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Certain Identities in Generalized R^h-Recurrent Finsler Space

Fahmi Yaseen Abdo Qasem
fahmiyaseen@hotmail.com,
fyaqasem@gmail.com

Adel Mohammed Ali AL-Qashbari
Dept.of Maths., Faculty of Educ. -Aden, Univ. of Aden,
Khormaksar, Aden, Yemen Adel_ma71@yahoo.com

***Abstract* - In this paper we define a Finsler space F_n in which Cartan's third curvature tensor R^i_{jkh} satisfies the generalized of recurrence condition with respect to Cartan's convection parameters Γ^{*i}_{kh} which given by the condition $R^i_{jkh|l} = \lambda_l R^i_{jkh} + \mu_l(\delta^i_h\, g_{jk} - \delta^i_k\, g_{jh})$, where $|l$ is h-covariant differentiation, λ_l and μ_l are non-null covariant vectors field is introduced and such space is called as *a generalized R^h-recurrent space* and denote it briefly by $G\,H^h$-$R\,F_n$. The Ricci tensor R_{kh} , the curvature vector R_k and the curvature scalar R of such space are non-vanishing. Finally, we obtained certain identities for a generalized R^h-recurrent space.**

***Keywords* - Finsler space, Generalized R[h] - Recurrent Space, Ricci Tensor.**

I. Introduction

Due the different connections of Finsler space the recurrent of different curvature tensors have been discussed by S. Dikshit [2], R.S.D. Dubey and A.k. Srivastava [3], P.N. Pandey ([7], [8]), R. Verma [16], M. Matsumoto [5], C.K. Mishra and G.Lodhi [6], P.N. Pandey and V.J. Dwivedi [10], P.N. Pandey and R.B. Misra [9], P.N. Pandey and S. Pal [11], Y.B. Maralebhavi and M. Rathnamma [4], P.N. Pandey, S. Saxena and A. Goswani [12], F.Y.A. Qasem and A.M.A. AL-Qashbari [14] and others.

Let us consider an n-dimensional Finsler space equipped with the metric function F satisfying the requisite conditions [15].

Let consider the components of the corresponding metric tensor g_{ij}, Cartan's connection parameters Γ^{*i}_{jk} and Berwald's connection parameters G^i_{jk} *. These are symmetric in their lower indices and positively homogeneous of degree zero in the directional arguments.

The two sets of quantities g_{ij} and its associate tensor g^{ij} are related by

(1.1) $g_{ij}\, g^{jk} = \delta^k_i = \begin{cases} 1\,, & if \quad i = k \;, \\ 0\,, & if \quad i \neq k \;. \end{cases}$

The vectors y_i and y^i satisfies the following relations

(1.2) a) $y_i = g_{ij}\, y^j$,

b) $y_i\, y^i = F^2$,

c) $g_{ij} = \dot\partial_i\, y_j = \dot\partial_j\, y_i$,

d) $g_{ij}\, y^j = \frac{1}{2}\, \dot\partial_i\, F^2 = F\, \dot\partial_i\, F$ and e) $\dot\partial_j\, y^i = \delta^i_j$.

The tensor C_{ijk} ^{*2} defined by

(1.3) $C_{ijk} = \frac{1}{2}\, \dot\partial_i\, g_{jk} = \frac{1}{4}\, \dot\partial_i\, \dot\partial_j\, \dot\partial_k\, F^2$

is known as *(h) hv - torsion tensor* [5] . It is positively homogeneous of degree -1 in the directional arguments and symmetric in all its indices.

The (v) hv-torsion tensor C^h_{ik} and its associate (h) hv-torsion tensor C_{ijk} are related by

(1.4) a) $C_{ijk} y^i = C_{kij} y^i = C_{jki} y^i = 0$,

b) $C^i_{jk}\, y^j = C^i_{kj}\, y^j = 0$

and c) $C^h_{ik} := g^{hj}\, C_{ijk}$.

