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Third Order Cnoidal Wave Solutions in Shallow Water and its Horizontal and Vertical Fluid Velocity Components

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***Abstract* – Third order cnoidal wave solutions in shallow water are developed where waves progress steadily without any change of form. Shallow water wave problems are solved at the bottom and at the free surface. The boundary conditions are also taken from Navier-Stokes equation of motion. Using these boundary conditions, three nonlinear ordinary differential equations are derived which can be solved using series expansion method. Taking Jacobi elliptic function, third order cnoidal wave solutions have been established from which wave elevation, horizontal wave velocity and acceleration due to gravity for cnoidal wave are expressed. Assuming Taylor expansion for the stream function about the bed, the fluid velocity components are derived in terms of Jacobi elliptic function.**

***Keywords* – Navier-Stokes Equation, Jacobi Elliptic Function, Cnoidal Wave, Elliptic Parameter, Velocity Components.**

I. Introduction

In recent years, the capabilities for calculating shallow water wave problems have advanced a great deal. The computation of higher order cnoidal wave solutions for steady, incompressible flow is an important aspect in coastal engineering. Several higher order approximations to irrotational water waves of constant form have appeared in recent years, often based on Fourier series, but where convergence is slow, if at all, for shallow water. These solutions are often numerical and of an inverse formulation, and are generally of such higher order, that it is difficult to obtain expressions for physical quantities as functions of position for practical use. The presentation of results has been limited to tables of integral quantities for a range of wave lengths and heights. However, these methods have achieved real success in obtaining numerically exact solutions for the first time Schwartz [20] and Cokelet [5]. A survey and comparison of the methods is given Cokelet [5].

Fenton [6] presented a fifth order cnoidal wave theory where boundary condition comes from Bernoulli's equation which was both apparently complicated, requiring the presentation of many coefficients as unattractive floating point numbers and also gave poor results for fluid velocities under high waves. However, in a later work, Fenton [7] showed that instead of fluid velocities being expressed as expansions in wave height, the original spirit of cnoidal theory were retained. Cnoidal theory obtained its name in 1895 when Korteweg and_de Vries [14] obtained their eponymous equation for the propagation of waves over a flat bed. The cnoidal solution shows the familiar long flat troughs and narrow crests of real wave in shallow water. Higher order solutions of cnoidal waves have been derived by Laitone [15], who provided a number of results, re-casting the series in terms of the wave height/depth. Tsuchiya and Yasuda [21] obtained a third order solution with the introduction of another definition of wave celerity based on assumptions concerning the Bernoulli constant. Nishimura et al. [16] devised procedures for generating higher order theories for both Stokes and cnoidal theories, making extensive use of recurrence relations. Nishimura et al. [17] continued the work of Nishimura et al. [16] and presented a unified view of Stokes and cnoidal theories. Karabut [12] solved an ordinary quadratic nonlinear differential difference equation of the first order containing an unknown function under certain conditions. Halasz [10] discussed on higher order corrections for shallow water solitary waves. Chen et al. [4] studied the cnoidal wave solutions of the Boussinesq systems in two different techniques using the Jacobi elliptic function series. Carter et al [3] discussed the kinematics and stability of solitary and cnoidal wave solutions of the Serre equations which are a pair of strongly nonlinear, weakly, dispersive, Boussinesq type partial differential equations. They [3] also described the model of the surface elevation and the depth averaged horizontal velocity of an inviscid, irrotational, incompressible shallow water. Jain et al. [11] derived coupled evolution equations for first and second order potentials using reductive perturbation method with appropriate boundary conditions. Oh and Watanabe [18] obtained the second order solution by taking into account the unstable and dissipative effects based on a cnoidal wave solution to the KdV equation. Xu et al. [22] calculated the cnoidal function in cnoidal wave theory based on the precise integration method and also provided a trigonometric function approximation for the cnoidal function. Parvin et al. [19] derived first and second order cnoidal wave solutions using Jacobi elliptic function. Khater et al. [13] used a suitable ansatz and Jacobi elliptic function expansion method to construct new exact cnoidal wave solutions of the modified fifth order KdV equation and generalized fifth order KdV equation which included as special cases, some well known equations. Fu et al. [8] applied Jacobi elliptic function in Jacobi elliptic function expansion method to construct the exact periodic solutions of nonlinear wave equations. In this paper, shallow water wave problems have been solved using boundary conditions, at the bottom $Y=0$ and at the free surface $Y=\eta(X)$. Also the boundary conditions from Navier-Stokes equation of motion generate third order

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cnoidal wave solutions. Then horizontal and vertical fluid velocity components have been established in terms of Jacobi elliptic function.

II. Mathematical Formulation

Consider the wave as shown in figure-1, with a stationary frame of reference (x, y), x in the direction of propagation of the waves and y vertically upwards with the origin on the flat bed. The waves travel in the x direction at speed c relative to this frame. Consider also a frame of reference (X, Y) moving with the waves at velocity c, such that $x = X + ct$, where t is time and $y = Y$. The fluid velocity in the (x, y) frame is (u, v) and that in the (X, Y) frame is (U, V). The velocities are related by $u = U + c$ and $v = V$.

In the (X, Y) frame, all fluid motion is steady and consists of a flow in the negative X direction, roughly of the magnitude of the wave speed, underneath the stationary wave profile. The mean horizontal fluid velocity in this frame, for a constant value of Y over one wavelength λ is denoted by -$\overline{U}$. It is negative because the apparent flow is in the -X direction. For the convenience of our calculation, the velocities in this frame are used to obtain the solutions.

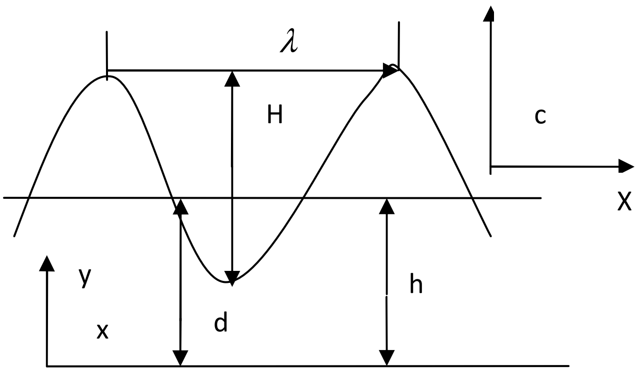

Fig. 1. Wave train, showing important dimensions and coordinates.

For irrotational flow, stream function satisfies Laplaces equation $\frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = 0$ (1)

The boundary conditions at the bottom $Y = 0$ is a stream line on which $\psi(X, Y)$ is constant and at the free surface $Y = \eta(X)$ is also a stream line.

$\therefore \psi(X, 0) = 0$ (Taking zero constant) and

$\psi(X, \eta(X)) = -Q$ (2)

where Q is the volume flux underneath the wave train per unit span. The negative sign is for the flow which is in the negative X-direction, such that the wave will also propagate in the positive X- direction.

For two components, Navier-Stokes equation of motion for steady incompressible flow is

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \upsilon\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$ on free surface $Y = \eta(X)$, pressure $p = 0$, (3)

and

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = g + \upsilon\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \quad (4)$$

We assume a Taylor expansion for ψ about the bed of the following form

$$\psi(X, Y) = -\left(\sin Y \frac{d}{dX}\right) f(X) = \left\{-Y\frac{d}{dX} + \frac{Y^3}{3!}\frac{d^3}{dX^3} - \frac{Y^5}{5!}\frac{d^5}{dX^5} + \ldots\ldots\right\} f(X) \quad (5)$$

as in Fenton [7], where $\frac{df}{dX}$ is the horizontal velocity on the bed.

