International Journal of Innovation in Science and Mathematics
Volume 2, Issue 1, ISSN (Online): 2347–9051

On Convergence of the Power Sequence of an Intuitionistic Fuzzy Matrix

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Abstract – **In this paper, we study upper (lower) triangular Intuitionistic Fuzzy Matrices and the convergence of the power sequence of Intuitionistic Fuzzy Matrices under min-max composition.**

Keywords – **Intuitionistic Fuzzy Matrix (IFM), Reflexive Irreflexive, Weakly reflexive and Ultrametric.**

I. Introduction

Atanassov [3] developed the concept of Intuitionistic Fuzzy Sets (IFSs) analogous to Fuzzy set. Im et., al [4] studied the determinant of square Intuitionistic Fuzzy Matrices (IFMs). Thomason[11] studied the convergence of powers of a fuzzy matrix. Jicheng Li and Wenxiu Zhang [5] gave out the convergence of the power sequence of min-max composition of fuzzy matrices. The author with Sriram [10] explored some results in sections of IFM and subinverse [9] of IFM. Meenakshi and Gandhimathi have studied the regularity idempotency, invertibility and symmetry of IFMs in terms of those of its membership and non-membership matrices in [2] and discussed the consistency of Intuitionistic fuzzy relational equations in [1]. The authors in [6,7] studied Intuitionistic fuzzy vector space over Intuitionistic Fuzzy algebra and in [8] obtained maximal and minimal solution for Intuitionistic fuzzy relational equation. In this paper convergence of power sequence of an IFM under minmax composition is discussed.

I. Preliminaries

The set of all IFMs of order m×n is denoted by $\mathcal{F}_{mn}$, for intuitionistic fuzzy square matrix of order n we write $\mathcal{F}_n$.

Definition 2.1.[8] An intuitionistic fuzzy matrix (IFM) is a matrix of pair $A = (\langle a_{ij}, a'_{ij}\rangle)$ of a non negative real numbers satisfying $a_{ij} + a'_{ij} \leq 1$ for all i, j. For any two elements $A = (\langle a_{ij},a'_{ij}\rangle)$, $B = (\langle b_{ij}, b'_{ij}\rangle) \in \mathcal{F}_{mn}$, define
$A \vee B=(\langle a_{ij} \vee b_{ij}, a'_{ij} \wedge b'_{ij}\rangle)$
$A \wedge B=(\langle a_{ij} \wedge b_{ij}, a'_{ij} \vee b'_{ij}\rangle)$, for all $1\leq i \leq m$ and $1\leq j \leq n$.
Further
$A\leq B \Longrightarrow a_{ij} \leq b_{ij}, a'_{ij} \geq b'_{ij}$

Definition 2.2. [8] If $A= (\langle a_{ij}, a'_{ij}\rangle) \in \mathcal{F}_{mn}$ and $B = (\langle b_{ij}, b'_{ij}\rangle) \in \mathcal{F}_{np}$ then the products of A and B denoted as AB is an IFM defined by
$AB=(\langle \vee_k\{\wedge \{a_{ik}, b_{kj}\}\}, \wedge_k\{\vee \{a'_{ik}, b'_{kj}\}\}\rangle)$, where $1\leq k \leq n$, $1\leq i \leq m$ and $1\leq j \leq p$. Here $A^2=AA$ and so $A^{k+1}=A^kA$.
For $\vee_k$ and $\wedge_k$ we can use $\sum_k$ and $\prod_k$ respectively.

Definition 2.3.[8] If $A= (\langle a_{ij}, a'_{ij}\rangle) \in \mathcal{F}_{mn}$ and $B = (\langle b_{ij}, b'_{ij}\rangle) \in \mathcal{F}_{np}$ then the min-max products of A and B denoted as $A * B$ is an IFM defined by
$A*B=(\langle \wedge_k \{\vee \{a_{ik}, b_{kj}\}\}, \vee_k \{\wedge \{a'_{ik}, b'_{kj}\}\}\rangle)$, where $1\leq k\leq n$, $1\leq i \leq m$ and $1\leq j \leq p$. Here $R^{[2]}=R*R$ and so $R^{[k+1]} = R^{[k]} * R$, $R^{[k]} = (\langle r_{ij}, r'_{ij}\rangle^k)$.

