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Quantum Theory of Systems with Friction and Electric Resistance of Circuits and Applications to Radiation Physics

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Abstract **– Friction incorporates a close connection between classical mechanics and irreversible thermodynamics. The translation to a quantum mechanical foundation is not trivial and needs a generalization of the Lagrangian. A change to electromagnetic circuits appears to more adequate, since the electric analogue (Ohm's law) is related to scatter of electrons at lattice vibrations. Important applications of quantized circuits with damping are electronic transitions of molecules (example: H_3PO_4), the emission of radiation by forced oscillators such as the design of a klystron and modeling of 'bremsstrahlung' created in a tungsten target or via synchrotron.**

Keywords **– Bremsstrahlung, Quantized Circuits and Resistance, Friction And Quantum Mechanics, Klystron, Synchrotron.**

I. INTRODUCTION

With regard to the motion of macroscopic systems friction plays a significant role in all dynamical processes of classical mechanics, since friction is connected to energy dissipation, which may be considered as a special case of the Langevin equation [1] - [3]:

$$M\cdot\ddot{q}+\gamma\cdot\dot{q}+\tfrac{\partial U}{\partial q}=0. \qquad (1)$$

It is usual to start with the Lagrange formalism and to write (1) in the following way:

$$Ł_0=\tfrac{M}{2}\cdot\dot{q}^2-U(q)\rightarrow \tfrac{d}{dt}\cdot\tfrac{\partial Ł_0}{\partial\dot{q}}-\tfrac{\partial Ł_0}{\partial q}=D(\gamma,\dot{q}). \quad (2)$$

$Ł_0$ is the 'standard' Lagrangian without friction, D a dissipation function and U the potential, and the free motion of a mass M (U = 0) suffers continuously slowing down with the range R given by (boundary conditions at t = 0: q = 0 and $\dot{q}=v_0$):

$$q(t)=R-(v_0\cdot M/\gamma)\cdot\exp(-\gamma\cdot t/M)$$

$$q(t\rightarrow\infty)\rightarrow \quad R=\tfrac{v_0\cdot M}{\gamma}. \qquad (3)$$

In the past decades, many attempts have been developed to describe friction properties by a nonlinear Schrödinger equation with a logarithmic nonlinearity [4 - 7]:

$$i\cdot\hbar\tfrac{\partial}{\partial t}\psi-H_0\psi=$$

$$\lambda\cdot[i\cdot\ln(\psi/\psi^*)\cdot\psi+i\cdot\langle\ln(\psi/\psi^*)\rangle]\psi. \qquad (4)$$

H_0 refers to the standard Hamiltonian, which appears in the linear Schrödinger equation. This extension leads to the problem, in which way this equation is applicable in the microscopic domain. Thus the stopping of fast particles, e.g. protons or electrons can be described by the Bethe-Bloch equation derived by quantum mechanical perturbation theory. It reads:

$$-\tfrac{dE(z)}{dz}=\tfrac{K}{v^2}\cdot[\ln(\tfrac{2mv^2}{E_I})-\ln(1-\beta^2)+\text{cor.terms}]$$

$$K=\tfrac{Z\cdot\rho}{2\cdot A_N\cdot m}\cdot 8\pi\cdot e_0^{\,4}\cdot q_p^2. \qquad (5)$$

$$\text{cor.terms: } a_{shell}+a_{Barkas}+a_{Bloch}. \qquad (5a)$$

All terms of (5) and (5a) and the integration-procedure have been presented in detail [8] - [9]. The contributions a_{shell}, a_{Barkas} and a_{Bloch} represent correction terms; ρ is the density of the medium (e.g. water: ρ =1 g/cm^3 is often a reference medium), Z and A_N are the effective charge/mass number of the medium, q_p is the charge of the projectile particle, e_0 the elementary charge, E_I an averaged ionization energy by inclusion of all possible energy levels of the medium and ß = v/c. All calculations of E(z) of projectile particles are based on (5). It appears that the conception of friction for slowing down of microscopic particles can hardly be included by the classical friction. It should be noted that a modified version of (1) with a completely different friction term can be used to derive empirical properties of slowing down of the motion of protons and neutrons:

$$M\cdot\ddot{q}+\delta\cdot\dot{q}^{-ß}=0. \qquad (6)$$

The integration of this equation is straightforward and leads to the range formula (*c*ontinuous slowing *d*own *a*pproximation → csda):