Now, the velocity components anywhere in the fluid are

$$U = \frac{\partial \psi}{\partial Y} = -\left(\cos Y \frac{d}{dX}\right) f'(X) \quad (6)$$

$$V = -\frac{\partial \psi}{\partial X} = \left(\sin Y \frac{d}{dX}\right) f'(X) \quad (7)$$

From Eqs. (3) and (4), we get

$$\left(\left(\cos Y \frac{d}{dX}\right) f'(X)\right)\left(\left(\cos Y \frac{d}{dX}\right) f''(X)\right) + \left(\left(\sin Y \frac{d}{dX}\right) f'(X)\right)\left(\left(\sin Y \frac{d}{dX}\right) f''(X)\right) = 0 \quad (8)$$

and

$$-\left(\left(\cos Y \frac{d}{dX}\right) f'(X)\right)\left(\left(\sin Y \frac{d}{dX}\right) f''(X)\right) + \left(\left(\sin Y \frac{d}{dX}\right) f'(X)\right)\left(\left(\cos Y \frac{d}{dX}\right) f''(X)\right) = g \quad (9)$$

At the free surface $Y = \eta(X)$, Eqs. (2), (8) and (9) become

$$\left(\sin \eta \frac{d}{dX}\right) f(X) = Q \quad (10)$$

$$\left(\left(\cos \eta \frac{d}{dX}\right) f'(X)\right)\left(\left(\cos \eta \frac{d}{dX}\right) f''(X)\right) + \left(\left(\sin \eta \frac{d}{dX}\right) f'(X)\right)\left(\left(\sin \eta \frac{d}{dX}\right) f''(X)\right) = 0 \quad (11)$$

and

$$-\left(\left(\cos \eta \frac{d}{dX}\right) f'(X)\right)\left(\left(\sin \eta \frac{d}{dX}\right) f''(X)\right) + \left(\left(\sin \eta \frac{d}{dX}\right) f'(X)\right)\left(\left(\cos \eta \frac{d}{dX}\right) f''(X)\right) = g \quad (12)$$

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Differentiating Eq. (10) and then substituting this value in Eqs. (11) and (12), we have

$$\left(\left(\cos\eta\frac{d}{dX}\right)f'(X)\right)\left(\left(\cos\eta\frac{d}{dX}\right)f''(X)\right)$$
$$-\frac{d\eta}{dX}\left(\left(\cos\eta\frac{d}{dX}\right)f'(X)\right)\left(\left(\sin\eta\frac{d}{dX}\right)f''(X)\right)=0 \quad (13)$$

and

$$\left(\left(\cos\eta\frac{d}{dX}\right)f'(X)\right)\left(\left(\sin\eta\frac{d}{dX}\right)f''(X)\right)$$
$$+\frac{d\eta}{dX}\left(\left(\cos\eta\frac{d}{dX}\right)f'(X)\right)\left(\left(\cos\eta\frac{d}{dX}\right)f''(X)\right)=-g \quad (14)$$

Eqs. (10), (13) and (14) are three nonlinear ordinary differential equations in the unknowns $\eta(X),\quad f'(X)\ and\ g$. These ordinary differential equations can be solved using power series method.

III. Power Series Solution

Assuming constant depth $\eta(X)=h\ and\ f'(X)=U$, the derived equations can be solved using series expansion method about the state of a uniform critical flow. Let the scaled horizontal variable be $X=\frac{\theta h}{\alpha}, \eta=\eta_* h\ and\ f=f_* Q$ Eqs. (10), (13) and (14) can be rewritten in terms of these dimensionless quantities

$$\frac{1}{\alpha}\left(\sin\eta_*\alpha\frac{d}{d\theta}\right)f_*(\theta)-1=0\ as\frac{1}{h}\approx 1 \quad (15)$$

$$\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right)f_*'(\theta)\right)\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right)f_*''(\theta)\right)$$
$$-\alpha\frac{d\eta_*}{d\theta}\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right)f_*'(\theta)\right)\left(\left(\sin\eta_*\alpha\frac{d}{d\theta}\right)f_*''(\theta)\right)=0 \quad (16)$$

and

$$\frac{d\eta_*}{d\theta}\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right)f_*'(\theta)\right)\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right).f_*''(\theta)\right)$$
$$+\frac{1}{\alpha}\left(\left(\cos\eta_*\alpha\frac{d}{d\theta}\right)f_*'(\theta)\right)\left(\left(\sin\eta_*\alpha\frac{d}{d\theta}\right)f_*''(\theta)\right)=-g_*$$
$$as\frac{1}{\alpha^2}\approx 1\ and\ \frac{gh}{Q^2}=g_* \quad (17)$$

Using α^2 as the expansion parameter, the series expansions are in the following

$$\eta_* = 1+\sum_{j=1}^{N}\alpha^{2j}Y_j(\theta) \quad (18)$$

$$f_*' = 1+\sum_{j=1}^{N}\alpha^{2j}F_j(\theta) \quad (19)$$

$$g_* = 1+\sum_{j=1}^{N}\alpha^{2j}g_j \quad (20)$$

where N is the order of solution required.

IV. Third Order Cnoidal Wave Solution

To derive third order cnoidal wave solutions, we need some values from Parvin et al. [19]

$$\left.\begin{aligned} &F_1+Y_1=0\\ &F_1'=0\\ &F_1''+g_1=0 \end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned} &F_2+Y_2-F_1^2-\frac{1}{6}F_1''=0\\ &F_1F_1'+F_2'-\frac{F_1'''}{2}=0\\ &F_2''-F_1'^2-\frac{1}{6}F_1^{iv}+g_2=0 \end{aligned}\right\} \quad (22)$$

and

$$\left.\begin{aligned} &Y_3+Y_2F_1+Y_1F_2+F_3-\frac{1}{6}F_2''-\frac{1}{2}Y_1F_1''+\frac{1}{120}F_1^{iv}=0\\ &F_3'-Y_1F_1'''-\frac{1}{2}F_2'''+\frac{1}{24}F_1^{v}+F_1F_2'-\frac{1}{2}F_1F_1'''+F_1'F_2\\ &-\frac{1}{2}F_1'F_1''-Y_1'F_1''=0\\ &F_1'Y_2'+F_2'Y_1'-\frac{1}{2}F_1'''Y_1'+F_1F_1'Y_1'+F_3''+Y_1F_2''\\ &+Y_2F_1''-\frac{1}{6}F_2^{iv}-\frac{1}{2}Y_1F_1^{iv}+\frac{1}{120}F_1^{vi}+F_1F_2''\\ &-\frac{1}{6}F_1F_1^{iv}+F_2F_1''+Y_1F_1F_1''-\frac{1}{2}F_1''^2+g_3=0 \end{aligned}\right\} \quad (23)$$

Using the series expansions (18), (19) and (20) and taking $o\left(\alpha^8\right)$, Eqs. (15), (16) and (17) can be written as

$$\alpha^2(Y_1+F_1)+\alpha^4\left(Y_2+F_1Y_1+F_2-\frac{1}{6}F_1''\right)$$
$$+\alpha^6\left(\begin{aligned}&Y_3+Y_2F_1+Y_1F_2+F_3-\frac{1}{6}F_2''\\&-\frac{1}{2}Y_1F_1''+\frac{1}{120}F_1^{iv}\end{aligned}\right)$$
$$+\alpha^8\left(\begin{aligned}&F_4+F_3Y_1+F_2Y_2+Y_3F_1+Y_4-\frac{1}{2}Y_2F_1''\\&-\frac{1}{2}Y_1^2F_1''-\frac{1}{2}Y_1F_2''-\frac{1}{6}F_3''+\frac{1}{120}F_2^{iv}\\&+\frac{1}{24}Y_1F_1^{iv}-\frac{1}{5040}F_1^{vi}\end{aligned}\right)=0 \quad (24)$$