Definition 2.4.[8] For $A \in \mathcal{F}_n$, if
(i) $A^2\leq A$, then A is called transitive.
(ii) $I_n\leq A$, then A is called reflexive.
(iii) $A^2 = A$, then Ais called idempotent.
(iv) $A \wedge A^T\leq I_n$, then A is called antisymmetric.
If A is reflexive and transitive, then A is a matrix representing an intuitionistic pre-order. Here A^T is the transpose of A.

Definition 2.5.[7] For $R \in \mathcal{F}_n$, if
(i) R is called reflexive if and only if $I_n\leq R$ or $\langle r_{ii}, r'_{ii}\rangle = \langle 1,0\rangle$ for all i = 1, 2, …..n. It is called $\langle \alpha, \alpha'\rangle$ - refelxive if and only if $\langle r_{ii}, r'_{ii}\rangle \geq \langle \alpha, \alpha'\rangle$ for all i = 1, 2, 3, …n where $\alpha, \alpha' \in [0, 1]$ with $\alpha + \alpha' \leq 1$.
(ii) It is called weakly reflexive if and only if $\langle r_{ii}, r'_{ii}\rangle \geq \langle r_{ij}, r'_{ij}\rangle$ for all i=1,2,…n.
(iii) It is called irreflexive if and only if $\langle r_{ii}, r'_{ii}\rangle = \langle 0, 1\rangle$ for all i=1,2,..n
(iv) It is called symmetric if and only if $\langle r_{ij}, r'_{ij}\rangle = \langle r_{ji}, r'_{ji}\rangle$ for all i, j.
(v) It is called compact if and only if $R^2 \geq R$.
(vi) It is called nilpotent if and only if $R^n = 0$. Note that, if $R\in \mathcal{F}_n$ with $R^m = 0$ for some positive integer m, then also R is nilpotent. If $R^m = 0$ and $R^{m-1}\neq 0$, $1 \leq m \leq n$,then R is nilpotent of degree m.

II. Some Results

Definition 3.1. An IFM $A \in \mathcal{F}_n$, is ultra metric if it satisfies
(i) A is symmetric
(ii) $\langle a_{ij}, a'_{ij}\rangle \geq \min \{\langle a_{ik}, a'_{ik}\rangle\}$, for all i, j, k = 1, 2, 3, …n
(iii) $\langle a_{ii}, a'_{ii}\rangle \geq \max \{\langle a_{ij}, a'_{ij}\rangle$, for any $j \neq i\}$

Definition 3.2.An IFM $A \in \mathcal{F}_n$, is upper tringular (lower triangular) if $\langle a_{ij}, a'_{ij}\rangle = \langle 0,1\rangle$ for i > j (i < j) and is strictly upper triangular(strictly lower triangular) if if $\langle a_{ij}, a'_{ij}\rangle = \langle 0, 1\rangle$ for $i \geq j$ $(i \leq j)$.

Definition 3.3.Let $\alpha, \beta \subseteq\{1, 2, 3, \ldots, n\}$ be subsets. We denote by $A_{[\alpha|\beta]}$the sub matrix of A containing rows numbered by α and columns numbered by β, $A_{(\alpha|\beta)} = A_{[\alpha^c|\beta^c]}$, where α^c and β^c denote their complements, respectively. If $\alpha = \beta$, then we denote $A_{[\alpha|\beta]}$and $A_{(\alpha|\beta)}$ by $A[\alpha]$ and $A(\alpha)$, respectively.

Definition 3.4. A $\in \mathcal{F}_n$, is nearly irreflexive if and only if $\langle a_{ii}, a'_{ii}\rangle \leq \langle a_{ij}, a'_{ij}\rangle$, for i , j=1,2,…n .

Theorem 3.1. An Intuitionistic fuzzy ultra metric matrix A$\in\mathcal{F}_n$ is transitive and weakly reflexive.

Proof: Weakly reflexive follows directly from the definition of ultra metric.

Let B= $(\langle b_{ij}, b'_{ij}\rangle) = (\langle a_{ij}, a'_{ij}\rangle)^2$

$\Longrightarrow(\langle b_{ij}, b'_{ij}\rangle) = \langle \sum_{k=1}^{n} a_{ik} a_{kj}, \prod_{k=1}^{n}(a_{ik} + a'_{kj})\rangle = \langle a_{ih} a_{hj}, a'_{il} + a'_{lj}\rangle$

for some h and l in {1, 2, ...n}. By the definition of ultra metric (ii) $a_{ij} \geq a_{ik}$ and $a'_{ij} \leq a'_{kj}$ for all i, j, k = 1, 2, 3, n.