$$R_{csda}=A\cdot E_0^{\,p}. \qquad (7)$$

A is a constant factor in (7), E_0 is the initial energy and p ≈1.74 for protons and p = 1.5 (ß = 1) for slow neutrons (Geiger rule). The empirical energy – range relation can be derived by the above formula. With regard to fast electrons ($E_{kinetic} > mc^2$) and fast protons ($E_{kinetic} > 200$ MeV) (6) has to be modified to include relativistic corrections. The derivation of the relativistic modification of (5) is given in [8]:

$$R_{csda}=A\cdot(E_0+E_0^{\,2}/2Mc^2)^p$$

$$p=1+ß/2;\ A=2^{p-1}/(\delta\cdot M^{\,p-1}). \qquad (8)$$

(5–7) show that fast micro-particles are stopped by passing through a medium (e.g. water) with an energy loss being proportional to a power of the inverse velocity, i.e. ≈ $1/v^ß$ with 1≤ ß < 2. The fact is, in particular, true for (5). However, this equation indicates that the stopping power - dE(z)/dz cannot be simply reduced to one power of the velocity *v* or the actual kinetic energy *E(z)*, since the correction terms denoted by '*cor.terms'* lead to intricate modifications of the stopping power. Thus a very accurate adaptation of the solutions given by (5) via (8) indicates that the power *p* is slightly depending on the initial energy E_0 of the projectile particle, which may vary between *p = 1.70* and *p = 1.76* [7]. It has also to be mentioned that (5 – 8) are restricted to '*csda'*. In order to determine the energy

loss - dE/dz of a projectile particle by passing through a medium energy straggling and lateral scatter have to be included [8], where further details can be obtained. Therefore the question arises, whether (1) is at all adequate in the molecular level. We mainly think of molecular oscillation with a reduced mass in the normal mode. Usually the energy of these oscillations can be restricted to the domain of thermal energy ($\approx k_B \cdot T$), and then the friction term of classical physics may be applicable. This motion can be damped by collisions with neighboring molecules or energy transfer via dipole-dipole interactions (van der Waals interactions). Classically this equation of motions reads:

$$M \cdot \ddot{q} + \gamma \cdot \dot{q} + M \cdot \omega_0^2 \cdot q = 0 \rightarrow \omega_0^2 = f / M. \quad (9)$$

Thus *f* represents the force constant, and the reduced frequency of (9) is given by:

$$\omega^2 = \omega_0^2 - \gamma^2 / (4 \cdot M^2). \quad (10)$$

Equations (9 - 10) make also sense in quantum mechanics; they require a modification of the Lagrange function $Ł_0$ we consider in the following section. It should be mentioned that the consideration of electric resistance (Ohm's law) in circuits appears to be more adequate for a quantum theoretical treatment.

II. Modification of Lagrangian $Ł_0$ - Hamilton Formalism and Schrödinger Equation with Irreversible Correction Terms

The required generalization of the Lagrange function is given by:

$$Ł = Ł_0 \cdot \exp(\gamma \cdot t / M). \quad (11)$$

Thus the application of the Lagrange formalism according to (2) with regard to (11) yields (1). The determination of the canonical momentum P due to (11) and using the Hamiltonian formalism yields:

$$H = \exp(-\gamma \cdot t / M) \cdot P^2 / 2M + V(q) \cdot \exp(\gamma \cdot t / M). \quad (12)$$

The Schrödinger equation of (12) reads:

$$-\frac{\hbar^2}{2M} \cdot \exp(-\gamma \cdot t / M) \frac{\partial^2}{\partial q^2} \psi(q,t) + V(q) \cdot \exp(\gamma \cdot t / M) \cdot \psi(q,t) = i \cdot \hbar \cdot \frac{\partial}{\partial t} \cdot \psi(q,t). \quad (13)$$

Although the physical sense may be questionable, we consider first the solution of a free particle with V(q) = 0. The particle mass M should be much bigger than that of a proton mass and the velocity v << c. Thus the motion of molecules in a medium appears to be adequate; the phenomenological friction term γ may be realized by interactions with the environment (e.g. a solvent).

