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Effect of Electrification of Particles on Boundary Layer Flow and Heat Transfer of A Steady Dusty Fluid Over An Inclined Stretching Sheet With Non - Uniform Heat Source/Sink

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***Abstract* – This paper focuses on electrification of particles, terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase and heat due to conduction and viscous dissipation in the energy equation of the particle phase in modeling the steady laminar boundary layer flow and heat transfer of a dusty fluid over an inclined stretching sheet. Viscous dissipation, internal heat generation/absorption and effect of electrification of particles are considered in the energy equation. The governing equations are reduced to a set of ordinary partial differential equation by using suitable similarity transformations along with pertinent boundary conditions. They are solved numerically by Runge Kutta forth order method along with shooting technique. Finally the effect of the pertinent physical parameters like fluid-particle interaction parameter, Grashof number, Prandtl number, Eckert number and diffusion parameter on the flow and heat transfer characteristics are computed and presented graphically and also in tabular form. The rate of heat transfer at the surface and skin friction increase with increasing value of electrification parameter.**
AMSW classification 76T10, 76T15

***Keywords* – Electrical Parameter, Non-uniform Heat Source/Sink, Angle of Inclination, Inclined Stretching Sheet, Steady Flow.**

Nomenclature
E_c Eckert number
q^{111} Space and temperature dependent internal heat generation/absorption
F_r Froud number
G_r Grashof number
P_r Prandtl number
T_∞ Temperature at large distance from the wall.
T_p Temperature of particle phase.
T_w Wall temperature
$U_w(x)$ Stretching sheet velocity
c_p & c_s Specific heat of fluid & particles
k_s Thermal conductivity of particle
u_p & v_p Velocity component of the particle along x- and y-axis
A Constant
c Stretching rate
g Acceleration due to gravity
k Thermal conductivity of fluid
l Characteristic length
T Temperature of fluid phase.
u, v Velocity component of fluid along x-axis and y-axis
x, y Cartesian coordinate
A*&B* The parameters of the space and temperature dependent internal heat source/sink.
K* Mean absorption co-efficient
Greek Symbols:
φ Volume fraction
β Fluid particle interaction parameter
β^* Volumetric coefficient of thermal expansion
σ^* The Stefan Bolzman constant
ρ &ρ_p Density of the fluid & particle phase
ρ_s Material density
η Similarity variable
θ Fluid phase temperature
θ_p Dust phase temperature
μ Dynamic viscosity of fluid
ν Kinematic viscosity of fluid
γ Ratio of specific heat
τ Relaxation time of particle phase
τ_T Thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.
τ_p Velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.
ε Diffusion parameter
ω Density ratio
α Angle of inclination

I. Introduction

The study of heat source / sink effects on heat transfer of dusty fluid flows over a continuously moving solid surface has many important applications including boundary layer flow over heat treated material between feed roll and a windup roll, rolling and manufacturing of plastic films, cooling of an infinite metallic plate in cooling bath, the boundary layer along a liquid film in condensation process, and aerodynamic extrusion of plastic sheets etc. The momentum and Heat transfer over a stretching sheet have been studied because of its ever increasing usage in polymer processing industry, manufacturing of artificial fibers, viscoelastic fluid flow and in some application of dilute polymer solution, such as the 5.4% solution of polyisobutylene in cetane, curing of plastics, manufacture of printed circuitry, pulp-insulated cables etc.