Therefore $\langle b_{ij}, b'_{ij}\rangle = \langle a_{ih} a_{hj}, a'_{il} + a'_{lj}\rangle \geq = \langle a_{ij}, a'_{ij}\rangle$ for all i, j $\Longrightarrow A^2 \leq A$, Therefore A is transitive.

Theorem 3.2. Let A,B and C,D be an n × n upper and lower triangular matrices,respectively. Then

(i) $(A * C)(1) = \langle 0, 1\rangle$

(ii) $(C * A)(n) = \langle 0, 1\rangle$

(iii) $(A*B)$ ({1, 2}|{n−1, n}) is upper triangular. In general, A∗B is not an upper triangular matrix;

(iv) $(C * D)$({n − 1, n}|{1, 2}) is lower triangular. In general, C ∗ D is not a lower triangular matrix.

In particular, if A,B,C and D are corresponding strictly triangular matrices, then

(i) $A * C = (\langle 0, 1\rangle)$;

(ii) $C * A = (\langle 0, 1\rangle)$;

(iii) A ∗ B is strictly upper triangular matrix;

(iv) C ∗ D is a strictly lower triangular matrix.

Proof: It is trivial from Definition 3.2 and 3.3.

Proposition 3.1. For IFMs $A=(\langle a_{ij}, a'_{ij}\rangle)$, $B = (\langle b_{ij}, b'_{ij}\rangle) \in \mathcal{F}_{mn}$, $C = (\langle c_{ij}, c'_{ij}\rangle) \in \mathcal{F}_{nl}$ and $D = (\langle d_{ij}, d'_{ij}\rangle) \in \mathcal{F}_{pm}$ we have the following:

(i) $(B * C)^T = C^T * B^T$;

(ii) if $A \leq B$, then $D * A \leq D * B$, $A * C \leq B * C$.

Proof:

(i) $(B*C)^T = \left(\langle \prod_{k=1}^{n}(b_{ik} + c_{kj}), \sum_{k=1}^{n} b'_{ik} c'_{kj}\rangle\right)^T$

$= \left(\langle \prod_{k=1}^{n}(b_{ki} + c_{jk}), \sum_{k=1}^{n} b'_{ki} c'_{jk}\rangle\right)$

$= \left(\langle \prod_{k=1}^{n}(c_{jk} + b_{ki}), \sum_{k=1}^{n} c'_{jk} b'_{ki}\rangle\right) = C^T * B^T$

(ii) If $A \leq B$, $D*A = \left(\langle \prod_{k=1}^{n}(d_{ik} + a_{kj}), \sum_{k=1}^{n} d'_{ik} a'_{kj}\rangle\right)$

$\leq \left(\langle \prod_{k=1}^{n}(d_{ik} + b_{kj}), \sum_{k=1}^{n} d'_{ik} b'_{kj}\rangle\right) = D * B$

Similarly we can prove the other part.

Theorem 3.3. Let R be a nearly irreflexive intuitionistic fuzzy matrix, then

(i) $R \geq R^{[2]} \geq R^{[3]} \geq \ldots \geq R^{[n-1]} \geq \ldots$, that is, the sequence $\{R^{|n|}\}$ is decreasing;

(ii) $\langle r_{ii}, r'_{ii}\rangle = \langle r_{ii}, r'_{ii}\rangle^{[2]} = \langle r_{ii}, r'_{ii}\rangle^{[3]} = \ldots = \langle r_{ii}, r'_{ii}\rangle^{[n]}$, i=1, 2, 3, …n, that is, the sequence of principal diagonal elements is stable;

(iii) $R^{[n-1]} = R^{[n]}$

Proof: (i). For any pair (i, j), we have

$\langle r_{ij}, r'_{ij}\rangle^{[2]} = \langle (r_{i1}+ r_{1j}) (r_{i2}+ r_{2j}) \ldots (r_{ii}+ r_{ij}) \ldots (r_{in}+ r_{nj}), (r'_{i1} r'_{1j}+ r'_{i2} r'_{2j}+\ldots+ r'_{ii} r'_{ij}\ldots+ r'_{in} r'_{nj})\rangle \leq \langle r_{ii}+r_{ij}, r'_{ii} r'_{ij}\rangle = \langle r_{ij}, r'_{ij}\rangle$, by the definition of nearlyirreflexive. Hence $R^{[2]} \leq R$.

