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Common Fixed Point Theorem in for Tree Maps

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***Abstract* – In this paper, we have proved fixed point theorem for three maps.**

***Keyword* – Complete Metric Space, Pair of Maps, Common Point, Fixed Point.**

I. Introduction

Common Fixed Point: Let S and T be a pair of maps of a metric space (X,d) then z is called common fixed point of S and T if Sz = Tz = z and if S , P and T be three maps of metric space. (X,d) then z is called a common fixed point of S, P and T if Sz = Pz = Tz = z.

A great deal of work on common fixed point have been done by Kannan [1] Gupta and Shrivastav [2] Jungck [3] Yen [4] Rus [5] Iseki [6].Fisher [7][8]Ray and Singh [9]Fisher [10]etc have also established results on common fixed point.

In this section we obtain some results on common fixed point.

II. Main Result

Theorem (1):- Let S , P , T are the three maps of a complete metric space (X,d) onto itself satisfied following conditions;

(I) ST = TS , PT = TP , SX⊂TX and PX⊂TX

(II) $$d(Sx,Py) \le \alpha \frac{d(Tx,Sx)\, d(Ty,Py)}{d(Tx,Py)+d(Ty,Sx)+d(Tx,Ty)} + \beta \frac{d(Tx,Ty)\left[1+\sqrt{d(Tx,Ty)d(Tx,Sx)}+\sqrt{d(Tx,Ty)d(Ty,Sx)}\right]}{\left[1+d(Tx,Ty)+d(Tx,Sx)\,d(Tx,Py)\,d(Ty,Sx)\,d(Ty,Py)\right]} + \gamma \frac{d(Ty,Py)\left[1+\sqrt{d(Tx,Py)+d(Ty,Sx)}\right]}{\left[1+\sqrt{d(Tx,Sx)+d(Ty,Py)}\right]}$$

for all x,y Ɛ X and α , β , γ ≥ O and α + β + γ < 1 with Tx ≠ Ty. Then S , P and T have a common fixed point z Ɛ X and z is a unique fixed point of S, P and T .

Proof : Let x_0 is any arbitrary point of a complete metric X since SX⊂TX we can choose a point x_1ƐX such that $Tx_1 = Sx_0$ also PX ⊂ TX , we can choose a point x_2 Ɛ X such that $Tx_2 = Px_1$. In general we can choose the point x_{2n+1} ,x_{2n+2} such that $Tx_{2n+1} = Sx_{2n}$ and $Tx_{2n+2} = Px_{2n+1}$ for n = 0 , 1 , 2 , 3 , ……………….. Now condition (II) we have

$$d(Tx_{2n+1}, Tx_{2n+2}) = d(Sx_{2n}, Px_{2n+1})$$

$$\le \alpha \frac{d(Tx_{2n},Sx_{2n})\, d(Tx_{2n+1},Px_{2n+1})}{d(Tx_{2n},Px_{2n+1})+d(Tx_{2n+1},Sx_{2n})+d(Tx_{2n},Tx_{2n+1}))}$$

$$+ \beta \frac{d(Tx_{2n},Tx_{2n+1})\left[1+\sqrt{d(Tx_{2n},Tx_{2n+1})d(Tx_{2n},Sx_{2n})}+\sqrt{d(Tx_{2n},Tx_{2n+1})d(Tx_{2n+1},Sx_{2n})}\right]}{\left[1+d(Tx_{2n},Tx_{2n+1})+d(Tx_{2n},Sx_{2n})d(Tx_{2n},Px_{2n+1})\, d(Tx_{2n+1},Sx_{2n})\, d(Tx_{2n+1},Px_{2n+1})\right]}$$

$$+ \gamma \frac{d(Tx_{2n+1},Px_{2n+1})\left[1+\sqrt{d(Tx_{2n},Px_{2n+1})+d(Tx_{2n+1},Sx_{2n})}\right]}{\left[1+\sqrt{d(Tx_{2n},Sx_{2n})+d(Tx_{2n+1},Px_{2n+1})}\right]}$$

$$\le \alpha \frac{d(Tx_{2n},Tx_{2n+1})\, d(Tx_{2n},Tx_{2n+1})}{d(Tx_{2n},Tx_{2n+2})+d(Tx_{2n+1},Tx_{2n+1})+d(Tx_{2n},Tx_{2n+1}))}$$