The behavior of boundary layer flow due to a moving flat surface immersed in an otherwise quiescent fluid was first studied by Sakiadis B.C. [26], who investigated it theoretically by both exact and approximate method. Then many researchers extended the above study with the effect of Heat Transfer. Tsou et.al [30] studied the effect of heat transfer and experimentally confirmed the numerical result of Sakiadis. Grubka et.al [8] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux. Anderson [9] discussed a new similarity solution for the temperature fields. Chen [4] investigated mixed convection of a power law fluid past a stretching surface in presence of thermal radiation and magnetic field. B.J.Gireesha et. al [3] have studied the boundary layer flow and heat Transfer of a dusty fluid over an unsteady stretching surface in presence of non uniform heat source/sink. Subhas et.al [29], have studied “heat transfer in MHD visco-elastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Robert et.al [25] were studied the convective heat transfer in a conducting fluid over a permeable stretching surface with suction and internal heat generation/absorption. M.S.Abel et.al [15] investigated Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non uniform heat source and radiation. Kai-Long Hsiao et.al [10] investigated viscoelastic fluid over a stretching sheet with electromagnetic effects and non –uniform heat source /sink. G.K.Ramesh et.al [5] have investigated the momentum and heat transfer characteristics in hydrodynamic flow of dusty fluid over an inclined stretching sheet with non uniform heat source/sink. Mohammad Ali et.al [21] have analyzed a steady hydro magnetic flow of an incompressible electrically conducting fluid over an inclined stretching sheet. M. Ali et.al [13] have emphasized the heat and mass transfer of a steady flow of an incompressible electrically conducting fluid over an inclined stretching plate under the influence of an applied uniform magnetic flied with heat generation and suction and the effect of Hall current. M. Ali et.al [22] have investigated a steady MHD free convection, heat and mass transfer of an incompressible electrically conducting fluid past an inclined stretching sheet under the influence of an applied uniform magnetic flied with radiation effect and hall current. M. Ali et.al [20] have studied the hall effect on the steady MHD boundary layer flow of an incompressible fluid combined heat and mass transfer over a moving inclined plate in a porous media with suction and viscous dissipation. M. S. Alam et.al [15] have studied the effect of viscous dissipation and nth order chemical reaction on MHD free convective heat and mass transfer flow along an inclined stretching sheet. Md. Shariful Alam [19] have investigated numerically the influence of chemical reaction and heat generation or absorption on MHD free convection and mass transfer flow of a viscous incompressible and electrically conducting fluid over an inclined stretching sheet.

Even though the study relating to flow and heat transfer in MHD dusty boundary layer flow over stretching sheet [1, 2, 6, 14, 18, 23, 31] are available, hardly any study is taken up by considering the base fluid as non –conducting and the particles are electrified. No consulted effort has been made to show the effect of electrification of particles and/or contribution of various physical aspects on two phase flow and heat transfer. Since tribo electrification occurs due to collision of particles with each other or impingement of particles with walls and since the electrification of particles have a pronounced effect on boundary layer characteristics like skin friction, heat transfer etc, it is essential to include this phenomena in the modeling of flow over a stretching sheet The forces and moments acting on a solid particle consist of those due to the net charge in the electric field due to the charged particles. As a general statement, any volume element of charge species, with charge "e" experiences an instantaneous force given by the Lorentz force law given by $\vec{f} = e\,\vec{E} + \vec{J} \times \vec{B}$ where $\vec{B}$ is the magnetic flux density. The current densities in corona discharge are so low that the magnetic force term $\vec{J} \times \vec{B}$ can be omitted, as this term is many orders of magnitude smaller than the Coulomb term $e\vec{E}$ .The ion drift motion arises from the interaction of ions, constantly subject to the Lorentz force with the dense neutral fluid medium. This interaction produces an effective drag force on the ions. The drag force is in equilibrium with the Lorentz force so that the ion velocity in a field $\vec{E}$ is limited to $k_m\vec{E}$, where k_m is the mobility of the ion species. The drag force on the ions has an equal and opposite reaction force acting on the neutral fluid molecules via this ion-neutral molecules interaction, the force on the ions is transmitted directly to the fluid medium, so the force on the fluid particles is also given by $\vec{f} = e\,\vec{E}$, Soo [28].

II. Flow Analysis of The Problem and Solution

Consider two dimensional steady laminar boundary layer of an incompressible viscous dusty fluid over a vertical stretching sheet which is inclined with an acute angle α and situated in the fluid of ambient temperatureT_∞. The x-axies moves along the stretching surface in the direction of motion with the slot as the origin. The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow. The sheet being stretched with the velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid. Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow.

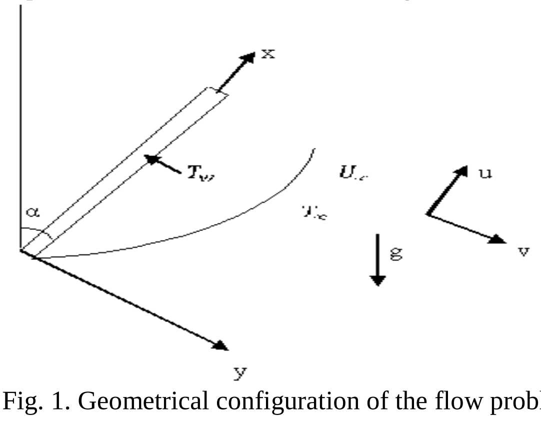

Fig. 1. Geometrical configuration of the flow problem

Here the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be

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taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase. The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of electrification, non uniform heat sour / sink, volume fraction of particles on skin friction, heat transfer and other boundary layer characteristics also have been studied.