(ii) By (i), we know that $R \geq R^{[2]} \geq R^{[3]} \geq \ldots \geq R^{[n-1]} \geq \ldots$ Hence,

$\langle r_{ii}, r'_{ii}\rangle \geq \langle r_{ii}, r'_{ii}\rangle^{[2]} \geq \langle r_{ii}, r'_{ii}\rangle^{[3]} \geq \ldots \geq \langle r_{ii}, r'_{ii}\rangle^{[n]}$ for any i=1,2,…n.

Since $\langle r_{ii}, r'_{ii}\rangle^{[2]} = \left(\langle \prod_{k=1}^{n}(r_{ik} + r_{ki}), \sum_{k=1}^{n} r'_{ik} r'_{ki}\rangle\right)$

$\langle r_{ii}, r'_{ii}\rangle^{[3]} = \langle r_{ii}, r'_{ii}\rangle^{[2]} * \langle r_{ii}, r'_{ii}\rangle = \left(\langle \prod_{k=1}^{n}(r_{ik} + r_{ki}), \sum_{k=1}^{n} r'_{ik} r'_{ki}\rangle\right) \langle r_{ii}, r'_{ii}\rangle$

$= \left(\langle \prod_{1\leq l,k\leq n}(r_{il} + r_{lk} + r_{ki}), \sum_{1\leq l,k\leq n} r'_{il} r'_{lk} r'_{ki}\rangle\right)$ and so

$\langle r_{ii}, r'_{ii}\rangle^{[s]} = \left(\langle \prod_{1\leq k_1,k_2,\ldots k_{s-1}\leq n}(r_{ik_1} + r_{k_1k_1} + \cdots + r_{k_{s-1}i}), \sum_{1\leq k_1,k_2,\ldots k_{s-1}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{s-1}i}\rangle\right)$

$\geq \left(\langle \prod_{1\leq k_i\leq n} r_{ik_i}, \sum_{1\leq k_i\leq n} r'_{ik_i}\rangle + \langle \prod_{1\leq k_j\leq n} r_{k_j}, \sum_{1\leq k_j\leq n} r'_{k_j}\rangle\right)$

$\geq \left(\langle \prod_{1\leq k_i\leq n} r_{ik_i}, \sum_{1\leq k_i\leq n} r'_{ik_i}\rangle\right) = \langle r_{ii}, r'_{ii}\rangle$,

by the definition of nearly irreflexive.

Hence (ii).

(iii) By (ii)

$\langle r_{ij}, r'_{ij}\rangle^{[n]} = \left(\langle \prod_{1\leq k_1,k_2,\ldots k_{n-1}\leq n}(r_{ik_1} + r_{k_1k_2} + \cdots + r_{k_{n-1}j}), \sum_{1\leq k_1,k_2,\ldots k_{n-1}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{n-1}j}\rangle\right)$

$= \langle (r_{ik_1} + r_{k_1k_1} + \cdots + r_{k_{n-1}i}), r_{ih_1} r_{h_1h_2} \ldots r_{h_{n-1}i}\rangle$ for some sequences $k_1, k_2 \ldots k_{n-1}$, $h_1, h_2, \ldots h_{n-1}$, statisfying $1 \leq k_1, k_2 \ldots k_{n-1} \leq n$ and $1 \leq h_1, h_2, \ldots h_{n-1} \leq n$;

$\langle r_{ij}, r'_{ij}\rangle^{[n-1]} = \left(\langle \prod_{1\leq m_1,m_2,\ldots m_{n-2}\leq n}(r_{im_1} + r_{m_1m_2} + \cdots + r_{m_{n-2}j}), \sum_{1\leq m_1,m_2,\ldots m_{n-2}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{n-2}j}\rangle\right)$

$= \langle (r_{ig_1} + r_{g_1g_1} + \cdots + r_{g_{n-1}i}), r'_{if_1} + r'_{f_1f_2} + \cdots + r'_{f_{n-1}i}\rangle$

for some sequences $g_1, g_2 \ldots g_{n-2}$, $f_1, f_2, \ldots f_{n-2}$, statisfying $1 \leq g_1, g_2 \ldots g_{n-2} \leq n$ and $1 \leq f_1, f_2, \ldots f_{n-2} \leq n$; Since the number of entries $1 \leq i, k_1,k_2, \ldots, k_{n-1}, j \leq n$ is n + 1, at least two entries are equal.

