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A DEA Model to Determine the Relative Weights of Evaluation Criteria and TOPSIS to Rank the Alternatives

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Abstract **– This paper processes a unification of TOPSIS and Data Envelopment Analysis (DEA) to select the units with most efficiency. This research is a two-stage modeled signed to fully rank the organizational alternatives, where each alternative has multiple inputs and outputs. First, the new methodology based on regression analysis to seek a common set of weights is applied to determine the relative weights of the evaluation criteria .In the second stage, the extension of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank the alternatives. Numerical examples are provided to illustrate the applications of the DEA models and the common set of weights in DEA ranking and evaluation DEA- TOPSIS ranking.**

Keywords **– CCR model, Common Set of Weights, DEA, MCDM, TOPSIS.**

I. Introduction

Data Envelopment Analysis (DEA) measuring the relative efficiency of peer decision-making units (DMUs) with multiple inputs and multiple outputs was introduced by Charges et al. [4]. This method is based on linear programming (LP), which gives it the ability to measure the decision units in a relative manner, though it has difficulties in measuring different scales and more than one scale, as well as in comparing entries or outputs that are in different units. Multi-Criteria Decision-Making (MCDM) is a modeling and methodological tool for dealing with complex engineering problem. However, the MCDM literature was entirely separate from DEA research until 1988, when Golany combined interactive, multiple-objective linear programming and DEA. Whilst the MCDM literature does not consider a complete ranking as their ultimate aim, they do discuss the use of preference information to further refine the discriminatory power of the DEA models. In this manner, the decision-makers could specify However; this could also be considered the weakness of this method, since additional knowledge on the part of the decision-makers is required. Golany [10], Kornbluth [13], Golany and Roll [11] each incorporated preferential information into the DEA models through, for example, a selection of preferred input/output targets or hypothetical DMUs. A separate set of papers reflected preferential information through limitations on the values of the weights, which can almost guarantee a complete DMU ranking [1]. DEA has been applied to DMUs in various forms, such as hospitals, cities, universities, business firms, and many others [5]. During the last decade, there have been attempts to fully rank units in the context of DEA. Cook and Kress [6], Cook et al. [7], and Green et al. [12] used subjective decision analysis. Norman and Stoker [15] asserted a step-by-step approach that uses the selected simple ratios between input and output couples. Ganley and Cubbin [9] improved the common weights, which maximizes the efficiency rates for all units. Sinuany-Stern et al. [19] ordered all units by using linear discriminated analysis that is based on the given DEA dichotomic classification. Friedman and Sinuany-Stern [8] used canonical correlation analysis (CCA/DEA) to order the units that are fundamental in common weights. Friedman and Sinuany-Stern [18] developed the discriminate analysis of ratios instead of traditional linear discriminate analysis. Also (DR/DEA) Oral et al. [16] used the cross-efficiency matrix for choosing R and D projects. There are deficiencies in all methods related to the nature of the methods themselves. Some of the deficiencies occur due to human faults, and some occur due to the presence of a large number of options. The DEA is a method for mathematically comparing difference in DMUs' productivity based on multiple inputs and outputs. The ratio of weighted inputs and outputs produces a single measure of productivity called relative efficiency. The DMUs that have a ratio of 1 are referred to as "efficient", given the required inputs and produced outputs. The units that have a ratio less than 1 are "less efficient" relative to the most efficient units. Because the weights for the input and the output variables of DMU's are computed to maximize the ratio and then compared to a similar ratio of the best-performing DMU's, the measuredproductivity is also referred to as "relative efficiency" [17]. DEA deals with classifying the units into two categories, efficient and in efficient, based on two sets of multiple outputs contributing positively to the overall evaluation [9] ,[23].The original DEA does not perform full ranking; it merely provides classification into two dichotomist groups: efficient and inefficient. It does not rank them; all efficient units are equally good in the pareto sense. In this study, our model integrates two well-known models, DEA and TOPSIS. The rest of the paper is organized as follows: Section 2,3 describes the literature review, Section 4 explains the materials and methods, Section 5 Applying the methodology: an Illustrative Problem.