The governing equations of steady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial}{\partial x}(\rho_p u_p)+\frac{\partial}{\partial y}(\rho_p v_p)=0 \quad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\mu\frac{\partial^2 u}{\partial y^2}-\frac{1}{(1-\varphi)\rho}\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+g\beta^*(T-T_\infty)\cos\propto+\frac{1}{1-\varphi}\frac{\rho_p}{\rho}\left(\frac{e}{m}\right)E \quad (3)$$

$$\varphi\rho_s\left(u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s(u-u_p)+\varphi(\rho_s-\rho)g+\rho_p\left(\frac{e}{m}\right)E \quad (4)$$

$$\varphi\rho_s\left(u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial y}\right)=\frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right)+\frac{1}{\tau_p}\varphi\rho_s(v-v_p) \quad (5)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\varphi\rho_s c_s}{(1-\varphi)c_p}\frac{1}{\rho\,\tau_T}(T_p-T)+\frac{\varphi\rho_s}{(1-\varphi)}\frac{1}{\rho c_p}\frac{1}{\tau_p}(u_p-u)^2+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{1}{\rho c_p}q'''+\frac{1}{1-\varphi}\frac{1}{\rho c_p}\left(\frac{e}{m}\right)EU_p \quad (6)$$

$$u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}=-\frac{1}{\tau_p}(T_p-T)+\frac{1}{\varphi\rho_s c_s}\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right)-\frac{1}{\tau_p}\frac{1}{c_s}(u-u_p)^2+\frac{\mu_s}{\rho_s c_s}\left[u_p\frac{\partial^2 u_p}{\partial y^2}+\left(\frac{\partial u_p}{\partial y}\right)^2\right]+\rho_p\left(\frac{e}{m}\right)EU_p \quad (7)$$

Where (u,v) *and* (u_p,v_p) are the velocity components of the fluid and dust particle phases along x and y directions respectfully. μ,ρ and ρ_p, N are the co-efficient of viscosity of the fluid, density of the fluid and particle phase, number density of the particle phase respectfully. Where g is the acceleration due to gravity, m is the mass of the dust particle with electric field .

With boundary conditions

$$\left.\begin{array}{c} u=U_\omega(x)=cx, v=0, T=T_w=T_\infty+A\left(\frac{x}{l}\right)^2 \\ at\ y=0 \\ \rho_p=\omega\rho, u=0, u_p=0, v_p\to v\ T\to T_\infty, T_p\to T_\infty\ as\ y\to\infty \end{array}\right\} \quad (8)$$

Where ω is the density ratio in the main stream. $U_\omega(x)=cx$ is the stretching sheet velocity, c>0;this is known as stretching rate.

Where A is a positive constant, $l=\sqrt{\frac{v}{c}}$ is a characteristic length. q''' is the space and temperature dependent internal heat generation/absorption (non-uniform heat source /sink) is given below

$$q'''=\left(\frac{kU_\omega(x)}{xv}\right)[A^*(T_w-T_\infty)f'+B^*((T-T_\infty)] \quad (9)$$

Where A^* and B^* are the parameters of the space and the temperature dependent internal heat source / sink respectfully. It is to be noted that A*and B* are positive heat source and negative to internal heat sink; ν is the kinematic viscosity.

For most of the gases $\tau_p\approx\tau_T$, $k_s=k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p}=\frac{2}{3P_r}$, $\varphi\rho_s=\rho_p$ Introducing the following non dimensional variables in equation (1) to (7)

$$u=cxf'(\eta), v=-\sqrt{cv}f(\eta), \eta=\sqrt{\frac{c}{\nu}}\,y,$$

$$u_p=cxF(\eta), v_p=\sqrt{cv}\,G(\eta), \rho_r=H(\eta)$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \theta_p(\eta)=\frac{T_p-T_\infty}{T_w-T_\infty}, \beta=\frac{1}{c\tau_p}, \epsilon=\frac{\nu_s}{\nu},$$

$$P_r=\frac{\mu c_p}{k}, E_c=\frac{c^2l^2}{Ac_p}, M=\frac{E}{c^2x}\left(\frac{e}{m}\right)$$

Where $T-T_\infty=A\left(\frac{x}{l}\right)^2\theta, T_p-T_\infty=A\left(\frac{x}{l}\right)^2\theta_p$

We get the following non dimensional form.