Case 1. If k_s=i, then

$\langle r_{ij}, r'_{ij}\rangle^{[n]} = \left(\langle \prod_{1\leq k_1,k_2,\ldots k_{n-1}\leq n}(r_{ik_1} + r_{k_1k_2} + \cdots + r_{k_{n-1}j}), \sum_{1\leq k_1,k_2,\ldots k_{n-1}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{n-1}j}\rangle\right)$

$$= \left(\langle \prod_{1\leq k_1,k_2,\ldots k_{n-1}\leq n} (r_{ik_1} + r_{k_1k_2} + \cdots + r_{k_{s-1}}k_s + r_{k_s}k_{s+1} + \cdots + r_{k_{n-1}j}\right),$$

$$\sum_{1\leq k_1,k_2,\ldots k_{n-1}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{s-1j}k_s} r'_{k_sk_{s+1}} \ldots r'_{k_{n-1j}}\rangle)$$

$\geq \langle r_{ij}, r'_{ij}\rangle^{[s]} * \left(\langle \prod_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n}(r_{ik_{s+1}} + \cdots + r_{k_{n-1j}}), \sum_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n} r'_{ik_{s+1}} \ldots r'_{k_{n-1j}}\rangle\right)$

Since $\langle r_{ik_1} + r_{k_1k_2}+\ldots+r_{k_{s-1i}}, r'_{ik_1} r'_{k_1k_2}\ldots r'_{k_{s-1j}}\rangle$ is one term of $\langle r_{ii}, r'_{ii}\rangle^{[s]}$.

By (ii), $\langle r_{ii}, r'_{ii}\rangle^{[s]} = \langle r_{ii}, r'_{ii}\rangle$ and R is nearly irreflexive, so $\langle r_{ii}, r'_{ii}\rangle \leq \langle r_{ij}, r'_{ij}\rangle$, $\langle r_{ii}, r'_{ii}\rangle + \langle r_{ik_{s+1}}, r'_{ik_{s+1}}\rangle = \langle r_{ik_{s+1}}, r'_{ik_{s+1}}\rangle$

Hence, $\langle r_{ij}, r'_{ij}\rangle^{[n]} \geq \langle r_{ii}, r'_{ii}\rangle^{[s]} * \left(\langle \prod_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n}(r_{ik_{s+1}} + \cdots + r_{n-1j}), \sum_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n} r'_{ik_{s+1}} \ldots r'_{k_{n-1j}}\rangle\right)$

$$= \left(\langle \prod_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n} (r_{ik_{s+1}} + \cdots + r_{n-1j}\right), \sum_{1\leq k_{s+1},k_{s+2},\ldots k_{n-1}\leq n} r'_{ik_{s+1}} \ldots r'_{k_{n-1j}}\rangle),$$

$\geq \langle r_{ij}, r'_{ij}\rangle^{[n-s]} \geq \langle r_{ij}, r'_{ij}\rangle^{[n-1]}$ as $s \geq 1$

Case 2. If k_s=j, then

$\langle r_{ij}, r'_{ij}\rangle^{[n]} = \left(\langle \prod_{1\leq k_1,k_2,\ldots k_{n-1}\leq n}(r_{ik_1} + r_{k_1k_2} + \cdots + r_{k_{n-1}j}), \sum_{1\leq k_1,k_2,\ldots k_{n-1}\leq n} r'_{ik_1} r'_{k_1k_2} \ldots r'_{k_{n-1}j}\rangle\right)$

$$= \left(\langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}j} + r_{jk_{s+1}} + \dots + r_{k_{n-1}j} \right),$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}j} r'_{jk_{s+1}} \dots r'_{k_{n-1}j} \rangle)$$

$$\geq \left(\langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}j} \right),$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}j} \rangle)$$

$\geq \langle r_{ij}, r'_{ij} \rangle^{[s]} \geq \langle r_{ij}, r'_{ij} \rangle^{[n-1]}$, since $s \leq n-1$