The (v) hv-torsion tensor C^h_{ik} is also positively homogeneous of degree -1 in the directional arguments and symmetric in its lower indices.

É. Cartan deduced the h-covariant derivative for an arbitrary vector filed X^i with respect to x^k [15]

(1.5) $X^i_{|k} := \partial_k\, X^i - (\,\dot\partial_r\, X^i\,)\, G^r_k + X^r\, \Gamma^{*i}_{rk}$.

The metric tensor g_{ij} and the vector y^i are covariant constant with respect to a above process, i.e.

(1.6) a) $g_{ij|k} = 0$,

b) $y^i_{|k} = 0$

and c) $g^{ij}_{|k} = 0$.

The process of h-covariant differentiation with respect to x^k commute with partial differentiation with respect to y^j for arbitrary vector filed X^i , according to

(1.7) $\dot\partial_j \left(X^i_{|k}\right) - \left(\dot\partial_j\, X^i\right)_{|k} = X^r\left(\dot\partial_j\, \Gamma^{*i}_{rk}\right) - \left(\dot\partial_r X^i\right) P^r_{jk}$,

where

(1.8) a) $\dot\partial_j \Gamma^{*r}_{hk} = \Gamma^{*r}_{jhk}$,

b) $P^i_{kh}\, y^k = 0 = P^i_{kh}\, y^h$

and c) $P^i_{jkh}\, y^j = P^i_{kh}$.

The tensor P^i_{kh} is called *v(hv) –torsion tensor* and its associate tensor P_{jkh} is given by

(1.9) a) $g_{rj}\, P^r_{kh} = P_{kjh}$.

The associate tensor P_{ijkh} of the (hv)-curvature tensor P^i_{jkh} is given by

(1.9) b) $g_{ir}\, P^r_{jkh} := P_{ijkh}$.

The quantities H^i_{jkh} and H^i_{kh} form the components of tensors and they called *h-curvature tensor of Berwald* (

$*$ The indices i , j , k , ... assume positive integral values from 1 to n .

$*_2$ Unless stated otherwise. Henceforth all geometric objects are assumed to be functions of line-elements.

Berwald curvature tensor) and *h(v) –torsion tensor,* respectively, and defined as follow:

(1.10) a) $H^i_{jkh} := \partial_j G^i_{kh} + G^r_{kh} G^i_{rj} + G^i_{rhj} G^r_k - h/k *$

and

b) $H^i_{kh} := \partial_h G^i_k + G^r_k C^i_{rh} - h/k$.

They are skew-symmetric in their lower indices, i.e. k and h . Also they are positively homogeneous of degree zero and one, respectively in their directional arguments. They are also related by

(1.11) a) $H^i_{jkh}\, y^j = H^i_{kh}$,

b) $H^i_{jkh} = \partial_j H^i_{kh}$

and c) $H^i_{jk} = \partial_j H^i_k$.

These tensors were constructed initially by mean of the tensor H^i_h , called the *deviation tensor*, given by

(1.12) $H^i_h := 2\,\partial_h G^i - \partial_r G^i_h\, y^r + 2\, G^i_{hs} G^s - G^i_s G^s_h$.

The deviation tensor H^i_h is positively homogeneous of degree two in the directional arguments.

In view of Euler's theorem on homogeneous functions and by contracting the indices i and h in (1.11) and (1.12), we have the following:

(1.13) a) $H^i_{jk}\, y^j = -H^i_{kj}\, y^j = H^i_k$

and b) $y_i H^i_j = 0$.

The quantities H^i_{jkh} and H^i_{kh} are satisfies the following [15]

(1.14) a) $H_{ijkh} := g_{jr} H^r_{ihk}$,

b) $H_{jk.h} := g_{jr} H^r_{hk}$

and c) $y_i H^i_{jk} = 0$.