II. CCR model for Target Efficiencies

Suppose there are n DMUs to be evaluated against m inputs and s outputs. Denote by x_{ij} (i=1,…,m)and y_{rj} (r=1,…,s)the input and output values of DMU_j(j=1,…,n) whose efficiencies are defined as

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$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad j = 1, \dots, m \quad (1)$$

Where v_i $(i = 1, \dots, m)$and u_r $(r = 1, \dots, s)$are input and output weights. Consider a DMU, say, DMU_k , $k \in \{1, \dots, n\}$, whose efficiency relative to the other DMUs can be measured by the following
CCR model [4]:

$$\text{Max} \quad \theta_k = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}$$
$$\text{S.t: } \theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad j = 1, \dots, n \quad (2)$$
$$u_r \ge 0, \quad r = 1, \dots, s,$$
$$v_i \ge 0 \quad i = 1, \dots, m,$$

Which aims to find a set of input and output weights that is most favorable to DMU_k .By using Charnes and Cooper transformation, model (2) can be equivalently transformed into the linear program (LP) below for solution:

$$\text{Max } \theta_k = \sum_{r=1}^{s} u_r y_{rk}$$
$$\text{S.t: } \sum_{i=1}^{m} v_i x_{ik} = 1 \quad (3)$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$$
$$u_r \ge 0, \quad r = 1, \dots, s,$$
$$v_i \ge 0 \quad i = 1, \dots, m,$$

Let u^*_r (r=1,…,s)and v^*_i (i=1,…,m) be the optimal solution to the above model. Then, $\theta^*_k = \sum_{r=1}^{s} u^*_r y_{rk}$ is referred to as the CCR-efficiency of DMU_k .

III. A DEA Model for Common Weights

Let θ^*_j (j =1, ..., n) be the target efficiencies of the n DMUs and $\theta_j$be the efficiencies computed by Eq. (3) with common weights. From the angle of regression analysis, θ_j defined by Eq. (1) can be seen as a nonlinear regression equation. It is most desirable thatθ_j be able to fit the target efficiency θ^*_j perfectly. That is,

$$\theta_j^* - \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} = 0 \quad j = 1, \dots, n \quad (4)$$
$$\sum_{r=1}^{s} u_r y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i x_{ij} = 0, \quad j = 1, \dots, n \quad (5)$$

However, since the target efficiencies are computed with different sets of input and output weights, it is nearly impossible that they can be perfectly fitted by the efficiencies computed with common weights unless there are only one input and one output. That is to say, the above Eqs. (4) and (5) do not hold in multiple input and multiple output cases. It is very natural that the two different efficiencies are expected to be as close to each other as possible. This enables us to build the following two new models for estimating the common weights for the n DMUs:

$$\text{Min} \quad Z = \sum_{j=1}^{n}\left(\theta_j^* - \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij}\right)^2 \quad (6)$$
$$\text{S.t:} u_r \ge 0 \quad r = 1, \dots, s$$
$$v_i \ge 0 \quad i = 1, \dots, m$$

$$\text{Max} \quad J = \sum_{j=1}^{n}\left(\sum_{r=1}^{s} u_r y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i x_{ij}\right)^2 \quad (7)$$
$$\text{S.t:} \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj}\right) + \sum_{i=1}^{m} v_i \left(\sum_{i=1}^{n} xij\right) = n$$
$$u_r \ge 0 \quad r = 1, \dots, s$$
$$v_i \ge 0 \quad i = 1, \dots, m$$

Model (6) is a direct fitting to the target efficiencies and the common weights are derived by minimizing the fitting errors between the two different efficiencies. Model (6) is an indirect fitting to the target efficiencies and requires an additional normalization constraint $\sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj}\right) + \sum_{i=1}^{m} v_i \left(\sum_{i=1}^{n} xij\right) = n$ to avoid the trivial solution, i.e. $u_r = 0$ and $v_i = 0$ for all r =1, ..., s and i =1, ..., m. The two new models are both solved to produce alternative common weights for fully ranking DMUs.