$$\alpha^2F_1'+\alpha^4\left(F_1F_1'+F_2'-\frac{1}{2}F_1'''\right)$$
$$+\alpha^6\left(\begin{aligned}&F_3'-Y_1F_1'''-\frac{1}{2}F_2'''+\frac{1}{24}F_1^{v}+F_1F_2'\\&-\frac{1}{2}F_1F_1'''+F_1'F_2-\frac{1}{2}F_1'F_1''-Y_1'F_1''\end{aligned}\right)$$

$$+\alpha^8\begin{pmatrix} F_4' - Y_2 F_1''' - Y_1 F_2''' + F_1 F_3' + F_2 F_2' + F_3 F_1' - F_1 Y_1 F_1''' - F_1' Y_1 F_1'' \\ -\frac{1}{2}\begin{pmatrix} F_1 F_2''' + F_2 F_1''' + F_1'' F_2' - \frac{1}{2} F_1'' F_1''' \\ + F_1' F_2'' + Y_1^2 F_1''' + F_3''' \end{pmatrix} \\ + \frac{1}{24}\left(4Y_1 F_1^{v} + F_2^{v} + F_1 F_1^{v} + F_1' F_1^{iv}\right) - \frac{1}{720} F_1^{vii} - Y_1 Y_1' F_1'' \\ - Y_1' F_2'' + \frac{1}{6} Y_1' F_1^{iv} - Y_2' F_1'' - F_1 F_1'' Y_1' \end{pmatrix} = 0 \tag{25}$$

and

$$\alpha^2 F_1'' + \alpha^4\left(F_1' Y_1' + Y_1 F_1'' + F_2'' - \frac{1}{6} F_1^{iv} + F_1 F_1''\right)$$
$$+\alpha^6\begin{pmatrix} F_2' Y_1' - \frac{1}{2} F_1''' Y_1' + F_1 F_1' Y_1' + F_1' Y_2' + Y_2 F_1'' \\ + Y_1 F_2'' + F_3'' - \frac{1}{6}\left(3Y_1 F_1^{iv} + F_2^{iv} + F_1 F_1^{iv}\right) \\ + \frac{1}{120} F_1^{vi} + F_1 F_2'' + Y_1 F_1 F_1'' + F_1'' F_2 - \frac{1}{2} F_1''^2 \end{pmatrix}$$
$$+\alpha^8\begin{pmatrix} F_3' Y_1' - Y_1 Y_1' F_1''' + F_1 F_2' Y_1' + F_2 F_1' Y_1' + F_1 F_1' Y' \\ {}_2 + F_1' Y_3' + F_2' Y_2' - \frac{1}{2}\begin{pmatrix} F_2''' Y_1' + F_1 F_1''' Y_1' + F' F_1'' Y_1' \\ + F_1''' Y_2' \end{pmatrix} \\ + \frac{1}{24} F_1^{v} Y_1' + F_4'' + Y_3 F_1'' + Y_2 F_2'' + Y_1 F_3'' \\ - \frac{1}{6}\left(3Y_2 F_1^{iv} + 3Y_1 F_2^{iv} + F_3^{iv}\right) + \frac{1}{120}\left(5Y_1 F_1^{vi} + F_2^{vi}\right) \\ - \frac{1}{5040} F_1^{viii} + Y_2 F_1 F_1'' + Y_1 F_1 F_2'' + F_1 F_3'' \\ - \frac{1}{6}\left(3Y_1 F_1 F_1^{iv} + F_1 F_2^{iv}\right) + \frac{1}{120} F_1 F_1^{vi} + Y_1 F_1'' F_2 \\ + F_2 F_2'' - \frac{1}{6} F_1^{iv} F_2 + F_1'' F_3 \\ - \frac{1}{2}\left(3Y_1 F_1''^2 + 2F_1'' F_2'' - \frac{1}{4} F_1'' F_1^{iv}\right) \end{pmatrix} = 0$$
$$= -\left(1 + \alpha^2 g_1 + \alpha^4 g_2 + \alpha^6 g_3 + \alpha^8 g_4\right) \tag{26}$$

Equating the coefficients of α^8 from Eqs. (24), (25) and (26), we have

$$F_4 + F_3 Y_1 + F_2 Y_2 + Y_3 F_1 + Y_4 - \frac{1}{2} Y_2 F_1'' - \frac{1}{2} Y_1^2 F_1''$$
$$- \frac{1}{2} Y_1 F_2'' - \frac{1}{6} F_3'' + \frac{1}{120} F_2^{iv} + \frac{1}{24} Y_1 F_1^{iv}$$
$$- \frac{1}{5040} F_1^{vi} = 0 \tag{27}$$

$$F_4' - Y_2 F_1''' - Y_1 F_2''' + F_1 F_3' + F_2 F_2' + F_3 F_1' - F_1 Y_1 F_1'''$$
$$- F_1' Y_1 F_1'' - \frac{1}{2}\begin{pmatrix} F_1 F_2''' + F_2 F_1''' + F_1'' F_2' - \frac{1}{2} F_1'' F_1''' + F_1' F_2'' \\ + Y_1^2 F_1''' + F_3''' \end{pmatrix}$$
$$+ \frac{1}{24}\left(4Y_1 F_1^{v} + F_2^{v} + F_1 F_1^{v} + F_1' F_1^{iv}\right) - \frac{1}{720} F_1^{vii}$$
$$- Y_1 Y_1' F_1'' - Y_1' F_2'' + \frac{1}{6} Y_1' F_1^{iv} - Y_2' F_1'' - F_1 F_1'' Y_1' = 0 \tag{28}$$

and

$$F_3' Y_1' - Y_1 Y_1' F_1''' + F_1 F_2' Y_1' + F_2 F_1' Y_1' + F_1 F_1' Y_2' + F_1' Y_3' + F_2' Y_2'$$
$$- \frac{1}{2}\left(F_2''' Y_1' + F_1 F_1''' Y' + F_1' F_1'' Y_1' + F_1''' Y_2'\right) + \frac{1}{24} F_1^{v} Y_1' + F_4'' + Y_3 F_1''$$
$$+ Y_2 F_2'' + Y_1 F_3'' - \frac{1}{6}\left(3Y_2 F_1^{iv} + 3Y_1 F_2^{iv} + F_3^{iv}\right)$$
$$+ \frac{1}{120}\left(5Y_1 F_1^{vi} + F_2^{vi}\right) - \frac{1}{5040} F_1^{viii} + Y_2 F_1 F_1'' + Y_1 F_1 F_2'' + F_1 F_3''$$
$$- \frac{1}{6}\left(3Y_1 F_1 F_1^{iv} + F_1 F_2^{iv}\right) + \frac{1}{120} F_1 F_1^{vi} + Y_1 F_1'' F_2 + F_2 F_2''$$
$$- \frac{1}{6} F_1^{iv} F_2 + F_1'' F_3 - \frac{1}{2}\left(3Y_1 F_1''^2 + 2F_1'' F_2'' - \frac{1}{4} F_1'' F_1^{iv}\right) = -g_4 \tag{29}$$

From Eqs. (27), (28) and (29), we have

$$\frac{1}{3} F_3^{iv} - F_1 F_3'' - \frac{1}{6} F_1^{iv} F_2 - \frac{1}{3} F_1'' F_2' - 2F_2'^2 - F_2 F_2''$$
$$- \frac{7}{6} F_1'' F_2'' - \frac{1}{6} F_1 F_2^{iv} - \frac{1}{30} F_2^{vi} + F_1 F_1''^2 + \frac{7}{60} F_1'' F_1^{iv}$$
$$+ \frac{1}{2} F_1^2 F_1^{iv} + \frac{11}{120} F_1 F_1^{vi} + \frac{1}{840} F_1^{viii} + g_4 = 0 \tag{30}$$

According to the Parvin et al. [19], the solutions for F_1 *and* F_2 in terms of $cn^2(\theta/m)$ are

$$F_1 = -5mcn^2(\theta/m) \tag{31}$$

and

$$F_2 = \frac{1}{675}\left(10192 - 55927m + 55927m^2\right)$$
$$+ \frac{1838}{45} m(1-2m) cn^2(\theta/m) + \frac{209}{15} m^2 cn^4(\theta/m) \tag{32}$$