In a first step, we start with the *'ansatz'* for a 'free' particle:

$$\psi(q,t) = A(t) \cdot \exp(-i \cdot k \cdot q). \quad (14)$$

Inserting (14) into (13) provides:

$$\psi(q,t) = A_0 \cdot \exp(-i \cdot k \cdot q) \cdot \exp[-\frac{i \cdot \hbar \cdot k^2 (1 - \exp(-\gamma \cdot t / M))}{2\gamma}]. \quad (15)$$

However, this solution (15) has the disadvantage that for $t \rightarrow \infty$ a comparison with (3) is hardly possible, whereas a low order expansion of $(1 - \exp(-\gamma \cdot t/M))/2\gamma$ makes sense:

$$\exp[-\frac{i \cdot \hbar \cdot k^2 (1 - \exp(-\gamma \cdot t / M))}{2\gamma}] \approx \exp(-\frac{i \cdot \hbar \cdot k^2 \cdot t}{2M}). \quad (16)$$

Equation (16) incorporates the initial behavior at rather small time intervals. In order to be free of this restriction, the integration over all possible '*k-values'* has to be performed:

$$\psi(q,t) = \frac{A_0}{2 \cdot \varepsilon \cdot \sqrt{\pi}} \cdot \sqrt{\frac{2\gamma}{i \cdot \hbar}} \exp(\frac{i \cdot \gamma \cdot q'^2}{2\hbar \cdot \varepsilon^2}). \quad (17)$$

$$\varepsilon = \sqrt{1 - \exp(-\gamma \cdot t / M)}; \; q' = q - \bar{q}. \quad (18)$$

The normalization of the wave-function yields:

$$A_0^2 = \frac{2\pi\varepsilon^2 \hbar}{\gamma \cdot \bar{q}}. \quad (19)$$

Now the range relation (3) results by forming the expectation value $<p>/M$ at $t \rightarrow \infty$.

A typical task is the damped harmonic oscillator **V(q) = $\mathbf{M \cdot \omega_0^2 \cdot q^2/2}$**. This type of potential is similar to the motion of charged molecules in constant magnetic fields. By that, we obtain the quantum mechanical analogue of (8) and (9). The Schrödinger equation of the damped harmonic oscillator reads:

$$-\frac{\hbar^2}{2M} \cdot \exp(-\gamma \cdot t / M) \cdot \frac{\partial^2}{\partial q^2} \psi(q,t) + V(q) \cdot \exp(\gamma \cdot t / M) \cdot \psi(q,t) = i\hbar \frac{\partial}{\partial t} \psi(q,t). \quad (20)$$

This problem is identical to the charge quantization problem of a circuit with a damping term (Ohm's). The ground state properties of the damped harmonic oscillator are given by:

$$\psi_0(q,t) = \exp[-\tfrac{1}{2}\xi^2] \cdot \exp(i \cdot \tfrac{1}{2} \cdot a \cdot t). \quad (21)$$

$$\xi^2 = \frac{M \cdot \omega}{\hbar} \cdot q^2 \cdot \exp(\gamma \cdot \cdot t / M)$$

$$\omega^2 = \omega_0^2 - \frac{\gamma^2}{4 \cdot M^2}; \; a = [\frac{i \cdot \gamma}{2M} \pm \omega] \; t / M). \quad (22)$$

Thus the general solution can readily be constructed from the ground state by introducing Hermite polynomials H_n:

$$\psi_n(q,t) = H_n(\xi) \cdot [\exp(-\tfrac{1}{2} \cdot \xi^2) \cdot \cdot \exp(i \cdot (n + 1/2) \cdot a \cdot t)]; (n = 0,1..). \quad (23)$$

Equations (21 - 23) possess the same reduced frequency ω as in the classical case (9), and due to friction stationary states do not exist. The behavior of these solutions can be characterized by switching on the friction at the initial condition t = 0. It should be mentioned that with regard to different solutions ψ_n and ψ^*_m the orthogonal relations exist. An important feature is the motion of a charged particle (e.g. an ionic molecule) in a constant magnetic with an additional damping. The Hamiltonian of this problem reads:

$$\frac{1}{2M} \exp(-\gamma \cdot t / M) \cdot (\frac{\hbar}{i} \nabla - \frac{q_0}{c} \vec{A})^2 \psi = i\hbar \frac{\partial}{\partial t} \psi. \quad (24)$$

Denoting the coordinates by *{x, y, z}* and q_0 the electric charge of a ion, we define the vector potential $\vec{A}$ by:

$$A_x = -B_0 \cdot y; \; B_x = B_y = 0; B_z = -\frac{\partial}{\partial y} A_x = B_0. \quad (25)$$