IV. DEA-TOPSIS Method

The priorities obtained from DEA and TOPSIS method are defined as a seven-step approach. The procedure for DEA and TOPSIS ranking model has been given as follows:

Step 1: In the first step, determine the common weights of (6 or 7) models.

Step2: Determine the weight (importance) of j_{th}criterion.

$$w_r = \frac{u_r}{\sum u_r + \sum v_i} r = 1, \dots, s, w_i = \frac{v_i}{\sum u_r + \sum v_i} i = 1, \dots, m (8)$$

Step 3: Develop the weighted decision matrix.

$$X_{ij} = w_i x_{ij}, \; Y_{rj} = w_r y_{rj} \quad (9)$$

Step 4: Determine the positive and the negative ideal solutions.

$$A^+ = \{X_1^+, \dots, X_m^+, Y_1^+, \dots, Y_r^+\}, \text{ where}$$
$$X_i^+ = \min(X_{ij}), Y_r^+ = \max(Y_{rj}) \quad (10)$$
$$A^- = \{X_1^-, \dots, X_m^-, Y_1^-, \dots, Y_r^-\}, \text{ where}$$
$$X_i^- = \max(X_{ij}), Y_r^- = \min(Y_{rj}) \quad (11)$$

Step 5: Calculate the separation measures for each alternative.

The separation from the ideal solution is computed as below:

$$s_j^+ = \sqrt{\sum\left(X_i^+ - X_{ij}\right)^2 + \sum\left(Y_r^+ - Y_{rj}\right)^2} \quad j=1,\dots,n \quad (12)$$

Similarly, the separation from the negative ideal solution is estimated using the following equation:

$$s_j^- = \sqrt{\sum\left(X_i^- - X_{ij}\right)^2 + \sum\left(Y_r^- - Y_{rj}\right)^2} \quad j=1,\dots,n \quad (13)$$

Step 6: Calculate the relative closeness to the ideal solution, C_j^*

$$C_j^* = s_j^- \Big/ (s_j^- + s_j^+) \quad (14)$$

Where$0 \le C_j^* \le 1$ and it is called the closeness coefficient.

Step 7: Select the alternative with C_j^*value closest to one. Ranking is done on the basis of descending order of the closeness coefficient values, i.e. the alternative with the highest closeness coefficient is assigned the topmost

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rank and the one with the lowest closeness coefficient is given the lowermost rank.

V. Numerical Examples

In this section, we provide numerical examples to illustrate the applications of the DEA model and the common weights evaluation in DEA ranking and evaluation DEA-TOPSIS ranking.

Example 1. Consider the numerical example provided by Andersen and Petersen [2] who proposed a super-efficiency procedure for ranking DMUs. The example is involved with five DMUs, two inputs and one virtual output, whose values were reproduced in Table 1. Banker and Chang [3] showed that the super efficiency procedure was for outlier detection rather than for ranking. Wang et al. [21] demonstrated that the ranking obtained by the super-efficiency procedure could not be realized by a common set of weights.

Table 1: Input and output data for five DMUs and their target and super efficiencies.

DMU	I_1	I_2	O	CCR efficiency	Super efficiency
1	2	12	1	1	1
2	2	8	1	1	1.36
3	5	5	1	1	1.2
4	10	4	1	1	1.25
5	10	6	1	.75	.75

To produce a full and realizable ranking for the five DMUs, we solve models (6) and (7) to get two alternative sets of common weights, which are shown in Table 2.Common set of weights is applied to determine the relative weights of the evaluation criteria and the extension of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank the alternatives,which are shown in Table 3.The efficiencies of the five DMUs computed with the common weights in Table 2 are presented in Table 3. As is clear from the table, both models (6) and (7) and TOPSIS successfully lead to a full and the same ranking for the five DMUs, that is, DMU3 $\geq$ DMU2 $\geq$ DMU4 $\equiv$ DMU1 $\geq$ DMU5 , where the symbol means that the former performs better than the latter.

Table 2: Common weights in Example 1 derived by different models.

Model	Input1	Input2	Output
6	.192705	.232416	2.467645
7	.038655	.041300	.486699

Table 3: Efficiencies of the five DMUs computed with the common weights in Table 2 and Topsis.