Now, assuming F_3 in terms of Jacobi elliptic function as Fenton [6], we have

$$F_3 = b_1 cn^2(\theta/m) + b_2 cn^4(\theta/m) + b_3 cn^6(\theta/m) \tag{33}$$

$$\therefore F_3^{iv}=8\left[\begin{array}{l}\left[3b_2\left(1-2m+m^2\right)-b_1\left(1-3m+2m^2\right)\right]\\+\left[\begin{array}{l}45b_3\left(1-2m+m^2\right)-30b_2\begin{pmatrix}1-3m\\+2m^2\end{pmatrix}\\+b_1\left(2-17m+17m^2\right)\end{array}\right]cn^2\left(\theta/m\right)\\+\left[\begin{array}{l}-195b_3\left(1-3m+2m^2\right)\\+b_2\left(32-212m+212m^2\right)\\+15b_1m(1-2m)\end{array}\right]cn^4\left(\theta/m\right)\\+\left[\begin{array}{l}b_3\left(162-1017m+1017m^2\right)\\+130b_2m(1-2m)+15b_1m^2\end{array}\right]cn^6\left(\theta/m\right)\\+105m\left[5b_3(1-2m)+b_2m\right]cn^8\left(\theta/m\right)\\+378b_3m^2cn^{10}\left(\theta/m\right)\end{array}\right] \tag{34}$$

Substituting these values in Eq. (30), we have

$$\frac{8}{3}\left[\begin{array}{l}\left[3b_2\left(1-2m+m^2\right)-b_1\left(1-3m+2m^2\right)\right]\\+\left[\begin{array}{l}45b_3\left(1-2m+m^2\right)-30b_2\left(1-3m+2m^2\right)\\+b_1\left(2-17m+17m^2\right)\end{array}\right]cn^2(\theta/m)\\+\left[\begin{array}{l}-195b_3\left(1-3m+2m^2\right)\\+b_2\left(32-212m+212m^2\right)\\+15b_1m(1-2m)\end{array}\right]cn^4(\theta/m)\\+\left[\begin{array}{l}b_3\left(162-1017m+1017m^2\right)\\+130b_2m(1-2m)+15b_1m^2\end{array}\right]cn^6(\theta/m)\\+105m\left[5b_3(1-2m)+b_2m\right]cn^8(\theta/m)\\+378b_3m^2cn^{10}(\theta/m)\end{array}\right]$$

$$+5mcn^2\left(\theta/m\right)\left[\begin{array}{l}2b_1\left(1-m\right)+\left[12b_2\left(1-m\right)-4b_1\left(1-2m\right)\right]cn^2\left(\theta/m\right)\\+\left[30b_3\left(1-m\right)-16b_2\left(1-2m\right)-6b_1m\right]cn^4\left(\theta/m\right)\\+\left[-36b_3\left(1-2m\right)-20b_2m\right]cn^6\left(\theta/m\right)-42b_3mcn^8\left(\theta/m\right)\end{array}\right]$$

$$+\frac{20}{3}m\left[\begin{array}{l}-\frac{1}{675}\begin{pmatrix}10192-86503m+244092m\\-279635m^3+111854m^4\end{pmatrix}\\+\frac{1}{675}\begin{pmatrix}20384-312688m+1373727m^2\\-2122078m^3+1061039m^4\end{pmatrix}cn^2\left(\theta/m\right)\\+\frac{1}{45}m\left(1-2m\right)\left(13868-87800m+87800m^2\right)cn^4\left(\theta/m\right)\\+\frac{2m^2}{45}\left(19508-88433m+88433m^2\right)cn^6\left(\theta/m\right)\\+\frac{2465}{3}m^3\left(1-2m\right)cn^8\left(\theta/m\right)+209m^4cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-\frac{16m^2}{27}\left[\begin{array}{l}1838\left(1-5m+8m^2-4m^3\right)cn^2\left(\theta/m\right)\\-2\left(1-2m\right)\left(919-7060m+7060m^2\right)cn^4\left(\theta/m\right)\\-2m\left(4303-19093m+19093m^2\right)cn^6\left(\theta/m\right)\\-10530m^2\left(1-2m\right)cn^8\left(\theta/m\right)-3762m^3cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-\frac{32}{2025}m^2\left[\begin{array}{l}844561\left(1-m\right)\left(1-2m\right)^2cn^2\left(\theta/m\right)\\-\left(1-2m\right)\left(844561-4530670m+4530670m^2\right)cn^4\left(\theta/m\right)\\-m\left(1996987-8381077m+8381077m^2\right)cn^6\left(\theta/m\right)\\-1545555m^2\left(1-2m\right)cn^8\left(\theta/m\right)\\-393129m^3cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-\frac{4}{45}m\left[\begin{array}{l}\frac{919}{675}\left(1-m\right)\left(1-2m\right)\left(10192-55927m+55927m^2\right)\\-\frac{1}{675}\begin{pmatrix}18732896-222233392m+839954703m^2\\-1235442622m^3+617721311m^4\end{pmatrix}cn^2\left(\theta/m\right)\\-\frac{1}{45}m\left(1-2m\right)\left(6955636-37176844m+37176844m^2\right)cn^4\left(\theta/m\right)\\-\frac{1}{45}m^2\left(12959624-56186114m+56186114m^2\right)cn^6\left(\theta/m\right)\\-\frac{604219}{3}m^3\left(1-2m\right)cn^8\left(\theta/m\right)-43681m^4cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$+\frac{140}{3}m^2\left[\begin{array}{l}\frac{919}{45}\left(1-m\right)^2\left(1-2m\right)-\frac{1}{45}\left(1-m\right)\begin{pmatrix}3676-16585m\\+16585m^2\end{pmatrix}cn^2\left(\theta/m\right)\\+\frac{1}{45}\left(1-2m\right)\left(3676-26488m+26488m^2\right)cn^4\left(\theta/m\right)\\+\frac{1}{15}m\left(5348-24318m+24318m^2\right)cn^6\left(\theta/m\right)\\+\frac{1471}{3}m^2\left(1-2m\right)cn^8\left(\theta/m\right)+209m^3cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-\frac{20}{3}m^2cn^2\left(\theta/m\right)\left[\begin{array}{l}\frac{1}{45}\left(1-m\right)\left(1838-9233m+9233m^2\right)\\-\frac{2}{45}\left(1-2m\right)\left(1838-25028m+25028m^2\right)cn^2\left(\theta/m\right)\\-\frac{2}{45}m\left(23817-121602m+121602m^2\right)cn^4\left(\theta/m\right)\\-2424m^2\left(1-2m\right)cn^6\left(\theta/m\right)-1463m^3cn^8\left(\theta/m\right)\end{array}\right]$$

$$-\frac{16}{675}m\left[\begin{array}{l}\left(1-m\right)\left(1-2m\right)\left(1838-25028m+25028m^2\right)\\-2\begin{pmatrix}1838-103831m+568749m^2\\-929836m^3+464918m^4\end{pmatrix}cn^2\left(\theta/m\right)\\-m\left(1-2m\right)\left(196050-1866000m+1866000m^2\right)cn^4\left(\theta/m\right)\\-10m^2\left(121989-606459m+606459m^2\right)cn^6\left(\theta/m\right)\\-2198700m^3\left(1-2m\right)cn^8\left(\theta/m\right)-1185030m^4cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-500m^3cn^2\left(\theta/m\right)\left[\begin{array}{l}\left(1-m\right)^2-4\left(1-3m+2m^2\right)cn^2\left(\theta/m\right)\\+2\left(2-11m+11m^2\right)cn^4\left(\theta/m\right)\\+12m\left(1-2m\right)cn^6\left(\theta/m\right)+9m^2cn^8\left(\theta/m\right)\end{array}\right]$$

