla u = f \qquad (4.2)$$

$$Where\; Lu = \frac{du}{dt} + Au \qquad (4.3)$$

L^{-1}***is*** a continuous operator fom H to D(A)

Multiplying the relation by L^{-1}

we have $u + L^1 Pu\nabla u = L^1 pf$ (4.4)

We define $M(u) = L^1 u\nabla u$ (4.5)

which is a nonlinear operator

Adomian polynomials

$M(u) = \sum_{n=0}^{\infty} A_{n(u_0, u_1, \dots u_n)}$ (4.6)

The operator A_n being linear operator from X into X.

$$u_0(t) = e^{-AT}u_0$$

We assume f=0 and $u(x,t) = \sum_{n=0}^{\infty} u_n(x,t)$

Substituting into the equation (4.8)

We can deduce the following u_0 (t) defined by the relation (2.7)

The An are formally defined by

$$An = \frac{d^n}{dt^n}\left[L^{-1}P\left(\sum_{i=0}^{\infty}\lambda^i u_i \nabla\left(\sum_{j=0}^{\infty}\lambda^j u_j\right)\right)\right]_{\lambda=0}$$

each u_k is assumed in D(A)

This can be established using results in part I

$u_{n+1} = L^{-1}(A_n)\; n \geq 1$

with zero initial value

We have

$$A_n = L^{-1}u_0\nabla u_0 \; and$$

$$u_1(t) = \int_0^t e^{-(t-s)A} A_0(u_0) sds$$

$$A_n = L^{-1}P[u_0\nabla u_1 + u_1\nabla u_0]$$

Lemma 4. 1

$$A_n = \sum_{i+j=n} L^{-1}Pu_i\nabla u_j$$

Proof

The calculations are obtained as the classical Adomian methods see Tchoua and Ita (2012)

Remark 3.4

It this paper the main steps and ideas of the new approach in solving nonlinear navier-Stokes is presented that we call Adomian-like Decomposition Method.

The full development is done in a forthcoming paper.

The method is semi-analytic. Some steps are numerical but easier to tackle than the classical algorithms on Navier-Stokes equations.

Theorem 4.1

The series (4.9) converges uniformly on any bounded time-interval to the unique and strong solution of problem (P_1) for small value in H1-norm of u_0.

Proof:

The proof is deduce from of the convergence of adomian sequence in [18] and the estimate

$$\|u_0\|_{H^1} \leq \theta < 1;\; \|A_n\|_{H^1} \leq Cn\|u_0\|_{H^1}^n$$

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V. Conclusion

In this paper we develop an Adomian scheme to compute the solution of the full nonlinear Navier-Stokes equations through a product formula in (12) very easy to simulate since product of linear operators.

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References

[1] Arikoglu A. : Solutions of differential equations by using differential transform Method.
Appl. Math. comput. 174(2006)pp1216 - 1228

[2] Avudainayagam et al: Wavelet-Galerkin method for integro-differential equations: appl. Numer. Meth. 32,(2000)247 - 254.

[3] ***Borhanifar*** A. Differential transform method for a class of integro-differential equations with derivative type kernel; Canadian Journal on computing in Mathematics. Vol 3 N.1 jan 2012.

[4] Corduneanu C. Integral equations and applications :cambridge university press Cambridge-Newyork-Sidney(1991).

[5] Hashim I :Adomian decomposition method in solving fourth order integro - dfferential equations
J of comput appl. math 193(2006)658 - 664.

[6] Hosseini S.M: tau numerical solution of fredholmintegro-differential equations with arbitrary polynombases. Appl math. models,27,(2003) 145 - 154.

[7] Malenegad K.Taylor polynomials solutions of higher order nonlinear voltera Fredholmintegro- differntial equations.
Appl math comput 145,(2003)641 - 653.

[8] Richard W. Johnson: A B-spline collocation method for solving the incompressible Navier-Stokes equations using an ad hoc method: the boundary residual method. Computer and Fluid 34 (2005) 121 - 149

[9] Tchoua P. and Ita .B.I: Numerical solution of a class of ordinary differential equations by the differential
transform method (2012 ***)(submitted)***

[10] Wazwaz A.M: A reliable algorithm for solving boundary value problems of higher order Appl math comput.118(2001)327 - 342.

[11] ***Yildiray Keskin: Numerical*** solutions of Sine-Gordon Equations by reduced transform method. Proceeding of the world congress on engineering (2011) Vol I, London UK.

[12] P. Tchoua ,J.S. Tayou; a product formula for linearized Navier-Stokes equations .JournFac SC UY(1994) HS no 4 pp 59-72

[13] ***Yosida*** K: ***Functional analysis*** Springger Verlag 1965

[14] Adomian G. : Solving Frontier Problem of physics:
The decomposition Method. Kluwer academicpress(1994)

[15] Ahmed H; Abdelzraza: Adomian decomposition Method Convergence analysis and numerical approximations
Thesis(2008)(Master university Hamilton, Ontario.)

[16] Rautman R: A product formula for linearized Navier-Stokes equations Zam(1989)pp23 - 33

[17] ***Temam***, R : Navier-Stokes equations, North Holland, Armsterdam 1980.

[18] K. Abbaoui, Y. Chernualt. Convergence of adomian's Method applied to nonlinear equations (20)(9) pp 69-75