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# Revisiting Twin Paradox in the Theory of Relativity

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***Abstract* – Instead of a general type of path, a particular track has been sugested. This consists of two semi-circles and two commontangents. The track was followed by one of the twins. However the journey was recorded by both twins.The recording was found to be totally symmetric even in dynamic conditions. The symmetrical nature of the two tracks recorded by the twins ensure that there is no age gap between twins whenever they meet.**

***Keywords* – Twins, Relativity.**

## I. What Is Twin Paradox?

I quote the Nobel Laureate Richard Fynman[1] from his book that "Peter and Paul who are supposed to be twins, born at same time. …..Peter, who is left on the ground, sees Paul flies away at the very high speed …all of Paul's clocks appear to go slower, his heart beats slower, his thoughts go slower, everything goes slower……… . But he travels around and about for a while and then comes back, he will be younger than Peter, the man on the ground. That isactually right. …………This is called "paradox" only by the people who believe that the principle of relativity means that all motion is relative……they say that from thepoint of view of Paul, can't we say that Peter was moving andtherefore appear to *age*more slowly? By symmetry, the only possible result is that both should be of the same age when they meet.

*Frames of Reference:*

Before we embark on discussion about twin paradox let us be clear about a term to be used as a'frame of reference' [2]. The space designated to describe the location of a particle at a given point of time is called a frame of reference.A covered space in a car, acabin in a train, a room constructed on the surface of earth may be cited as a few examples of such frames They can be classifed as.

(i) Non-inertial fraames
(ii) Inertial frames

As it is obvious from their names, the inertial frames are those frames in whichNewton's first law, the law of inertia, holds good, while in the non-inertial frames it does not hold good.In fig.2all the trains are divided into three types. 'A' trains have velocity gradient equal to zero i.e. their value of velocitiesincluding zero remains constant. Its passengers do not feel any force. As such these trains are frames belonging to inertial frames. Trains whose velocity gradients are not zero their passengers feel a kind of force inside their frames.Such frames are called non-inertial frames.

## II. Behaviour of Inertial Frames

Velocity of an inertial frame keeps changing with respect to the velocities of other inertial frames. Let us consider a case to make the concept clearer. Suppose an observer 'P' is standing on a railway station and watching that three trains 'X','Y' and 'Z' are passing this railway station with velocities 80, 60 and 40 mph. The velocity of railway station with respect to observer P is zero. Thus we can form the first row of the matrix as velocities of P, X,Y and Z observed by P as 0, 80, 60 and 40 mph as given in fig.1. The second row of the matrix is the velocities of P, X, Y and Z as observed by a passenger siting in train X. then.element XX is jero. Similarly 3rd and 4th rows can be obtained making dioganal elements tobe ero. The matrix obtained as such is shown in fig.1

P X Y Z

P 0 80 60 40

X -80 0 -20 - 40

Y -60 20 0 -20

Z –40 40 20 0

Fig. 1. Matrix of relative velocities

Scientists were trying hard to find out a basic frame which may be at absolute zero velocity so that they may find out absolute velocities of other frames with respect to that basicframe. However it was found impossible todifferentiate whether the frame is at rest or it is moving withsomeconstant velocity. Soon it wass realized thatthere is no experiment which could be performed within an inertial frame to divulge its own velocity. For example, three experiments are shown in fig.2which failed to find outwhich of the 'A' type of trains are at rest or moving with a constant velocity. The three experiments are: dropping balls from the ceiling, oscillating a pendulum from the top and throwing balls up and see the direction in which they are going after bouncing back fromFig.2:

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## III. Motion of Paul

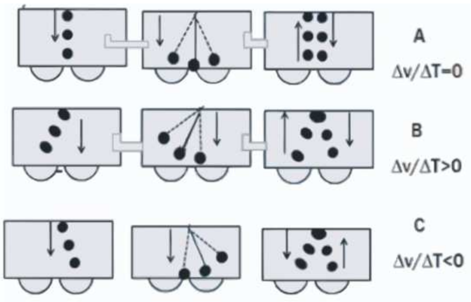


Fig. 2. Three experiments are being perfoemed

Ultimately it was realized that the concept of absolute velocity is non-existant. Albert Einstein grabbed this idea and proclaimed that all the inertial frames are equivalent and we can talk of only relative velocities.He put this idea as the first postulate of the special theory of relativity.. As has been discussed above that the track of the journey of Paul includesboth types ofmotioninertial as well as non-inertial.The effect of non-inertial forces is out of the scope of this presentation[3]. Howeveron the basis of thought experiments and