$$-\frac{140}{3}m^2\left[\begin{array}{l}\left(1-m\right)^2\left(1-2m\right)-\left(1-m\right)\left(4-25m+25m^2\right)cn^2\left(\theta/m\right)\\+4\left(1-2m\right)\left(1-13m+13m^2\right)cn^4\left(\theta/m\right)\\+6m\left(6-31m+31m^2\right)cn^6\left(\theta/m\right)\\+75m^2\left(1-2m\right)cn^8\left(\theta/m\right)+45m^3cn^{10}\left(\theta/m\right)\end{array}\right]$$

$$-500m^3cn^4\left(\theta/m\right)\left[\begin{array}{l}-\left(1-3m+2m^2\right)+\left(2-17m+17m^2\right)cn^2\left(\theta/m\right)\\+15m\left(1-2m\right)cn^4\left(\theta/m\right)+15m^2cn^6\left(\theta/m\right)\end{array}\right]$$

$$+\frac{55}{3}m^2cn^2\left(\theta/m\right)\left[\begin{array}{l}2\left(2-19m+34m^2-17m^3\right)\\-8\left(1-2m\right)\left(1-31m+31m^2\right)cn^2\left(\theta/m\right)\\-252m\left(1-6m+6m^2\right)cn^4\left(\theta/m\right)\\-840m^2\left(1-2m\right)cn^6\left(\theta/m\right)-630m^3cn^8\left(\theta/m\right)\end{array}\right]$$

$$+\frac{16}{21}m\left[\begin{array}{l}\left(1-m\right)\left(1-33m+93m^2-62m^3\right)\\-\left(2-259m+1641m^2-2764m^3+1382m^4\right)cn^2\left(\theta/m\right)\\-5m\left(51-738m+1908m^2-1272m^3\right)cn^4\left(\theta/m\right)\\-315m^2\left(7-37m+37m^2\right)cn^6\left(\theta/m\right)\\-4725m^3\left(1-2m\right)cn^8\left(\theta/m\right)-2835m^4cn^{10}\left(\theta/m\right)\end{array}\right]+g_4=0 \tag{35}$$

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Now equating the coefficients of like power of $cn^2(\theta/m)$ from Eq. (35), we obtain the values of b_1, b_2 *and* b_3, i. e.,

$$b_1 = -\frac{1}{4039875}\left(2274828166 - 12828611911m + 12828611911m^2\right)m \quad (36)$$

$$b_2 = -\left(\frac{86755418}{269325}\right)m^2(1-2m) \quad (37)$$

and

$$b_3 = -\left(\frac{3769211}{89775}\right)m^3 \quad (38)$$

Substituting the values b_1, b_2 *and* b_3 in Eq. (33), we get

$$F_3 = -\frac{1}{4039875}m\begin{pmatrix}2274828166 - 12828611911m \\ +12828611911m^2\end{pmatrix}cn^2(\theta/m) - \left(\frac{86755418}{269325}\right)m^2(1-2m)cn^4(\theta/m) - \left(\frac{3769211}{89775}\right)m^3cn^6(\theta/m) \quad (39)$$

From Eq. (23), the value of Y_3 is

$$Y_3 = \frac{1793}{135}m(1-m)(1-2m) + \frac{1}{4039875}m\begin{pmatrix}1557525916 - 9086341036\,m \\ +9086341036\,m^2\end{pmatrix}cn^2(\theta/m) - \frac{24954607}{269325}m^2(1-2m)cn^4(\theta/m) + \frac{7739261}{89775}m^3cn^6(\theta/m) \quad (40)$$

Substituting these value in Eqs. (18) and (19), we obtain

$$\eta_* = 1 + 5m\alpha^2cn^2(\theta/m) - \alpha^4\begin{Bmatrix}\frac{1}{675}\left(10192 - 54802\,m + 54802\,m^2\right) \\ +\frac{1688}{45}m(1-2m)cn^2(\theta/m) \\ -\frac{241}{15}m^2cn^4(\theta/m)\end{Bmatrix} + \alpha^6\begin{bmatrix}\frac{1793}{135}m(1-m)(1-2m) \\ +\frac{1}{4039875}m\begin{pmatrix}1557525916 - 9086341036\,m \\ +9086341036\,m^2\end{pmatrix}cn^2(\theta/m) \\ -\frac{24954607}{269325}m^2(1-2m)cn^4(\theta/m) + \frac{7739261}{89775}m^3cn^6(\theta/m)\end{bmatrix} \quad (41)$$

$$f'_* = 1 - 5m\alpha^2cn^2(\theta/m) + \alpha^4\begin{Bmatrix}\frac{1}{675}\left(10192 - 55927\,m + 55927\,m^2\right) \\ +\frac{1838}{45}m(1-2m)cn^2(\theta/m) \\ +\frac{209}{15}m^2cn^4(\theta/m)\end{Bmatrix} + \alpha^6\begin{bmatrix}-\frac{1}{4039875}m\begin{pmatrix}2274828166 - 1282861191\ 1m \\ +1282861191\ 1m^2\end{pmatrix}cn^2(\theta/m) \\ -\left(\frac{86755418}{269325}\right)m^2(1-2m)cn^4(\theta/m) \\ -\left(\frac{3769211}{89775}\right)m^3cn^6(\theta/m)\end{bmatrix} \quad (42)$$

Also, from Parvin et al. [19], Eq. (22) can be written

$$g_* = 1 - \frac{16}{3}\alpha^2\left(m^2 - m + 1\right) - \frac{40}{3}\alpha^4m(1-2m)(1-m) - \frac{64}{27}\alpha^6m(1-m)\left(20 - 173m + 173m^2\right) \quad (43)$$

Eqs. (41), (42) and (43) are the third order cnoidal wave solutions i.e. which express wave elevation, horizontal wave velocity and acceleration due to gravity for cnoidal wave.

V. Results and Discussion

In Eqs. (41), (42) and (43) we have obtained the third order cnoidal wave solutions. These solutions are obtained for elliptic parameter which is less than one (assuming five numbers say 0.1, 0.2, 0.3, 0.4 and 0.5) and Jacobi elliptic function which tends to one. In Eqs. (41), (42) and (43) taking expansion parameter as positive and negative and so on, it is noted that y1, y2, y3, y4 and y5 are solution curves of Eqs. (41), (42) and (43) shown in Fig. 2-7. In Fig. 2-7, we observe that the variations of elliptic parameter (m=0.1, 0.2, 0.3, 0.4 and 0.5) cause significant changes in the third order cnoidal wave solutions. In Figs. 2 and 3, it is mentioned that the first two curves of third order cnoidal wave increase with the increases of elliptic parameter and the last three curves of third order cnoidal wave decrease with the increases of elliptic parameter. In Figs. 4 and 5, the first two curves of third order cnoidal wave decrease with the increases of elliptic parameter and the last three curves of third order cnoidal wave increase with the increases of elliptic parameter. In Figs. 6 and 7, the first curve of third order cnoidal wave decreases with the increases of elliptic parameter and the last four curves firstly decrease and finally increase rapidly with the increases of elliptic parameter.