Cartan's third curvature tensor R^i_{jkh} satisfies the identity known as Bianchi identity [15]

(1.15) a) $R^i_{jkh|s} + R^i_{jsk|h} + R^i_{jhs|k} + (R^r_{mhs} P^i_{jkr} + R^r_{mkh} P^i_{jsr} + R^r_{msk} P^i_{jhr})\, y^m = 0$

and

b) $R_{ijhk} + R_{ihkj} + R_{ikjh} + (C_{ijs} K^s_{rhk} + C_{ihs} K^s_{rkj} + C_{iks} K^s_{rjh})\, y^r = 0$,

where

c) $P^r_{ijs} = \dot{\partial}_s \Gamma^{*r}_{ij} - C^r_{is|j} + C^r_{im} C^m_{js|k}\, y^k$.

The Ricci tensor R_{jk} , the deviation tensor R^r_h and the curvature scalar R of the curvature tensor R^i_{jkh} are given by

(1.16) a) $R^i_{jkh}\, y^j = H^i_{hk} = K^i_{jhk}\, y^j$,

b) $R_{ijhk} = g_{rj} R^r_{ihk}$,

c) $R_{jkhm}\, y^j = H_{kh.m}$,

d) $R^r_{ihk} = g^{jr} R_{ijhk}$

and e) $R^i_{jkh}\, g^{jk} = R^i_h$,

* $-h/k$ means the subtraction from the former term by interchanging the indices h and k .

The contracted tensor R_{kh} (Ricci tensor) and R_k (Curvature vector) are also connected by

(1.17) a) $R_{jk}\, y^k = R_j$,

b) $R_{jk}\, y^j = H_k$

and c) $R^i_{jki} = R_{jk}$.

Also this tensor satisfies the following relation too

(1.18) a) $R^i_{jkh} = K^i_{jkh} + C^i_{js} K^s_{rhk}\, y^r$

and b) $R_{ijkh} = K_{ijkh} + C_{ijs} H^s_{kh}$.

where R_{ijkh} is the associate curvature tensor of R^i_{jkh}. Cartan's fourth curvature tensor K^i_{jkh} and its associate curvature tensor of K_{ijkh}satisfy the following known as Bianchi identities

(1.19) a) $K^i_{jkh} + K^i_{hjk} + K^i_{khj} = 0$

and b) $K_{jrkh} + K_{hrjk} + K_{krhj} = 0$.

F.Y.A. Qasem and A.M.A. AL-Qashbari [14] discussed a Generalized H^h- Recurrent space whose Berwald curvature tensor H^i_{jkh} satisfies the generalized recurrence property in the sense of Cartan.

II. On Generalized R^h-Recurrent Space

Let us consider a Finsler space F_n whose Cartan's third curvature tensor R^i_{jkh} satisfies the following condition

(2.1) $R^i_{jkh|l} = \lambda_l R^i_{jkh} + \mu_l (\delta^i_h g_{jk} - \delta^i_k g_{jh})$, $R^i_{jkh} \neq 0$,

where λ_l and μ_l are non-null covariant vectors field. We shall call such space as *a generalized R^h-recurrent space.*

We shall denote it briefly by $G\,R^h$- $R\,F_n$.

Transvecting of (2.1) by the metric tensor g_{ip} , using (1.6a), (1.16b) and in view of (1.1), we get

(2.2) $R_{jpkh|l} = \lambda_l R_{jpkh} + \mu_l (g_{hp}\, g_{jk} - g_{kp}\, g_{jh})$.

Conversely, the transvection of the condition (2.2) by the associate tensor g^{ip} of the metric tensor g_{ip} , yields the condition (2.1). Thus, the condition (2.2) is equivalent to the condition (2.1). Therefore a generalized R^h-recurrent space characterized by the condition (2.2).

Therefore, we have

Theorem 2.1. *An $G\,R^h$- $R\,F_n$ may characterized by the condition (2.2).*

Let us consider $G\,R^h$- $R\,F_n$ characterized by the condition (2.2).