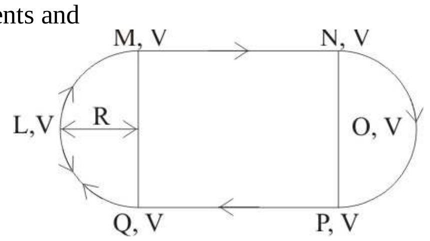


Fig. 3. The track to be folloed by Paul

The track to be followed by Paul is shown in fig.3. It consists two semi circles (LMQ and NOP) connected by two common tangens (NM) and QP). First Paul is moving and Peter is at rest and he is recording the motion of Paul. In the beginning both are sitting at the location 'R' the center of first circle. Then Paul moves to the location 'L' on the periphery of the circle. This motion is radial and therefore it will be orthogonal to the tangential motion.

When Paul reaches atlocation'L' on the periphery he stops. This motion from R to L can't be added or subtracted to tangential motion. Therefore the actual journey starts from here.Paul starts from velocity 'zero' and accelerates so that by the time it covers first quarter of the circle,he attains velocity 'V' at the location 'M'. 'V' is comparable to 'C', the velocity of light. Paul moves on with constant velocity and covers very long journey up to location'N' where it undergoes a central force so that after moving on this semi-circular path it completes 'U' turnand at the same time keeping its speed 'V' constant and finallyleaves the circle tangentially at location 'P'. He, again, covers a long distance PQ with constant velocity

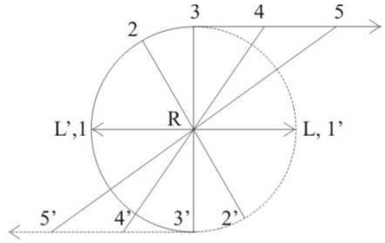


Fig. 4. conservaton of linear and circular motions

'V'. After location Q' he decelerates to velocity zero..By the time it coverslast quarter of the circular motion, its velocity reduces to jero. Paul takes radial motion LR and meets Peter at location 'R' from where he had departed. The trackisshownin fig.3. This is observed by Petersitting at location R. Nowlet us consider that the same journey is recorded by Paul. When he moves from R to L. Paul thinks that he himself is at rest and Peter is moving awayfrom him reaching at location L'. The distance RL' is equal and opposite to that of RL. Another four locations namely 2',3',4' and 5' have been tracedout corresponding topoints 2, 3, 4 and 5. Using this method whole path can betraced out. The method is based on the conservation of angular momentum. Asshown in fig.4 each pair (2-2', 3-3' etc) have equal and opposite angular momentum The central part of the journey has been shown in fig.4. We will discuss three other methods which may be considred as corolaries to what has been dicussed above. They may be termed as second, third and fourth methods.In thesecond method one can change the lenghths of the tracks of the inertial part of the journey i.e.the length of the tracks MN and QP. If the age-gap does not change, it is convincingt that there is noage-gap because of special theory of relativity.

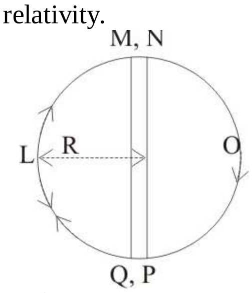


Fig. 5. The track followed by Peter in the 3rd method

In the third method, while Paul is away on his journey as has been discussed above, instead of waitng for Paul, Peter also goes for a space walk, but he covers only non-inertial parts of the journey as has been shown in fig.5. In vieew of symetrical tracks again the age gap is zero, we can concludeagain that there is no age gap because of special theory of relativity. Finaly in 4th method we can change 1st and 4th quarters into a semi circle which may cause a U-turn with velocity V, just like a mirror image of the previous semicircle at position 'O' So both the twins follow exacly same track.as shown in Fig.6. This type of motion is impossible if their age gap is not jero.

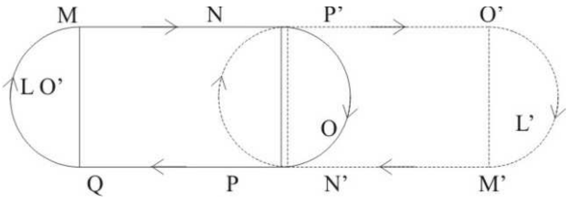


Fig. 6. Two tracks (solid and dotted) one traversed by Paul and the other recorded by Paul

The coresponding locations on the two tracks (LMNOPQL) and (LMNOPQL)' are shown.

## IV. Conclusion

In view of symmetrical tracks again the age gap is zero, we can conclude again that there is no age gap because of special theory of relativity.

## Reference

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