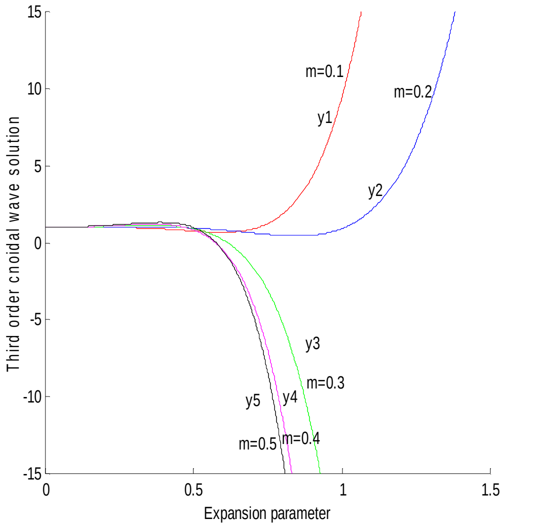

Fig. 2: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$

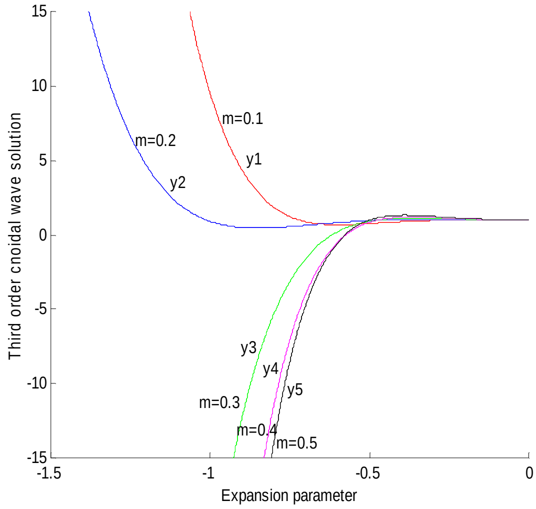

Fig. 3: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$.

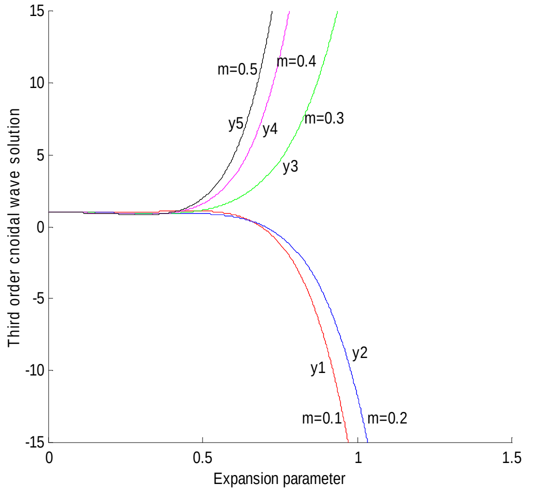

Fig. 4: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$.

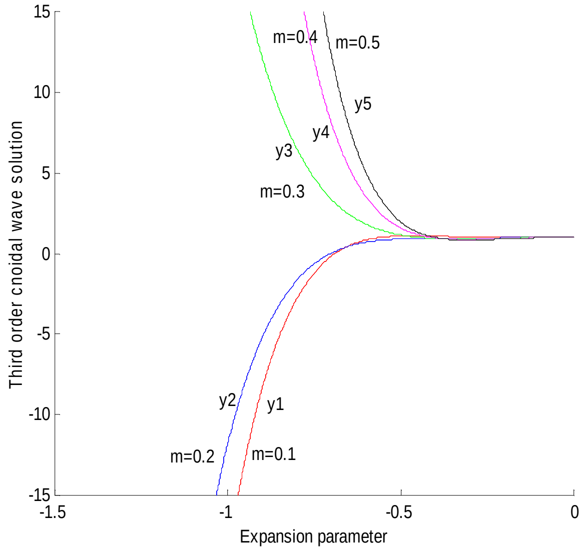

Fig. 5: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$.

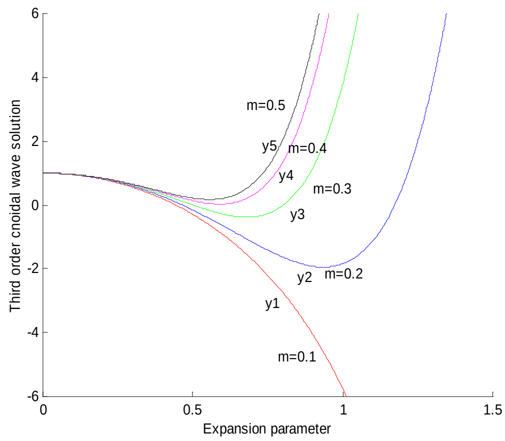

Fig. 6: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$.

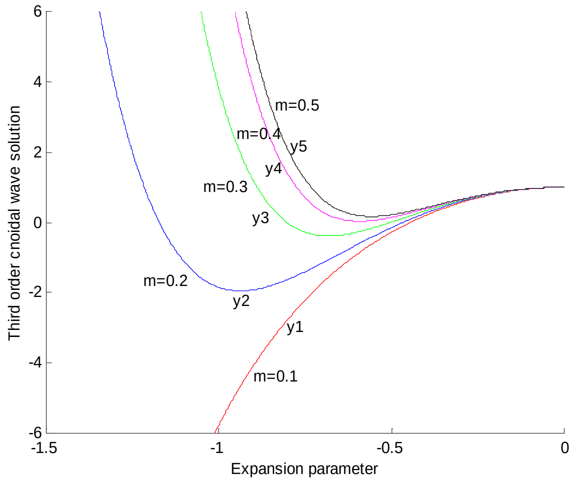

Fig. 7: Third order cnoidal wave solution due to Jacobi elliptic function $cn^2(\theta/m)=1$.

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VI. Velocity Components

To derive velocity components in fluid, Taylor expansion for the stream function is assumed. Then from Eq. (6), we obtain for dimensionless form, consider $X = hX_*,\ Y = hY_*,\ f' = Qf_*'$,

$$\frac{U}{\sqrt{g/h}} = -Q_*\left(\cos Y_* \frac{d}{dX_*}\right) f_*'(X_*)$$

$$= -(g_*)^{-\frac{1}{2}} \left[f_*'(X_*) - \frac{1}{2}\left(\frac{Y}{h}\right)^2 f_*'''(X_*) + \frac{1}{24}\left(\frac{Y}{h}\right)^4 f_*^{v}(X_*) - \dots\dots\dots \right] \quad (44)$$

Hence from Eq. (43) and neglecting higher order term, we get

$$g_*^{-\frac{1}{2}} = 1 + \frac{8}{15}\delta\left(m^2 - m + 1\right) + \frac{4}{75}\delta^2\left(8 - 11m + 9m^2 - 6m^3 + 8m^4\right) + \frac{32}{3375}\delta^3\left(\begin{array}{l} 40 - 55m - 133m^2 + 336m^3 \\ -158m^4 - 30m^5 + 40m^6 \end{array}\right)$$

Since $X = hX_* = \frac{\theta h}{\alpha}$, then Eq. (42), by differentiating in four times and substituting these necessary values in Eq.(44) , we have