By that, (24) assumes the shape:

$$\exp(-\gamma \cdot t/M)\cdot H = i\hbar\frac{\partial}{\partial t}\psi$$

$$H = [-\frac{\hbar^2}{2M}\Delta - \frac{i\cdot\hbar\cdot q_0}{M\cdot c}B_0\cdot y\cdot\frac{\partial}{\partial x} + \frac{{q_0}^2}{2Mc^2}{B_0}^2\cdot y^2]. \quad (26)$$

The solution of (26) can be found in every textbook of quantum mechanics, if the γ = 0 (stationary case), and the Larmor frequency is given by:

$$\omega_0 = \frac{q_0\cdot B_0}{M\cdot c}. \quad (27)$$

By taking account of γ ≠ 0 we have to modify the 'standard' solution by:

$$\psi(x,y,z,t) = A(t)\cdot\exp(i\cdot(\alpha\cdot x+\beta\cdot z))\cdot\varphi(y',t)$$

$$y' = y - \frac{\hbar\cdot\alpha\cdot c}{q_0\cdot B_0}. \quad (28)$$

Equation (28) provides the well-known oscillator equation with inclusion of friction:

$$\exp(-\gamma\cdot t/M)\cdot[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial y'^2} + \frac{M\cdot\omega^2}{2}\cdot y'^2]\varphi$$

$$= i\hbar\frac{\partial}{\partial t}\varphi. \quad (29)$$

Equation (29) can be solved by the previously elaborated methods to yield:

$$\phi_n(\xi,t) = A_0\cdot H_n(\xi)\cdot\exp(-\frac{\xi^2}{2})\cdot$$

$$\cdot\exp[\frac{M}{i\cdot\hbar\cdot\gamma}\cdot(1-\exp(-\frac{\gamma\cdot t}{M}))];\ \omega^2 = \omega_0^2 - \frac{\gamma^2}{4M^2}$$

$$\xi^2 = \frac{M\cdot\omega}{\hbar}\cdot y'^2 = \frac{M\cdot\omega}{\hbar}\cdot(y+\frac{\hbar\cdot\alpha\cdot c}{q_0\cdot B_0})^2. \quad (30)$$

The stationary case provides the Landau levels of charged particles, which cannot be satisfied by the time-depending damping function. The terms referring to exp(i·α·x) and exp(i·β·z) have already been given by the previous methods (i.e. *motion of free particles with friction)* and can readily be added to obtain the complete wave-function ψ.

By the substitutions M → L, ω_0^2→ 1/LC, q(position) →Q (electric charge), γ(friction) → R(Ohm's resistance) the treatment of damped circuits is straightforward. Therefore it is possible to simplify the following section by adopting the methods already worked out.

III. Quantized Electromagnetic Circuits with Damping (Ohm's Law)

The extension of Lagrange formalism to electromagnetic circuits is more promising. A main reason comes from molecular physics/biophysics, since the charge distribution in a molecule (or molecular environment) can be treated as a capacity *C*, whereas transitions to different states represent electric currents associated with the inductivity *L*. Very impressive examples are the H bonds between the DNA base pairs, where the exchange protons represent a tunnel current and simultaneously induce a weak local magnetic field [10] - [11].

At first, according to Fig. 1A with the case R ≠ 0 the generalized Lagrange function **Ł** is given by:

$$Ł = Ł_0\cdot\exp(R\cdot t/L)$$

$$Ł_0 = \frac{L}{2}\cdot\dot{Q}^2 - \frac{L\cdot\omega_0^2}{2}\cdot Q^2;\ \omega_0^2 = 1/(LC). \quad (31)$$

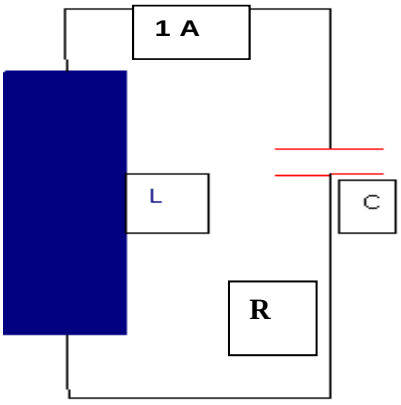

Fig.1A. One single electric circuit oscillator with the following abbreviations L: inductivity, C: capacity, and R: Ohm's resistance. These abbreviations are also applicable in the following figures.