DMU	Model6	Model7	DEA-TOPSIS
1	.7774(4)	.8495(4)	.5020(3)
2	1.0993(2)	1.1937(2)	.685(2)
3	1.1609(1)	1.2174(1)	.71(1)
4	.8638(3)	.8821(3)	.497(4)
5	.7429(5)	.7672(5)	.4146(5)

Example 2. Six nursing homes (DMUs) were evaluated in terms of two inputs and two outputs that were defined below:

Table 4: Input and output data for six nursing homes and their target efficiencies.

DMU	x_1	x_2	y_1	y_2	CCRfficiency
A	150	.2	14000	3500	1
B	400	.7	14000	21000	1
C	320	1.2	42000	10500	1
D	520	2.0	28000	42000	1
E	350	1.2	19000	25000	.9775
F	320	.7	14000	15000	.8675

StHr (x_1): staff hours per day, including nurses, physicians, etc.

Supp (x_2): suppliers per day, measured in thousands of dollars.

MCPD (y_1): total Medicare-plus Medicaid-reimbursed patient days.

PPPD (y_2): total privately paid patient days.

The input and output data for the six nursing homes that are takenrom Sexton et al. [20] are reproduced together with the target efficiencies of the six nursing homes in Table 4, from which it is seen that the traditional CCR model evaluates nursing homes A–D all as DEA efficient and cannot discriminate them from one another. For this example, we solve models (6) and (7) and DEA-Topsis, respectively, to obtain alternative sets of common weights, which are shown in Table 5. The corresponding efficiencies computed from these alternative common weights are provided in Table 7, from which it is seen that the common weights obtained by models (6) and (7) andDEA-TOPSIS produce a full ranking for the six nursing homes. It is also noted that non-DEA efficient nursing home E is ranked higher than DEA efficient nursing homes B and C. This makes sense according to previous research (Friedman & Sinuany-Stern, [8]; Liu & Peng [14]; Sinuany-Stern & Friedma[18]). When input and output weights cannot be selected very freely, some of DEA efficient units will be no longer DEA efficient and receive lower efficiencies than some of non-DEA efficient units. In DEA-TOPSISmethod,efficient nursing home A is ranked lower than efficient nursing homes D and E and B the reason can be related to the size of this unite in comparison to others .

Table 5: Common weights in Example 1 derived by different models.

Model	x_1	x_2	y_1	y_2
6	0.009684	2.948961	0.000101	0.000201
7	0.000680	0.271169	8.87E-06	1.55E-05

Table 6: The weighted and distance measurementsix nursing homes

DMU	x_1	x_2	y_1	y_2	s_i^+	s_i^-
A	.4905	.2	.4774	.2376	2.78	1.5696
B	1.308	.7	.4774	1.4259	1.96	1.3476
C	1.0464	1.2	1.4322	.7129	2.425	1.25
D	1.7004	2.0	.9548	2.8518	2.16	2.7
E	1.1445	1.2	.6479	1.6975	1.836	1.49
F	1.308	.7	.4774	1.0185	2.278	1.0068

Table 7: Efficiencies of the six nursing homes computed with the common weights in Table 5and their rankings.

DMU	Model6	Model7	DEA-Topsis
A	1.0382(1)	1.1419(1)	.36(4)
B	.9490(5)	.9730(5)	.407(3)
C	.9584(4)	.9855(4)	.327(5)
D	1.0308(2)	1.0030(2)	.55(1)
E	1.0024(3)	.9862(3)	.4479(2)
F	.8580(6)	.8749(6)	.3064(6)

VI. Conclusion

The present study explored the use of DEA and TOPSIS in solving a MCDM problem. This study aimed at searching an improved solution to MCDM problems. When the criteria weights and performance ratings are vague and inaccurate, then the DEA-TOPSIS are the preferred techniques.For this purpose, we use DEA to determine the relative weights of evaluation criteria and TOPSIS to rank the alternatives.In addition, there exists other worth investigating MADM methods for a MADM problem. This becomes one of the future research opportunities in this classical, yet important, research area.

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