$$\frac{U}{\sqrt{g/h}} = -\left[\begin{array}{l} -1 + \delta\left\{\frac{8}{15}\left(m^2 - m + 1\right) - mcn^2(\alpha X_*/m)\right\} \\ + \delta^2 \left\{\begin{array}{l} \frac{4}{16875}\left(\begin{array}{l} 17392 - 65827m + 64027m^2 \\ -5400m^3 + 7200m^4 \end{array}\right) \\ + \frac{2}{1125}m\left(619 - 1538m - 300m^2\right)cn^2(\alpha X_*/m) \\ + \frac{209}{375}m^2 cn^4(\alpha X_*/m) \\ - \frac{1}{10}\left(\frac{Y}{h}\right)^2 \left\{\begin{array}{l} -2m(1-m) \\ +4m(1-2m)cn^2(\alpha X_*/m) \\ +6m^2 cn^4(\alpha X_*/m) \end{array}\right\} \end{array}\right\} \\ -\delta^3 \left[\begin{array}{l} \frac{8}{253125}\left(\begin{array}{l} 22192 - 82619m + 82146m^2 \\ -11054m^3 + 8527m^4 - 9000m^5 \\ +12000m^6 \end{array}\right) \\ - \frac{m}{504984375}\left(\begin{array}{l} 2050270966 \\ -11804817811m \\ +11750952811m^2 \\ +718439400m^3 \\ +215460000m^4 \end{array}\right) cn^2(\alpha X_*/m) \\ - \frac{2m^2}{33665625}\left(\begin{array}{l} 38374249 - 81751958m \\ -5003460m^2 \end{array}\right) cn^4(\alpha X_*/m) \\ - \frac{3769211}{11221875} m^3 cn^6(\alpha X_*/m) \\ - \frac{1}{10}\left(\frac{Y}{h}\right)^2 \left[\begin{array}{l} \frac{4}{1125}\left(619 - 1538m - 300m^2\right)m(1-m) \\ - \frac{4}{1125}\left(\begin{array}{l} 1238 - 7433m + \\ 7433m + 1200m^3 \end{array}\right) mcn^2(\alpha X_*/m) \\ - \frac{4}{25}\left(97 - 214m - 20m^2\right)m^2 cn^4(\alpha X_*/m) \\ - \frac{836}{75} m^3 cn^6(\alpha X_*/m) \end{array}\right] \\ + \frac{1}{75}\left(\frac{Y}{h}\right)^4 \left[\begin{array}{l} m(1-m)(1-2m) \\ - m\left(2 - 17m + 17m^2\right)cn^2(\alpha X_*/m) \\ -15m^2(1-2m)cn^4(\alpha X_*/m) \\ -15m^3 cn^6(\alpha X_*/m) \end{array}\right] \end{array}\right] \end{array}\right] \quad (45)$$

$$
\begin{aligned}
&= 2\alpha\left(\frac{Y}{h}\right)cn(\alpha X_*/m)sn(\alpha X_*/m)dn(\alpha X_*/m) \\
&\left[\begin{aligned}
&\delta m - \delta^2\left\{\begin{aligned}&\frac{2}{1125}m\left(619-1538m-300m^2\right)\\ &+\frac{418}{375}m^2cn^2(\alpha X_*/m)\\ &-\frac{1}{30}\left(\frac{Y}{h}\right)^2\left\{4m(1-2m)+12m^2cn^2(\alpha X_*/m)\right\}\end{aligned}\right\}\\
&+\delta^3\left\{\begin{aligned}
&\frac{m}{504984375}\begin{pmatrix}2050270966-11804817811m\\ +11750952811m^2+718439400m^3\\ +215460000m^4\end{pmatrix}\\
&+\frac{4m^2}{33665625}\begin{pmatrix}38374249-81751958m\\ -5003460m^2\end{pmatrix}cn^2(\alpha X_*/m)\\
&+\frac{3769211}{3740625}m^3cn^4(\alpha X_*/m)\\
&-\frac{1}{30}\left(\frac{Y}{h}\right)^2\left[\begin{aligned}&\frac{4m}{1125}\left(1238-7433m+7433m+1200m^3\right)\\ &+\frac{8}{25}\left(97-214m-20m^2\right)m^2cn^2(\alpha X_*/m)\\ &+\frac{836}{25}m^3cn^4(\alpha X_*/m)\end{aligned}\right]\\
&+\frac{1}{375}\left(\frac{Y}{h}\right)^4\left[\begin{aligned}&m\left(2-17m+17m^2\right)\\ &+30m^2(1-2m)cn^2(\alpha X_*/m)\\ &+45m^3cn^4(\alpha X_*/m)\end{aligned}\right]
\end{aligned}\right\}
\end{aligned}\right]
\end{aligned}
$$

To obtain the vertical component of fluid velocity, we can use the equation that the fluid motion is incompressible,

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$\therefore \frac{V}{\sqrt{g/h}} = -\int_0^{Y_*}\frac{\partial}{\partial X_*}\left(\frac{U}{\sqrt{g/h}}\right)dY_* \qquad (46)$$

VII. Conclusion

Cnoidal wave theory is useful for studying wave motion in shallow water. Here, third order cnoidal wave solutions are formulated using boundary conditions at the bottom and at the free surface. Also the boundary conditions are used from Navier-Stokes equation of motion. Then taking Jacobi elliptic function, third order cnoidal wave solutions have been derived. Finally, the horizontal and vertical fluid velocity components are established using Taylor expansion for the stream function about the bed.

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ACADEMIC QUALIFICATION

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Year :2002 (Held in 2004) **Post Graduation:** Master of Science (M. Sc.) in Applied Mathematics(Thesis)
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

Year : 2001 (Held in 2002) **Graduation :** B. Sc. Honors in Mathematics
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

FIELD OF SPECIALIZATION

- **Current Main Research Field**: Water Waves, Fluid Dynamics, Meteorology.

Awards/Scholarships

1. Scholarship in B.Sc. Program given by the University of Rajshahi, Bangladesh.
2. Scholarship in M.Sc. Program given by the University of Rajshahi, Bangladesh.
3. Scholarship for highest marks in Science Faculty given by Saida Khatun Merit Scholarship, University of Rajshahi, Bangladesh.
4. Agrani Bank Gold Medal award for M.Sc. Program, University of Rajshahi, Bangladesh.

Published Articles

- Published 4(Four) research papers in National and International Journals

Computer Literacy

- MATLAB
- FORTRAN 77, 90
- PROGRAMMING WITH C
- C++.

Languages

- Writing and speaking (fluently): English and Bengali.
- Arabic: Proficiency-Good

PERSONAL DETAILS

Name:	Mst. Shahana Parvin
Date of Birth :	25th June, 1979
Present Address:	**Assistant Professor,** Department of Applied Mathematics, Univrsity of Rajshahi, Rajshahi-6205, Bangladesh.
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ACADEMIC QUALIFICATION

Year :2013 **Doctor of Philosophy (Ph. D.)**
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

Year :2004 **Master of Philosophy (M. Phil)**
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

Year :1997 **Post Graduation:**
Master of Science (M. Sc.) in Applied Mathematics
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

Year : 1996 **Graduation :**
B. Sc. Honors in Mathematics
Institution : University of Rajshahi, Rajshahi-6205, Bangladesh

FIELD OF SPECIALIZATION

- Teaching at Graduate Level: **Calculus, Mathematical Methods, Classical Mechanics**
- Teaching at Post Graduate Level: **Magneto-hydrodynamics and Turbulence, Water Waves.**
- Current Main Research Field: Water Waves.

Awards/Scholarships

1. Fellowships for M. Phil degree from Rajshahi University..
2. Gold Medal for the Faculty first awarded by Rajshahi University.
3. Gold Medal for the 1st class 1st position in B.Sc (Hons.) Degree awarded by Rajshahi University
4. Awarded scholarship for the 1st class 1st position in B.Sc (Hons.) Degree.
5. Awarded the ‘Rajshahi University Prize’ along with a Certificate for M.Sc. Degree.
6. Awarded the ‘Rajshahi University Prize’ along with a Certificate for B. Sc. Degree.
7. Awarded “Engineer Akbar Hossain Scholarship” for B.Sc. (Hons.) Degree.

THESIS SUPERVISION

- 6 M. Sc. Thesis students

PUBLICATIONS

- Published 12(Twelve) research papers in National and International Journals

Languages

- Writing and speaking (fluently): English and Bengali.
- Arabic: Proficiency-Good

PERSONAL DETAILS

Name:	Dr. Mst. Shamima Sultana
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Present Address:	**Associate Professor** Department of Applied Mathematics, Univrsity of Rajshahi, Rajshahi-6205, Bangladesh.
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ACADEMIC QUALIFICATION

Year :1992 **Doctor of Philosophy (Ph. D.)**
Institution : Department of Mathematics, Banaras Hindu University, India.
Thesis Title : **Some Theoretical Investigations in Magneto-hydrodynamic Turbulence.**

Year :1981 **Post Graduation:**
Master of Science (M. Sc.) in Applied Mathematics
Institution : University of Rajshahi
Result : First Class First

Year : 1979 **Graduation :**
Bachelor of Science (B. Sc.)
Institution : Gaibandha Govt. College, Gaibandha
University : Rajshahi
Result : **First Division (5th Place)**

ACADEMIC QUALIFICATION FIELD OF SPECIALIZATION

- Fluid dynamics (Turbulence, MHD Turbulence and Laminar Flow)
- Teaching at Graduate Level: **Business Mathematics and Hydrodynamics.**

- Teaching at Post Graduate Level: **Fluid dynamics.**
- Also conducting research on **Turbulence, MHD Turbulence and Laminar Flow**.