Transvecting the condition (2.1) by y^j, using (1.6b), (1.16a) and (1.2a), we get

(2.3) $H^i_{kh|l} = \lambda_l H^i_{kh} + \mu_l (\delta^i_h\, y_k - \delta^i_k\, y_h)$.

Further, transvecting (2.3) by y^k, using (1.6b), (1.13a), (1.2b) and in view of (1.1), we get

(2.4) $H^i_{h|l} = \lambda_l H^i_h + \mu_l (\delta^i_h F^2 - y_h\, y^i)$.

Therefore, we have

Theorem 2.2. *In $G\,R^h$-$R\,F_n$, the h-covariant derivative of the h(v)-torsion tensor H^i_{kh} and the deviation tensor H^i_h is given by the conditions (2.3) and (2.4), respectively .*

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Contracting the indices i and h in the condition (2.1), using (1.17c) and (1.1), we get

(2.5) $R_{jk|l} = \lambda_l R_{jk} + (n-1)\mu_l g_{jk}$.

Transvecting (2.5) by y^k , using (1.6b), (1.17a) and (1.2a), we get

(2.6) $R_{j|l} = \lambda_l R_j + (n-1)\mu_l y_j$.

Further, transvecting the condition (2.1) by the associate tensor g^{jk} of the metric tensor g_{jk} , using (1.6c), (1.16e) and in view of (1.1), we get

(2.7) $R^i_{h|l} = \lambda_l R^i_h + \mu_l (n-1)\delta^i_h$.

Contracting the indices i and h in condition (2.7) and using (1.1), we get

(2.8) $R_{|l} = \lambda_l R + n(n-1)\mu_l$.

where $R^r_r = R$.

The conditions (2.5), (2.6), (2.7) and (2.8), show that the Ricci tensor R_{jk} , the curvature vector R_j , the deviation tensor R^i_h and the curvature scalar R of a generalized R^h-recurrent space can not vanish, because the vanishing of them imply the vanishing of the covariant vector field μ_l , i.e. $\mu_l = 0$, a contradiction.

Thus, we conclude

Theorem 2.3. *In $G R^h$-$R F_n$, the Ricci tensor R_{jk} , the curvature vector R_j , the deviation tensor R^i_h and the curvature scalar R are non-vanishing.*

III. Certain Identities

In this section we shall obtain some identities in $G R^h$-$R F_n$.

Taking h-covariant differentiation of the formula (1.15b) with respect to x^l in the sense of Cartan and transvecting (1.15b) by the associate tensor g^{jp} of the metric tensor g_{jp} , using (1.6c), (1.16a), (1.16d) and (1.4a), we get

(3.1) $$R^p_{ihk|l} + g^{jp} R_{ihkj|l} + g^{jp} R_{ikjh|l} + \left(C^p_{is} H^s_{hk} + g^{jp} C_{ihs} H^s_{kj} + g^{jp} C_{iks} H^s_{jh} \right)_{|l} = 0 \quad .$$

Transvecting (3.1) by y^i, using (1.6b), (1.16a), (1.16c), (1.4b) and (1.4a), we get

(3.2) $$H^p_{hk|l} + g^{jp} H_{hk.j|l} + g^{jp} H_{kj.h|l} = 0 \quad .$$

Thus, we conclude

Theorem 3.1. *In $G R^h$-$R F_n$, the identities (3.1) and (3.2) hold good.*

Using (1.16b) and (1.16a) in the identity (1.15b), we get

(3.3) $$g_{rj} R^r_{ihk} + g_{rh} R^r_{ikj} + g_{rk} R^r_{ijh} + C_{ijs} H^s_{hk} + C_{ihs} H^s_{kj} + C_{iks} H^s_{jh} = 0 \quad .$$

Now, transvecting (3.3) by y^i, using (1.16a) and (1.4b), we get

(3.4) $g_{rj} H^r_{hk} + g_{rh} H^r_{kj} + g_{rk} H^r_{jh} = 0$.