The Hamilton operator and the Schrödinger equation in the charge space are obtained with the help of the canonical momentum:

$$P = \frac{\partial}{\partial\dot{Q}}\cdot Ł(Q,\dot{Q},t). \quad (32)$$

It should be pointed out that the canonical momentum P is equivalent to the magnetic flux Φ. The time-dependent Schrödinger now assumes the shape:

$$-\frac{\hbar^2}{2\cdot L}\cdot\exp(-R\cdot t/L)\cdot\frac{\partial^2}{\partial Q^2}\cdot\psi(Q,t) +$$

$$\frac{L\cdot\omega_0^2}{2}\cdot Q^2\cdot\exp(R\cdot t/L)\cdot\psi(Q,t) =$$

$$i\cdot\hbar\frac{\partial}{\partial t}\cdot\psi(Q,t). \quad (33)$$

According to previous results the general solution is:

$$\psi_n(Q,t) = H_n(\xi)\cdot\exp(-\frac{1}{2}\cdot\xi^2)\cdot\exp(i\cdot(n+1/2)\cdot a\cdot t)$$

$$(n=0,1,,,,.)$$

$$\xi^2 = \frac{L\cdot\omega}{\hbar}\cdot Q^2\cdot\exp(R\cdot t/L);\ \omega_0^2 = \frac{1}{LC};$$

$$\omega^2 = \omega_0^2 - \frac{R^2}{4\cdot L^2};\ a = [\frac{i\cdot R}{2L}\pm\omega]. \quad (34)$$

The case with two magnetically coupled electric circuits is presented in Fig. 1B (the mutual inductivity between the circuits is denoted by *M* and the electric charge by *Q*). It should also be pointed out that the electric resistance R (Ohm's law) and the related heat production can be regarded as an obvious microscopic problem, since it results from collisions of electrons with lattice vibrations. Therefore the quantum mechanical consideration is justified.

In absence of an electric resistance (R = 0) the equations for the two coupled circuits are given by:

$$L\cdot\ddot{Q}_1 + M\cdot\ddot{Q}_2 + Q_1/C = 0$$

$$L\cdot\ddot{Q}_2 + M\cdot\ddot{Q}_1 + Q_2/C = 0. \quad (35)$$

The normal modes are given by the substitutions:

$$q_1 = Q_1 - Q_2;\ \ q_2 = Q_1 + Q_2. \quad (36)$$

The Lagrangian of the two resulting equations for the normal modes is given by:

$$Ł_1 = \tfrac{\lambda_1}{2}\cdot \dot{q}_1^{\,2} - \tfrac{\lambda_1\cdot\omega_1^2}{2} q_1^{\,2};\ Ł_2 = \tfrac{\lambda_2}{2}\cdot \dot{q}_2^{\,2} - \tfrac{\lambda_2\cdot\omega_2^2}{2} q_2^{\,2}$$

$$\lambda_1 = L - M;\ \lambda_2 = L + M;\quad \omega_{1,2}^{\,2} = \tfrac{1}{C\cdot\lambda_{1,2}}. \qquad (37)$$

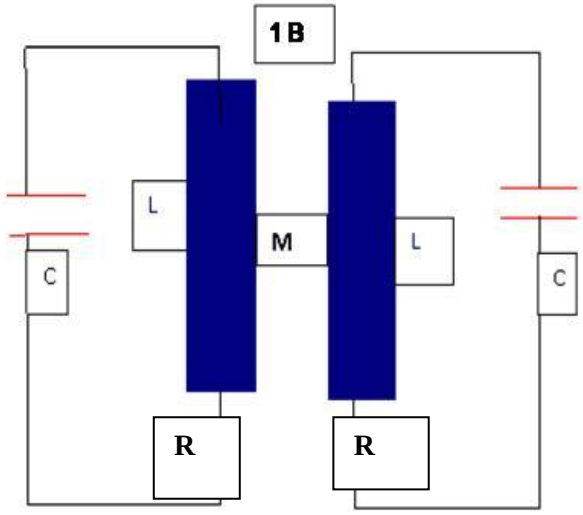

Fig.1B. Two identical oscillators with mutual coupling M of the currents (magnetic coupling).