ACADEMIC EXPERIENCES

1. **Professor,** Dept. of Applied Mathematics, Rajshahi University, Rajshahi, from July 19, 2004 to date.
2. **Professor,** Dept. of Mathematics, Rajshahi University, from June 11,1999 to July 18, 2004
3. **Associate Professor,** Dept. of Mathematics, Rajshahi University, from Nov.6,1993 to June 10,1999.
4. **Assistant Professor,** Dept. of Mathematics, Rajshahi University, from July 16, 1989 to Nov. 1993.
5. **Lecturer,** Dept. of Mathematics, Rajshahi University, from July 16, 1986 to July 15, 1989.
6. Lecturer in Mathematics, Sundargonj D.W. College, Sundargonj, Gaibandha, from 25 Sept., 1983 to July, 11, 1984.
7. Adjunct Faculty Member :

(i) Northern University, Bangladesh (Rajshahi Campus). From February 2005 to Dec. 2010.

(ii) Ahsanullah University of Science and Technology,Bangladesh (Rajshahi Campus). From April, 2006 to September, 2007 and April 2008 to September, 2008.

(iii) Part time course teacher: Institute of Business Administration (IBA), Rajshahi University. From July,2002 to December 2004 and July, 2007 to Nov.2008

ADMINISTRATIVE EXPERIENCES

1. Senior Officer, Janata Bank, Lalmonirhat Br. from July 12, 1984 to July12, 1986.
2. **House Tutor,** Shaheed Suhrawardi Hall, Rajshahi University, from March 15, 1992 to Dec. 04, 1994.
3. **Provost**, Sher-E-Bangla Fazlul Haque Hall, Rajshahi University, from Oct. 17, 1996 to Oct. 16, 1999.
4. **Convener,** Provost Council, Rajshahi University, from January 1999 to Oct. 1999.
5. **Proctor**, Rajshahi University, from March 09, 2003 to March 08, 2006.
6. **Chairman**, Department of Applied Mathematics, Rajshahi University, Bangladesh. From October 1, 2008 to Sept.30, 2011.
7. **Senate Member,** Rajshahi University Senate, from Feb. 26, 2005 to date.
8. **Member,** Academic Council, Rajshahi Univ., Rajshahi, from June 11, 1999 to date
9. Member, Faculty of science, Rajshahi University, Rajshahi, from June 11, 1999 to date
10. **Member,** Planning and Academic Committee, Dept. of Applied Math. Rajshahi Univ .from July 19, 2004 to date.
11. **Member,** Appeal and Arbitration Council, Board of Intermediate and Secondary Education, Rajshahi, from Dec. 11, 2004 to Dec., 10, 2007.

OTHER EXPERIENCES

1. Member, Editional Board, Ganit, Journal of Bangladesh Mathematics Society, January, 2008 to December, 2009.
2. Member, Editorial Board, Journal of Science, Rajshahi university studies, from April 8, 2006 to date.
3. Expert Member, Selection Board for Lecturer/ Assistant Professor, in Mathematics, Shahjalal University of Science & Technology, Sylhet,(01/10/2005 to 29/03/2009) Haji Danesh Univ. of Science & Technology, Dinajpur,(01/03/2004 to 21/11/2010) Rajshahi Univ. of Science & technology(RUET), Rajshahi,(01/07/2005 to 30/06/2009), Jahangirnagar University (06/07/2002 to 05/07/2004) and Bangladesh Open University, Dhaka(2006).
4. Expert Member, Selection Board for Professor/ Associate Professor in Mathematics, Dhaka University, Dhaka,(18/03/2003 to 6/02/2011) Shahjalal Univ. of Science & Tech.(SUST), Sylhet,(26/12/2006 to 29/03/2009) and Khulna Univ. of Science & Tech.(KUET) Khulna (07/03/ 2006 to 11/03/ 2008 and 11/03/ 2010 to 10/03/2012)
5. Member, Selection Committee in Applied Mathematics, Accounting & Information System and Population Science & Human Resource Dev. Departments, Rajshahi University (01/11/2006 to 30/10/2007)
6. External Examiner of M. Phil. and Ph. D. in Mathematics, Bangladesh Univ. of Science & Tech.(BUET) Dhaka, Khulna Univ. of Science & Tech.(KUET) Khulna and RCMPS, Chittagong Univ. Chittagong.
7. Reviewer, Ganit, Journal of Bangladesh Math. Soc., Dhaka Univ., Khulna Univ. Journal, Khulna and Islamic univ. Journal, Khustia etc.
8. Reviewer, Journal of Scientific Research, Faculty of Science, Rajsahhi University, Bangladesh
9. Member, Faculty of Applied Sciences and Technology, Islamic University, Kustia, from Sept. 9, 2004 to Sept. 8, 2006.
10. Member, Project Evaluation Committee, University Grants Commission, from July 2006 to June 2007 and July 2008-June 2009.
11. Member, Governing Body, Dental College, Rangpur (Nominatated by Rajshahi University), from 27/08/ 2002 to 29/07/.2010.
12. **Expert Member, Selection Board for Assistant Professor, Associated Professor & Professor in Applied Mathematics, South Asian University, New Delhi, India, July 02-03, 2012**

CONFERENCE/ SEMINAR ATTENDED

1. Summer Science Institute in Mathematics, Dept. of Mathematics, Dhaka Univ. (1987). (Secured Second Position in Performance).
2. Summer Science Institute in Mathematics, Dept. of Mathematics, Rajshahi Univ. (1988). (Secured First Position in Performance)
3. Bangladesh Association for Science (BAAS) International Conference, Jahangirnagar University, Dhaka (1987)
4. BHU Mathematical Conference, Dept. of Mathematics, Banaras Hindu Univ. India (1989)
5. Bangladesh Association for Science (BAAS) International Conference, Bangobandhu Sheikh Mujib Agriculture University, Gazipur, 1992
6. Bangladesh International Mathematical Conference, Dept. of Mathematics, Rajshahi Univ. (1993)
7. Bangladesh International Mathematical Conference, Dept. of Mathematics, Dhaka University Univ., Dhaka (1995)
8. Bangladesh Association for Advancement of Sciences (BAAS) International Conference, Jahangirnagar University, Dhaka (1996)
9. Bangladesh International Mathematical Conference, RCMPS, Chittagong, Univ. (1999)
10. 25th International Nathiagali Summer College on Physics and Contemporary Needs, Islamabad, Pakistan (2000)
11. International Conference on Geometry, Analysis and Applicatoions, Banaras Hindu Univ. India (2000).
12. Bangladesh International Mathematical Conference, Dept. of Mathematics, Dhaka University Univ., Dhaka (2007).
13. Bangladesh International Mathematical Conference, Dept. of Mathematics, Bangladesh University of Engineering and Technology (BUET) Dhaka (2009)

MEMBERSHIP

1. Vice-President, Bangladesh Mathematical society, from Jan-2008-Dec-2009.
2. Life-Member, Bangladesh Mathematical Society.
3. Member, Bangladesh Association for Advancement of Science(BAAS).

THESIS SUPERVISION

- Supervised 7(Seven) Ph. D.
- 3(Three) M. Phil. and 10(Ten) M. Sc. Students.
- At present supervising 1(one) Ph. D. and 1(M. Sc.) M. Sc. student.

PUBLICATIONS

- Published 65(Sixty five) research papers in National and International Journals

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