By using (1.14b), the equation (3.4) yields to

(3.5) $H_{hj.k} + H_{kh.j} + H_{jk.h} = 0$.

Transvecting (3.4) by y^h , using (1.13a) and (1.2a), we get

(3.6) $g_{rj} H^r_k = g_{rk} H^r_j$.

Thus, we conclude

Theorem 3.2. *In $G R^h$-$R F_n$, the identities (3.4), (3.5) and (3.6) hold good.*

Using the equation (1.16a) in the identity (1.15a), we get

(3.7) $$R^r_{ijk|h} + R^r_{ihj|k} + R^r_{ikh|j} + \left(H^s_{kh} P^r_{ijs} + H^s_{jk} P^r_{ihs} + H^s_{hj} P^r_{iks} \right) = 0 \quad .$$

In view of the condition (2.1), the identity (3.7), may be written as

(3.8) $$\lambda_h R^r_{ijk} + \lambda_k R^r_{ihj} + \lambda_j R^r_{ikh} + \mu_h(\delta^r_k g_{ij} - \delta^r_j g_{ik}) + \mu_k(\delta^r_j g_{ih} - \delta^r_h g_{ij}) + \mu_j(\delta^r_h g_{ik} - \delta^r_k g_{ih}) + \left(H^s_{kh} P^r_{ijs} + H^s_{jk} P^r_{ihs} + H^s_{hj} P^r_{iks} \right) = 0 \quad .$$

Transvecting (3.8) by y^i , using (1.16a), (1.2a) and (1.8c), we get

(3.9) $$\lambda_h H^r_{jk} + \lambda_k H^r_{hj} + \lambda_j H^r_{kh} + \mu_h(\delta^r_k y_j - \delta^r_j y_k) + \mu_k(\delta^r_j y_h - \delta^r_h y_j) + \mu_j(\delta^r_h y_k - \delta^r_k y_h) + \left(H^s_{kh} P^r_{js} + H^s_{jk} P^r_{hs} + H^s_{hj} P^r_{ks} \right) = 0 \quad .$$

Transvecting (3.9) by y^j, using (1.13a), (1.2b), (1.1) and (1.8b), we get

(3.10) $$\lambda_h H^r_k - \lambda_k H^r_h + \lambda H^r_{kh} + \mu_h(\delta^r_k F^2 - y_k y^r) + \mu_k(y_h y^r - \delta^r_h F^2) + \mu(\delta^r_h y_k - \delta^r_k y_h) + \left(H^s_k P^r_{hs} - H^s_h P^r_{ks} \right) = 0 \quad ,$$

where $\lambda_j y^j = \lambda$ and $\mu_j y^j = \mu$.

Thus, we conclude

Theorem 3.3. *In $G R^h$-$R F_n$, the identities (3.8), (3.9) and (3.10) hold good.*

Further, transvecting (3.9) and (3.10) by the vector y_r , using (1.14c), (1.1),(1.13b) and (1.2b), we get

(3.11) $$\left(H^s_{kh} P^r_{js} + H^s_{jk} P^r_{hs} + H^s_{hj} P^r_{ks}\right) y_r = 0$$

and

(3.12) $H^s_k y_r P^r_{hs} = H^s_h y_r P^r_{ks}$.

respectively. Thus, we conclude

Theorem 3.4. *In $G R^h$-$R F_n$, the identity (3.11) holds good.*

Theorem 3.5. *In $G R^h$-$R F_n$, we have the identity (3.12).*

Transvecting (3.8), (3.9) and (3.10) by the metric tensor g_{rm} , using (1.16b), (1.1), (1.9b), (1.14b), (1.9a) and (1.2a), we get

(3.13) $$\lambda_h R_{imjk} + \lambda_k R_{imhj} + \lambda_j R_{imkh} + \mu_h (g_{km} g_{ij} - g_{jm} g_{ik}) + \mu_k (g_{jm} g_{ih} - g_{hm} g_{ij}) + \mu_j (g_{hm} g_{ik} - g_{km} g_{ih}) + \left(H^s_{kh} P_{imjs} + H^s_{jk} P_{imhs} + H^s_{hj} P_{imks} \right) = 0 \quad ,$$