$$+ \beta \frac{d(Tx_{2n},Tx_{2n+1})\left[1+\sqrt{d(Tx_{2n},Tx_{2n+1})d(Tx_{2n},Tx_{2n+1})}+\sqrt{d(Tx_{2n},Tx_{2n+1})d(Tx_{2n+1},Tx_{2n+1})}\right]}{\left[1+d(Tx_{2n},Tx_{2n+1})+d(Tx_{2n},Tx_{2n+1})d(Tx_{2n},Tx_{2n+2})\, d(Tx_{2n+1},Tx_{2n+1})\, d(Tx_{2n+1},Tx_{2n+2})\right]}$$

$$+ \gamma \frac{d(Tx_{2n+1},Tx_{2n+2})\left[1+\sqrt{d(Tx_{2n},Tx_{2n+2})+d(Tx_{2n+1},Tx_{2n+1})}\right]}{\left[1+\sqrt{d(Tx_{2n},Tx_{2n+1})+d(Tx_{2n+1},Tx_{2n+2})}\right]}$$

$$\le \alpha \frac{d(Tx_{2n},Tx_{2n+1})\left[1+d(Tx_{2n},Tx_{2n+1})+d(Tx_{2n},Tx_{2n+2})\right]}{\left[d(Tx_{2n},Tx_{2n+1})+d(Tx_{2n},Tx_{2n+2})\right]}$$

$$+ \beta\, d(Tx_{2n}, Tx_{2n+1}) + \gamma\, d(Tx_{2n+1}, Tx_{2n+2})$$

$$\le (\alpha+\beta)\, d(Tx_{2n}, Tx_{2n+1}) + \gamma\, d(Tx_{2n+1}, Tx_{2n+2})$$

$$d\,(Tx_{2n+1}, Tx_{2n+2}) \le \left[\frac{\alpha+\beta}{1-\gamma}\right] d\,(Tx_{2n+1}, Tx_{2n+1})$$

Similarly , if can be show that

$$d\,(Tx_{2n+1}, Tx_{2n}) \le \left[\frac{\alpha+\beta}{1-\gamma}\right] d\,(Tx_{2n}, Tx_{2n+1})$$

Thus

$$d\,(Tx_{2n+1}, Tx_{2n+2}) \le \left[\frac{\alpha+\beta}{1-\gamma}\right] d\,(Tx_{2n}, Tx_{2n+1})$$

$$\le \left[\frac{\alpha+\beta}{1-\gamma}\right]^{2n+1} d\,(Tx_0, Tx_1)$$

Further

$$d\,(Tx_n, Tx_{n+k}) \le \sum_{i=1}^{k}. \ d\,(Tx_{n+i-1}, Tx_{n+i})$$

$$\le \left[\frac{\alpha+\beta}{1-\gamma}\right]^{n} d\,(Tx_0, Tx_1) \to 0 \text{ as } n \to \infty \text{ as } \alpha+\beta+\gamma < 1$$

Hence $\{Tx_n\}$ is a Cauchy sequence, as X is complete , So it has a limit point since $\{Sx_{2n}\}$ and $\{Px_{2n+1}\}$ are subsequences of $\{Tx_n\}$, they will also converge to the some limit point z Ɛ X.

Now next we prove the fixed point z is unique. Suppose w(≠z) be a another fixed point, such that Sz = Pz = Tz = z and Sw = Pw = Tw = w then we have

d (z,w) = d (Sz , Pw)

d (z.w) = d (Sz , Pw)

$$\le \alpha \frac{d\,(Tz,Sz)\ d\,(Tw,Pw)}{d\,(Tz,Pw)+ d\,(Tw,Sz)+ d\,(Tz,Tw)}$$

$$+ \ \beta \frac{d\,(Tz,Tw)\left[1+ \sqrt{d\,(Tz,Tw)\ d\,(Tz,Sz)} + \sqrt{d\,(Tz,Tw) d\,(Tw,Sz)}\right]}{[\,1+d\,(Tz,Tw)+ d\,(Tz,Sz)\ d\,(Tz,Pw)\ d\ (Tw,Sz)\ d\,(Tw,Pw)\,]}$$

$$+ \ \gamma \frac{d\ (Tw,Pw)\left[1+ \sqrt{d\,(Tz,Pw)+ d\,(Tw,Sz)}\ \right]}{\left[1+ \sqrt{d\,(Tz,Sz)+ d\,(Tw,Pw)}\ \right]}$$

$\le \beta \ d\,(z, w)$

$< d\,(z , w)$ as $\beta < 1$

Which is contradiction . Hence z =w.

III. Conclusion

Aim of this paper prove that fixed point theorems with new untractive condition involving three maps are discussed.

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