$$HF+HG'+GH'=0 \quad (10)$$

$$f'''(\eta)+f(\eta)f''(\eta)-[f'(\eta)]^2+\frac{1}{(1-\varphi)}\beta H(\eta)[F(\eta)-f'(\eta)]+Gr\theta\cos\propto+\frac{H(\eta)}{1-\varphi}M=0 \quad (11)$$

$$G(\eta)F'(\eta)+[F(\eta)]^2=\epsilon F''(\eta)+\beta[f'(\eta)-F(\eta)]+\frac{1}{Fr}\left(1-\frac{1}{\gamma}\right)+M \quad (12)$$

$$GG'=\epsilon G''-\beta[f+G] \quad (13)$$

$$\theta''=Pr(2f'\theta-f\theta')-\frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p-\theta]-\frac{1}{1-\varphi}PrE_c\beta H[F-f']^2-PrE_cf''^2-[A^*f'+B^*\theta(\eta)]-\frac{1}{(1-\varphi)}H(\eta)ME_c\,Pr\,F(\eta) \quad (14)$$

$$\theta_p''(\eta)=(2F\theta_p+G\theta_p'+\beta[\theta_p-\theta]+\beta EcPr[f'-F]^2-\frac{3}{2}\epsilon EcPr[FF''+(F')^2]-\frac{3}{2}MEcPrF(\eta))/\frac{\epsilon}{Pr} \quad (15)$$

Where $Gr=g\frac{\beta^*(T_\omega-T_\infty)}{c^2x}$ *is the local Grashof number*

With boundary conditions

$$\left.\begin{array}{l} G'(\eta)=0, f(\eta)=0, f'(\eta)=1, F'(\eta)=0, \\ \theta(\eta)=1, \theta_p'=0\ as\ \eta\to 0 \\ f'(\eta)=0, F(\eta)=0, G(\eta)=-f(\eta), H(\eta)=\omega, \\ \theta(\eta)\to 0, \theta_p(\eta)\to 0\ as\ \eta\to\infty \end{array}\right\} \quad (16)$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which defined as

$C_f=\frac{\tau_w}{\rho U_w^2}$, $Nu_x=\frac{xq_w}{X(T_w-T_\infty)'}$ where the surface shear stress τ_w and surface heat flux q_w are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

Using the non dimensional variables, we obtained

$$C_f Re_x^{1/2}=f''(0), \frac{Nu_x}{Re_x^{1/2}}=-\theta'(0)$$

Solution of the Problem

Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty)=0, F(\infty)=0, G(\infty)=-f(\infty), H(\infty)=\omega, \theta(\infty)=0, \theta_p(\infty)=0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$. We have supplied $f''(0)=\propto_0$ and $f''(0)=\propto_1$. The improved value of $f''(0)=\propto_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\propto_2,\infty)$ is determined by using Runge-Kutta method. If $f'(\propto_2,\infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0=\propto_1$ *and* $\propto_1=\propto_2$ until a correct $\propto_2$ is obtained. The same procedure described above is adopted to

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determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The essence of shooting technique to solve a boundary value problem is to convert the boundary value problem into initial value problem. In this problem the missing value of $\theta'(0)$ and $f''(0)$ for different set of values of parameter are chosen on hit and trial basis such that the boundary condition at other end i.e. the boundary condition at infinity (η_∞) are satisfied. A study was conducting to examine the effect of step size as the appropriate values of step size Δη was not known to compare the initial values of $\theta'(0)$ and $f''(0)$. If they agreed to about 6 significant digits, the last value of η_∞ used was considered the appropriate value; otherwise the procedure was repeated until further change in η_∞ did not lead to any more change in the value of $\theta'(0)$ and $f''(0)$. The step size Δη =0.1 has been found to ensure to be the satisfactory convergence criterion of 1×10^{-6}. The solution of the present problem is obtained by numerical computation after finding the infinite value for η. It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as η =5.0 with a step length of 0.1 beginning from η = 0.0 Depending upon the initial guess and number of steps N. the values of $\theta'(0)$ and $f''(0)$ are obtained from numerical computations which are given in table –2 for different parameters.

III. Results And Discussion

The equations (11) to (16) with boundary conditions (17) were solved numerically, in double precision, by shooting method using the Runge-Kutta fourth order algorithm. The computations were done by the computer language FORTRAN-77. The results of heat transfer and skin friction coefficient characteristics are shown in Table-2, the effect of various parameters on the velocity profiles and temperature profiles also demonstrated graphically. In order to check the accuracy of our present numerical solution procedure used a comparison of wall temperature gradient θ′(0) is made with those reported by with G.K. Ramesh [5] & Tsai [24] for various values of Pradtl number Pr & B* in absence of other parameters which are given in table-1.Our present results are in a good agreement with the previous results.