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(3.14) $\lambda_h H_{jm.k} + \lambda_k H_{hm.j} + \lambda_j H_{km.h} + (H^s_{kh} P_{jms} + H^s_{jk} P_{hms} + H^s_{hj} P_{kms}) = 0$

and

(3.15) $g_{rm} (\lambda_h H^r_k - \lambda_k H^r_h + \lambda H^r_{kh}) + \mu_h (g_{km} F^2 - y_k y_m) + \mu_k (y_h y_m - g_{hm} F^2) + \mu (g_{hm} y_k - g_{km} y_h) + (H^s_k P_{hms} - H^s_h P_{kms}) = 0$,

respectively. Thus, we conclude

Theorem 3.6. *In $G R^h$-R F_n, the identities (3.13), (3.14) and (3.15) hold good.*

Using (1.18b) and (1.16a) in the identity (1.15b), we have

(3.16) $K_{ijhk} + K_{ihkj} + K_{ikjh} + 2 (C_{ijs} H^s_{hk} + C_{ihs} H^s_{kj} + C_{iks} H^s_{jh}) = 0$.

In view of (1.19b), the identity (3.18) becomes

(3.17) $C_{ijs} H^s_{hk} + C_{ihs} H^s_{kj} + C_{iks} H^s_{jh} = 0$.

Taking h- covariant differentiation of the identity (3.17) with respect to x^l in the sense of Cartan, we get

(3.18) $C_{ijs|l} H^s_{hk} + C_{ijs} H^s_{hk|l} + C_{ihs|l} H^s_{kj} + C_{ihs} H^s_{kj|l} + C_{iks|l} H^s_{jh} + C_{iks} H^s_{jh|l} = 0$.

Now, transvecting (3.18) by the associate tensor g^{rj}, using (1.6c) and (1.4c), we get

(3.19) $C^r_{is|l} H^s_{hk} + C^r_{is} H^s_{hk|l} + C_{ihs|l} H^s_{kj} g^{rj} + C_{ihs} H^s_{kj|l} g^{rj} + C_{iks|l} H^s_{jh} g^{rj} + C_{iks} H^s_{jh|l} g^{rj} = 0$.

Transvecting (3.18) and (3.19) by y^h , using (1.6b), (1.13a) and (1.4a), we get

(3.20) $C_{ijs|l} H^s_k + C_{ijs} H^s_{k|l} = C_{iks|l} H^s_j + C_{iks} H^s_{j|l}$

and

(3.21 $C^r_{is|l} H^s_k + C^r_{is} H^s_{k|l} = (C_{iks|l} H^s_j + C_{iks} H^s_{j|l}) g^{rj}$,

respectively. Thus, we conclude

Theorem 3.7. *In $G R^h$-R F_n, the identities (3.18), (3.19), (3.20) and (3.21) hold good.*

Further, transvecting (3.18) and (3.19) by g_{rs} , using (1.6a) and (1.14b), we get

(3.22) $C_{ijs|l} H_{hr.k} + C_{ijs} H_{hr.k|l} + C_{ihs|l} H_{kr.j} + C_{ihs} H_{kr.j|l} + C_{iks|l} H_{jr.h} + C_{iks} H_{jr.h|l} = 0$

and

(3.23) $C^r_{js|l} H_{hr.k} + C^r_{js} H_{hr.k|l} + (C_{ihs|l} H_{kr.j} + C_{ihs} H_{kr.j|l} + C_{iks|l} H_{jr.h} + C_{iks} H_{jr.h|l}) g^{rj} = 0$,

respectively. Thus, we conclude

Theorem 3.8. *In $G R^h$-R F_n, the identities (3.22) and (3.23) hold good.*

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