Case 3. If $i \neq k_s = k_t \neq j$, with out loss of generality, we suppose $s < t$, then

$$\langle r_{ij}, r'_{ij} \rangle^{[n]} = \left(\langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}k_s} + r_{k_sk_{s+1}} + \dots + r_{k_{t-1}k_t} + r_{k_tk_{t+1}} \dots + r_{k_{n-1}j}, \right.$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}k_s} r_{k_sk_{s+1}} \dots r'_{k_{t-1}k_t} r'_{k_tk_{t+1}} \dots r_{k_{n-1}j} \rangle)$$

$$= \left(\langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}k_s} + r_{k_sk_{s+1}} + \dots + r_{k_{t-1}k_s} + r_{k_sk_{t+1}} + \dots + r_{k_{n-1}j} \right),$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}k_s} r'_{k_sk_{s+1}} \dots r'_{k_{t-1}k_s} r'_{k_sk_{t+1}} \dots r'_{k_{n-1}j} \rangle)$$

$$\geq \langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}k_s} + r_{k_sk_s} + r_{k_sk_{t+1}} + \dots + r_{k_{n-1}j},$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}k_s} r'_{k_sk_s} \dots r'_{k_sk_{t+1}} \dots r'_{k_{n-1}j} \rangle)$$

Since R is nearly irreflexive, $\langle r_{k_sk_s}, r'_{k_sk_s} \rangle \leq \langle r_{k_sk_{t+1}}, r'_{k_sk_{t+1}} \rangle$, $\langle r_{k_sk_s}, r'_{k_sk_s} \rangle + \langle r_{k_sk_{t+1}}, r'_{k_sk_{t+1}} \rangle = \langle r_{k_sk_{t+1}}, r'_{k_sk_{t+1}} \rangle$, Thus,

$$\langle r_{ij}, r'_{ij} \rangle^{[n]} = \left(\langle \prod_{1 \le k_1, k_2, \dots k_{n-1} \le n} (r_{ik_1} + r_{k_1k_2} + \dots + r_{k_{s-1}k_s} + r_{k_sk_{t+1}} + \dots + r_{k_{n-1}j}, \right.$$

$$\sum_{1 \le k_1, k_2, \dots k_{n-1} \le n} r'_{ik_1} r'_{k_1k_2} \dots r'_{k_{s-1}k_s} r_{k_sk_{t+1}} \dots r_{k_{n-1}j}$$

$\geq \langle r_{ij}, r'_{ij} \rangle^{[n-t+s]} \geq \langle r_{ij}, r'_{ij} \rangle^{[n-1]}$, since $t > s$, $n - t + s \leq n - 1$.

REFERENCES

[1] AR. Meenakshi and T. Gandhimathi., Intuitionistic Fuzzy Relational Equations, Advances in Fuzzy Mathematics, Vol. 5(3), (2010), 239-244.

[2] AR. Meenakshi and T. Gandhimathi., On Regular Intuitionistic Fuzzy Matrices, The Journal of Fuzzy Mathematics, Vol. 19(2), (2011), 599-605.

[3] Atanassov K., Intuitionistic Fuzzy Sets., VII ITKR's Section, Sofia, June 1983.

[4] Im Young Bim, Eun Pyo Lee and S.W.Park., The determinant of Square Intuitionistic Fuzzy Matrix, Far East Journal of Mathematical Sciences, Vol.3(5), (2001), 789-796.

[5] Jicheng Li and Wenxiu Zhang.,On Convergence of the Min-Max Composition of Fuzzy matrices, Southeast Asian Bulletin of Mathematics,Vol.24(2000), 389-33.

[6] S. Sriram and P. Murugadas., On Semiring of Intuitionistic Fuzzy Matrices, Applied Mathematical Science, Vol. 4(23), (2010), 1099-1105.

[7] S. Sriram and P. Murugadas., Rank and Regularity of Intuitionistic Fuzzy Matrices, Advances in Fuzzy Mathematics, Vol. 5(3), (2010), 335-347.

[8] S. Sriram and P. Murugadas., Contributions to a Study on Generalized Fuzzy Matrices, Ph.D Thesis, Department of Mathematics, Annamalai University, Tamil Nadu, India, (2011).

[9] S. Sriram and P. Murugadas., Sub-inverses of Intuitionistic Fuzzy Matrices, Acta Ciencia Mathematics,Vol. XXXVII M,No.1,41(2011).

[10] S. Sriram and P. Murugadas., ⟨ α, α′⟩ -cut of Intuitionistic Fuzzy Matrices, Int.Journal of Fuzzy Mathematics,Vol.20(2),(2012),307-318.

[11] M.G.Thomason, Convergence of powers of a fuzzy matrix, J.Math.Anal.Appl.Vol.57,(1977), 476-480.

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