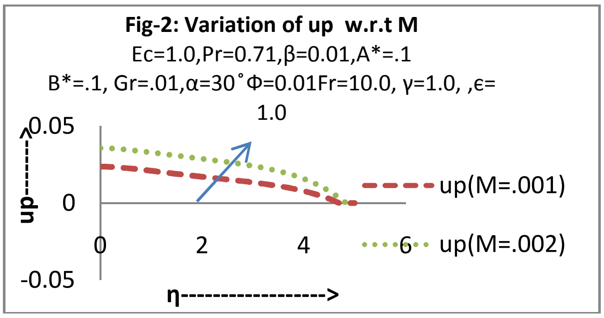

Fig-2: Variation of up w.r.t M

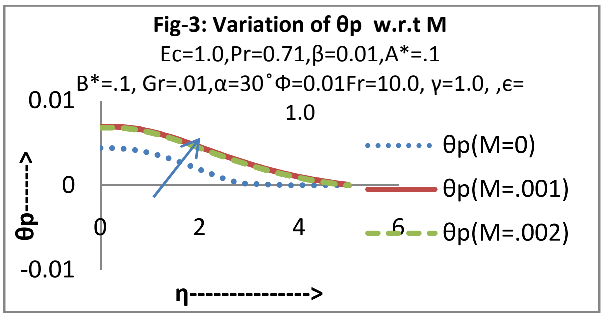

Fig-3: Variation of θp w.r.t M

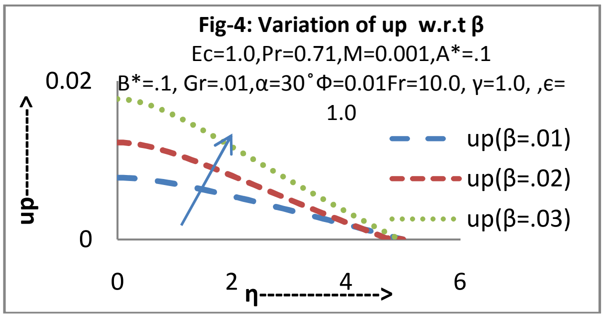

Fig-4: Variation of up w.r.t β

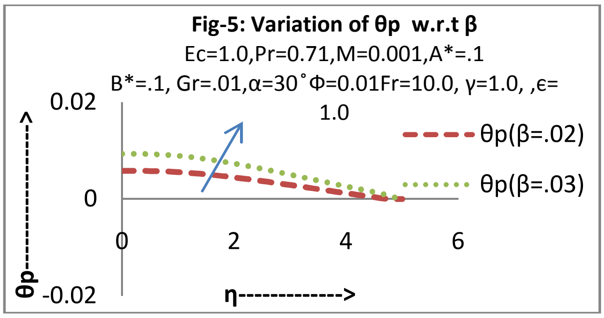

Fig-5: Variation of θp w.r.t β

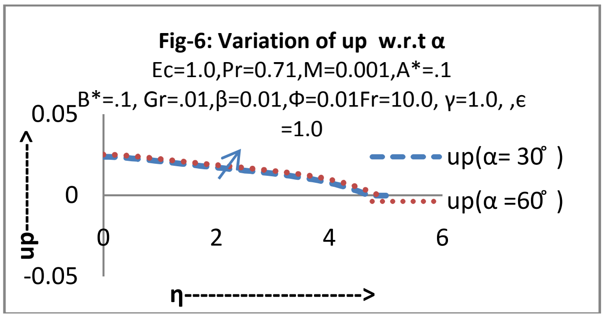

Fig-6: Variation of up w.r.t α

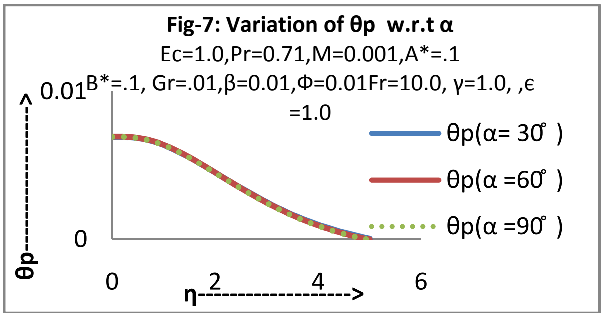

Fig-7: Variation of θp w.r.t α

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Fig-8: Variation of up w.r.t Ec

