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Relations between Special Polygonal Numbers Generated through the Solutions of Pythagorean Equation

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***Abstract* – Employing the solutions of the Pythagorean equation, we obtain the relations between the pairs of special polygonal numbers such that the difference in each pair is a perfect square.**

***Keywords* – Ternary Quadratic Equation, Integer Solutions, Pythagorean Equation, Polygonal Numbers Centered Polygonal Numbers.**

I. Introduction

In [1,2,4-8,10], employing the integral solutions of special binary quadratic Diophantine equation, special patterns of Pythagorean triangles are generated. In [3], the relations among the pairs of special m-gonal numbers generated through the solutions of the binary quadratic equation $y^2 = 2x^2 - 1$ are determined. In[9], the relations among special figurate numbers through the equation $y^2 = 10x^2 + 1$ are obtained. In[11,12] employing the solutions of the Pythagorean equation, the relations between Triangular number and Pentagonal number, Octagonal number, Hexagonal number, Heptagonal number, Decagonal number, Square number, Dodecagonal number, Pentagonal number and Hexagonal number, Octagonal number such that the difference in each pair is a perfect square are obtained. In this communication, employing the solutions of the Pythagorean equation, we obtain the relations between the pairs of special polygonal numbers which are not mentioned in [11,12] such that the difference in each pair is a perfect square.

NOTATIONS:

$t_{m,n}$ =*Polygonal number of rank n with sides m*

$ct_{m,n}$ =*Centered polygonal number of rank n with sides m*

II. Method of Analysis

Consider the Pythagorean equation

$$x^2 + y^2 = z^2 \qquad (1)$$

whose solutions are x=2rs; y= $r^2 - s^2$; z= $r^2 + s^2$ (2)

Relation: 1

We illustrate below the process of obtaining relations between the pairs of special polygonal numbers such that each relation is a perfect square.

The choices 8N-3= $r^2 + s^2$, 2M+1= $r^2 - s^2$ (3)
in (1) leads to the relation $16t_{10,N} - 8t_{3,M} + 8$ is a square integer

From (3), the values of ranks of the Decagonal and triangular numbers are respectively given by

$$N=\frac{r^2+s^2+3}{8},\ M=\frac{r^2-s^2-1}{2}$$

It is seen that N and M are integers when
(a) r=4k-2, s=4k-3 and (b) r=4k-1, s=4k-2.

For case (a), the corresponding values of M and N are given by M=4k-3, N= $4k^2 - 5k + 2$

Thus the value of $t_{10,N}$ and $t_{3,M}$ are respected by

$$t_{10,N} = 64k^4 - 160k^3 + 152k^2 - 65k + 10$$

$$t_{3,M} = 8k^2 - 10k + 3$$

Note that $16t_{10,N}$ -$8t_{3,M}$ +8= $[2(16k^2 - 20k + 6)]^2$

For case (b), the corresponding values of M and N are given by M=4k-2, N= $4k^2 - 3k + 1$

Thus the values of $t_{10,N}$ and $t_{3,M}$ are respected by $t_{10,N} = 64k^4 - 96k^3 + 56k^2 - 15k + 1$

$$t_{3,M} = 8k^2 - 6k + 1$$

Note that $16t_{10,N}$ - $8t_{3,M}$ +8= $[2(16k^2 - 12k + 2)]^2$

Relation2:

The choices 8N-3= $r^2 + s^2$, 5M-2= $r^2 - s^2$ (4)
in (1) leads to the relation

$16t_{10,N} - 5t_{12,M} + 5$ is a square integer (5)

From (4), the values of ranks of the Decagonal and Dodecagonal numbers are respectively given by

$$N=\frac{r^2+s^2+3}{8},\ M=\frac{r^2-s^2+2}{5}$$

which are integers for the following choices of r, s namely,
r=5k-3; s=5k-4

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and we have N= $\frac{1}{8}[50k^2-70k+28]$ and
M=2k-1

Thus the Decagonal number and Dodecagonal number are represented as follows,

$t_{10,N}=4[\frac{1}{8}(50k^2-70k+28)]^2-3[\frac{1}{8}(50k^2-70k+28)]$

$t_{12,M}=[5(2k^2-1)^2-4(2k-1)]$

Note that $16t_{10,N}-5t_{12,M}+5=[2(25k^2-35k+12)]^2$

Relation 3:

The choices 2N+1= r^2+s^2, M= r^2-s^2 (6)

in (1) leads to the relation $8t_{3,N}-t_{4,M}+1$ is a square integer (7)

From (6), the values of ranks of the Triangular and square numbers are respectively given by

$$N=\frac{r^2+s^2-1}{2},\ M=r^2-s^2$$

which are integers for the following choices of r, s namely,

r=k+1; s=k

and we have N= k^2+k and
M=2k+1

Thus the Triangular number and square number are represented as follows,

$t_{3,N}=\frac{1}{2}[(k^2+k)(k^2+k+1)]$

$t_{4,M}=k^2$

Note that $8t_{3,N}-t_{4,M}+1=[2k^2+2k]^2$

Relation 4:

The choices 2N+1= r^2+s^2, 14M-5= r^2-s^2 (8)

in (1) leads to the relation $8t_{3,N}-56t_{9,M}-24$ is a square integer (9)

From (6), the values of ranks of the Triangular and Nonagonal numbers are respectively given by

$$N=\frac{r^2+s^2-1}{2},\ M=\frac{r^2-s^2+5}{14}$$

which are integers for the following choices of r, s namely,

r = 7k-2; s = 7k-3

and we have N= $49k^2-35k+6$ and
M=k

Thus the Triangular number and Nonagonal number are represented as follows,

$t_{3,N}=\frac{1}{2}[(49k^2-35k+6)(49k^2-35k+7]$

$t_{9,M}=\frac{1}{2}[7k^2-5k]$

Note that, $8t_{3,N}-56t_{9,M}-24=[2(49k^2-35k+6)]^2$

For simplicity, a few examples are presented below in a tabular form:

Table 1; M= r^2-s^2, 5N-2= r^2+s^2

M	N	$5t_{12,N}-t_{4,M}+4$
10k-5	$10k^2-10k+3$	$[2(25k^2-25k+6)]^2$

Table 2: 4 M-1= r^2-s^2, 2N+1= r^2+s^2

M	N	$4ct_{6,N}-24t_{6,M}-4$
K	$4k^2-2k$	$3[4k(2k-1)]^2$

Table 3: 12 M-5= r^2-s^2, 2N+1= r^2+s^2

M	N	$8t_{3,N}-24t_{14,M}-24$
K	$36k^2-30k+6$	$[2(36k^2-30k+6)]^2$

Table 4: 10 M-3= r^2-s^2, 6N-1= r^2+s^2

M	N	$40t_{7,M}-24t_{5,N}+8$
3k	$75k^2-15k+1$	$[2(225k^2-45k+2)]^2$

Conclusion

In this paper, we have presented relations between the pairs of special polygonal numbers by employing the solutions of Pythagorean equation. To conclude one may consider different choices of ternary quadratic diophantine equation and may attempt to obtain various relations between special polygonal and centered polygonal numbers.

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