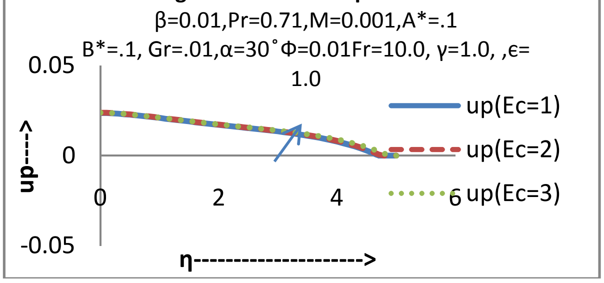


Fig-9: Variation of θ w.r.t Ec

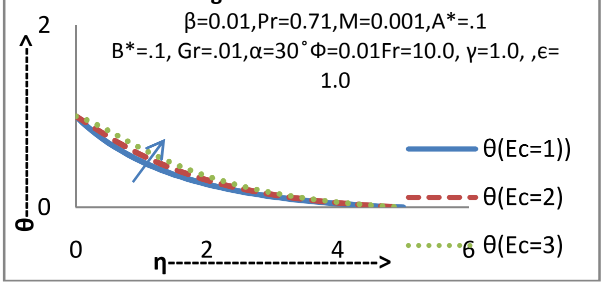


Fig-10: Variation of θp w.r.t Ec

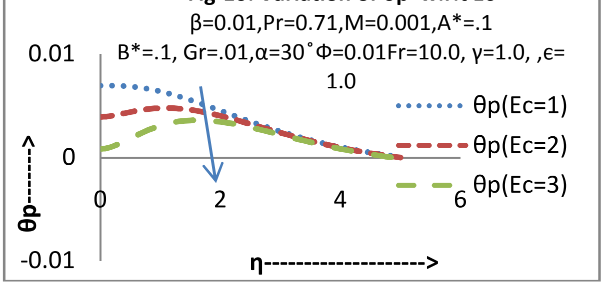


Fig-11: Variation of up w.r.t Pr

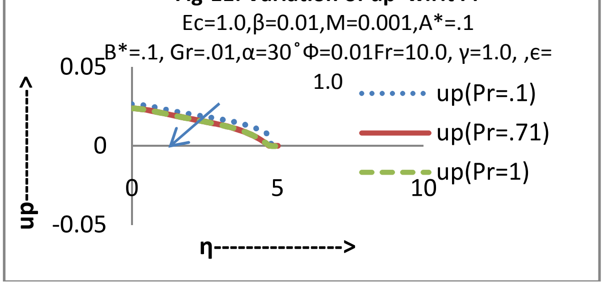


Fig12: Variation of θ w.r.t Pr

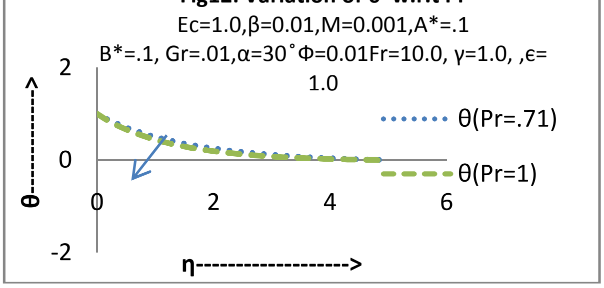


Fig-13: Variation of θp w.r.t Pr

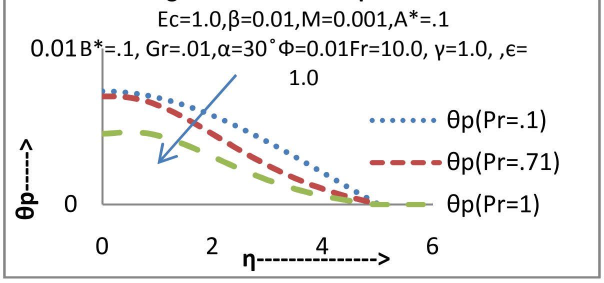


Fig-14: Variation of up w.r.t A*

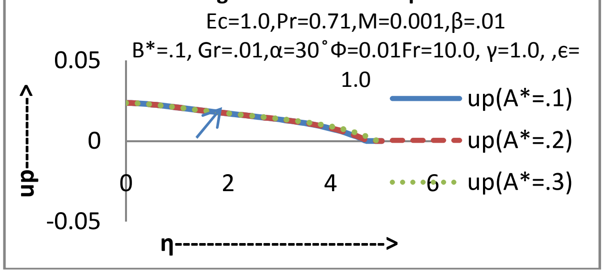


Fig-15: Variation of θ w.r.t A*

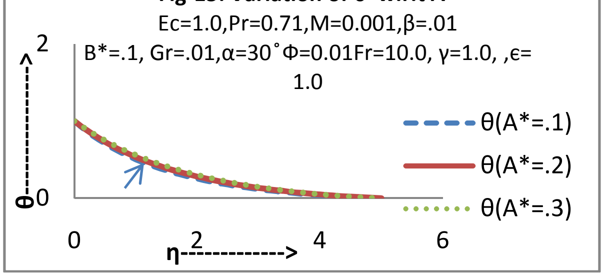


Fig-16: Variation of θp w.r.t A*

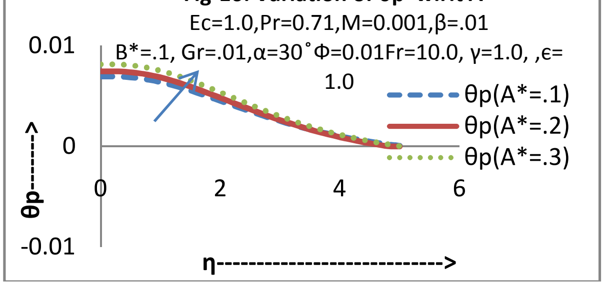
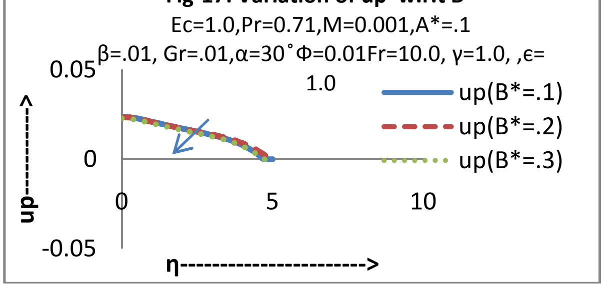


Fig-17: Variation of up w.r.t B*



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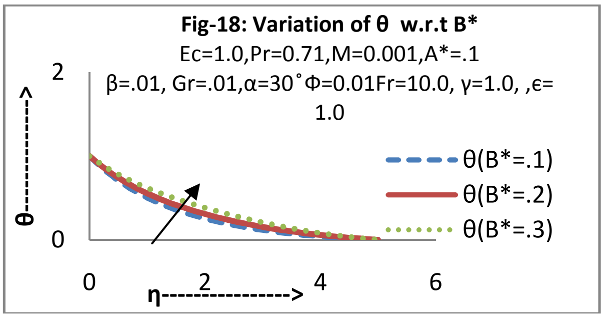

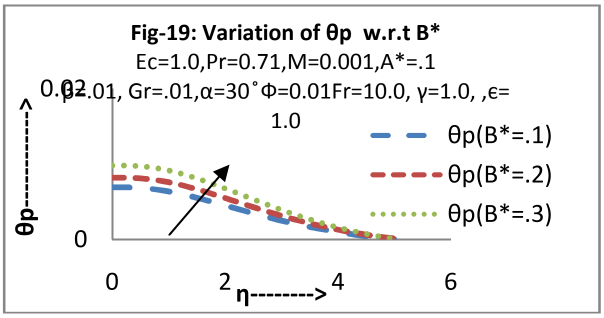

Figure 2 & 3 depict the velocity profiles u_p and the temperature profiles $\theta_p$versus η for different values of M. This figures show that velocity u_p and temperature θ_p increase for increasing value of electrical parameter M. Figure-4&5 illustrate the velocity profiles u_p and the temperature profiles θ_p w.r.t. η for different values of β. It is observed that the increase in the value of β produces significant increase in the velocity distribution u_p and temperature distribution θ_p of dust phase.Figure-6&7 depict the velocity profiles u_p and the temperature profiles θ_p versus η for different values of α. The increasing value α produces a significant increase in velocity distribution u_p but no significant change in the temperature distribution θ_p . Figure-8, illustrates the velocity profiles u_p versus η for different values of Ec. There is no significant change in u_p with the increasing value of Eckert number Ec. Figure 9 & 10, illustrate temperature profiles θ and θ_p of both phases w.r.t. η for different values of Ec. The fluid temperature distribution increases but dust temperature θ_p decreases for the increasing value of Eckret number Ec. The velocity distribution u_p decreases with the increasing of Pr which is illustrated in figure-11.From figure-12and 13 we observed that the fluid phase temperature θ as well as particle phase temperature θ_p decreases with the increase value of Prandtl number Pr. The velocity distribution u_p of particle phase increases with the increase A* which is demonstrated in figure-14. From figure-15 and 16 we observed that both fluid phase temperature θ and particle phase temperature θ_p increase with increasing values of A*. It is observed from figure-17 that the velocity distribution u_p increases with increasing value of B*.From figure-18 can be observed that the thermal boundary layer thickness increases with the increasing of B*.Figure-19, demonstrates the increasing the particle phase temperature θ_p with the increasing value of B*.

Table 1: We investigate the comparison value of $\theta'(0)$ for various values of Pr and *B**when *β*= 0, Gr = 0, *A** = 0, *Q*/*M* = 0, and Ec = 0

B*	Pr	Tsai etal (24)	Ramesh et.al(5)	Present study θ'(0)
-2	2	-2.4859	-2.4859	-2.4860
-3	3	-3.0281	-3.0281	-3.0280
-4	4	-3.5851	-3.5851	-3.5852

IV. Conclusion

In this paper, numerical analysis is presented to investigate the steady boundary flow and heat transfer of a dusty fluid over an inclined stretching sheet with heat source/sink. Numerical solutions are obtained by using shooting technique and FORTRAN software. Influence of physical parameters found to effect the problem under consideration are the fluid particle interaction parameter, local Grashof number, Prandtl number and Eckert number, electric parameter, volume fraction and non-uniform heat source/sink. On this basis of the above study we have the following observations:

1. The velocity of dust phase increases with increasing value of β, Ec, Gr, M B*, A* and α but decreases with Pr.
2. No significant change in particle phase velocity u_p with increasing value of Ec,
3. The fluid phase temperature θ decreases with increasing value Pr but increases with B*, A* and Ec.
4. The particle phase temperature θ_p increases with increasing value of β, A*, B*, and M but decreases with Ec and Pr.
5. No significant change in temperature profile θ_p with increasing value of Gr and α.
6. The values of wall velocity gradient $f''(0)$ increases with the increase of Ec, Gr, M and B* but decreases with the increase of Pr, A* and α.
7. The rate of heat transfer decreases with the increase of α, Ec, A* and B* but increases with the increase M, Pr and Gr.
8. We have investigated the problem assuming the values, $\varphi = 0.01, \epsilon = 1.0$, γ=1.0 and $F_r = 10.0$.

Table-2. Values of wall velocity gradient -$f''(0)$, temperature gradient -$\theta'(0)$, F(0), G(0), H(0) and $\theta_p(0)$ for different values of β, E_c, G_r, P_r, M, A*, B* and α.

β	Ec	Pr	Gr	M	A*	B*	α	$-f''(0)$	F(0)	-G(0)	H(0)	$-\theta'(0)$	θp(0)
.01	1.0	0.71	0.01	.001	0.1	0.1	30˚	0.998425	0.07794	0.95313	.199321	.699036	-.00187
.02								0.997869	0.012273	0.989472	.195735	.697655	.005824
.03								0.998347	0.017744	0.977508	.193557	.701825	.009315
.01	1.0	0.71	0.01	.001	0.1	0.1	30˚	0.996818	0.023672	1.007141	.193680	.700435	.006934
	2.0							0.996278	0.02348	1.007711	.189671	.441411	.00392

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β	Ec	Pr	Gr	M	A*	B*	α	$-f''(0)$	F(0)	-G(0)	H(0)	$-\theta'(0)$	$\theta p(0)$
	3.0							0.995761	0.023836	1.011457	.190150	.183510	.000826
.01	1.0	0.1	0.01	.001	0.1	0.1	30˙	0.993490	0.026459	1.002459	.189054	.069895	.007259
		0.71						0.996818	0.023672	1.007141	.193680	.700435	.006934
		1.0						0.997265	0.023647	1.007597	.192085	.829723	.00453
.01	1.0	0.71	.010	.001	0.1	0.1	30˙	0.996818	0.023672	1.007141	.193680	.700435	.006934
			.015					0.994161	0.024175	1.010633	.196013	.702025	.006909
			.017					0.993108	0.023664	1.013176	.196136	.703016	.006818
.01	1.0	0.71	0.01	000	0.1	0.1	30˙	0.997137	0.002316	0.997870	.206225	.696256	.004381
				.001				0.996818	0.023672	1.007141	.193680	.700435	.006934
				.002				0.996465	0.035657	0.018091	.184578	.700048	.006827
.01	1.0	0.71	0.01	.001	0.1	0.1	30˙	0.996818	0.023672	1.007141	.193680	.700435	.006934
					0.2			1.002056	0.023561	1.005895	.194083	.634620	.007441
					0.3			1.002060	0.023614	1.009998	.190813	.573766	.008119
.01	1.0	0.71	0.01	.001	0.1	0.1	30˙	0.996818	0.023672	1.007141	.193680	.700435	.006934
						0.2		0.996783	0.023621	1.012408	.193500	.613572	.00818
						0.3		0.995864	0.022819	1.014546	.191706	.509773	.007959
.01	1.0	0.71	0.01	.001	0.1	0.1	30˙	0.996818	0.023672	1.007141	.193680	.700435	.006934
							60˙	0.999049	0.025212	0.998500	.198386	.698349	.006941
							90˙	1.002109	0.023295	1.012162	.197351	.696055	.006921

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