Thus the Lagrange functions Ł$_1$ and Ł$_2$ provide the tools to determine the canonical momentum p_1 and p_2 and the application of canonical commutation relations: $[p_1, q_1] = [p_2, q_2] = -i\cdot\hbar$. We do not consider here the Schrödinger equation in the charge space. However, with the help of creation/annihilation operators the problem can be treated as known from the oscillators in the position space:

$$b_k = \alpha_k \cdot q_k + i\beta_k p_k$$

$$b_k^+ = \alpha_k \cdot q_k - i\beta_k p_k$$

$$\alpha_k^{\,2} = \tfrac{\lambda_k\cdot\omega_k}{2\cdot\hbar},\ \beta_k^{\,2} = \tfrac{1}{2\cdot\hbar\cdot\lambda_k\cdot\omega_k},\ (k = 1, 2). \qquad (38)$$

The algebra of the operators b_k and b_k^+ is well-known, i.e. $[b_k, b_k^+] = 1$, (k = 1, 2), and the Hamilton operator of the normal modes reads:

$$H = \hbar\cdot\omega_1\cdot(n + 1/2)\cdot b_1^+\cdot b_1$$

$$+\hbar\cdot\omega_2\cdot(m + 1/2)\cdot b_2^+\cdot b_2,\ (n, m = 0,1,2\ldots). \qquad (39)$$

In the presence of the Ohm's resistance with R ≠ 0 we may either solve the Schrödinger equation for each mode with k = 1, 2 (12, 13) or modify the creation/annihilation operators. The Hamiltonian now becomes:

$$H = \sum_{k=1}^{2}\left(\tfrac{1}{2\cdot\lambda_k}\cdot p_k^2\cdot\exp\left(-\tfrac{R\cdot t}{\lambda_k}\right) + \tfrac{\lambda_k\cdot\omega_k^2}{2}\cdot q_k^2\cdot\exp\left(\tfrac{R\cdot t}{\lambda_k}\right)\right). \qquad (40)$$

In order to formulate (40) according to (38) we have to perform the following modification:

$$b_k'(t) = \alpha_k\cdot q_k\cdot\exp\left(\tfrac{R\cdot t}{2\cdot\lambda_k}\right) + i\beta_k p_k\cdot\exp\left(-\tfrac{R\cdot t}{2\cdot\lambda_k}\right)$$

$$b_k^{+\prime}(t) = \alpha_k\cdot q_k\cdot\exp\left(\tfrac{R\cdot t}{2\cdot\lambda_k}\right) - i\beta_k p_k\cdot\exp\left(-\tfrac{R\cdot t}{2\cdot\lambda_k}\right). \qquad (41)$$

$$b_k'(t)\,b_k^{+\prime}(t) - b_k^{+\prime}(t)\,b_k'(t) = 1.\quad (k = 1,2)$$

$$\omega_k^2 = \tfrac{1}{C\cdot\lambda_k} - \tfrac{R^2}{4\cdot\lambda_k^2},\quad (k = 1, 2). \qquad (42)$$

Since a stationary form of (39) does not exist, we have to write the problem similar to the time-dependent Schrödinger equation:

$$i\hbar\,\tfrac{\partial}{\partial t}\psi = H\cdot\psi$$

$$H = \tfrac{\hbar}{2}\sum_{k=1}^{2}\omega_k\cdot[b_k'(t)\,b_k^{+\prime}(t) + b_k^{+\prime}(t)\,b_k'(t)]. \qquad (43)$$

Thus the handling of (43) according to the algebra (41) is a little more difficult to calculate excited states and transitions, since the time-dependent version has to be accounted for each excited state $\psi_n(b_k'(t), b_k^{+\prime}(t), t)$. The procedure is equivalent to that of (34).

IV. Coupled Oscillator Circuits

Fig.2 presents three identical oscillators, which are mutually coupled by the interaction inductivity M. The well-known dynamical equations are:

$$L\cdot\ddot{Q}_1 + M\cdot(\ddot{Q}_2 + \ddot{Q}_3) + Q_1/C = 0$$

$$L\cdot\ddot{Q}_2 + M\cdot(\ddot{Q}_1 + \ddot{Q}_3) + Q_2/C = 0$$

$$L\cdot\ddot{Q}_3 + M\cdot(\ddot{Q}_1 + \ddot{Q}_2) + Q_3/C = 0. \qquad (44)$$

With the help of the following substitutions we obtain the normal modes:

$$q_1 = Q_1 - Q_3;\ q_2 = Q_2 - Q_1;\ q_3 = Q_1 + Q_2 + Q_3. \qquad (45)$$

These substitutions imply the Lagrange function of the normal modes:

$$Ł_1 = \tfrac{1}{2}\lambda_1\cdot\dot{q}_1^2 - \tfrac{\lambda_1}{2}\omega_1^2\cdot q_1^2;\quad (\lambda_1 = L - M;\omega_1^2 = \tfrac{1}{\lambda_1\cdot C})$$

$$Ł_2 = \tfrac{1}{2}\lambda_2\cdot\dot{q}_2^2 - \tfrac{\lambda_2}{2}\omega_2^2\cdot q_2^2;\ (\lambda_2 = L - M;\omega_2^2 = \tfrac{1}{\lambda_2\cdot C})$$

$$Ł_3 = \tfrac{1}{2}\lambda_3\cdot\dot{q}_3^2 - \tfrac{\lambda_3}{2}\omega_3^2\cdot q_3^2;\ (\lambda_3 = L + 2\cdot M;\omega_3^2 = \tfrac{1}{\lambda_3\cdot C}). \qquad (46)$$

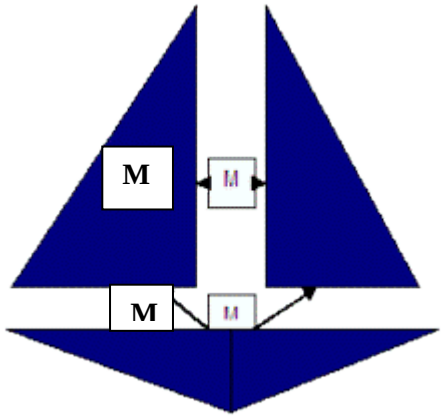

Fig.2. Three identical oscillators according to Fig. 1A and Fig. 1B, which are mutually coupled via M.

Using the canonical momentum of the three Lagrange functions we can either define the Hamilton operator (and Schrödinger equation) for q_1, q_2, q_3 or introduce creation- and annihilation operators as previously carried out. In the latter case, the Hamiltonian reads:

$$H = \tfrac{\hbar}{2}[\omega_1\cdot(b_1 b_1^+ + b_1^+ b_1) + \omega_2\cdot(b_2 b_2^+ + b_2^+ b_2)$$

$$+\omega_3\cdot(b_3 b_3^+ + b_3^+ b_3)]. \qquad (47)$$

$$[b_k, b_l^+] = \delta_{k,l}\ (k, l = 1,..,3). \qquad (48)$$

It should be pointed out that the case of 3 independent oscillators (M = 0) implies the symmetry group SU_3 [9]. However, without any coupling a transfer of energy from one mode to another one cannot occur (the degeneracy is threefold). Due to the mutual coupling with M ≠ 0 the symmetry is reduced to SU_2 and an additional hypercharge, i.e. $SU_2 \otimes U(1)$, denoted b λ_3 and ω_3, since $\lambda_1 = \lambda_2$ and $\omega_1 = \omega_2$. In similar fashion, we can treat four mutually coupled oscillators with M ≠ 0 (Fig. 3):

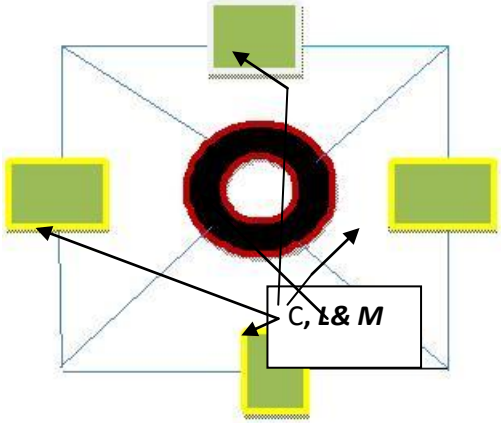

Fig.3. Schematic representation of four coupled electromagnetic circuits referring to (49 – 52). The inductivity L of each circuit is placed in the center and each oscillator is coupled to all other oscillators via M. The rectangles represent the capacity C of each circuit.

Fig. 3 may also stand for those cases, where more than 3 oscillator circuits are mutually coupled. We consider these cases in (49 - 54).

$$L\cdot\ddot{Q}_1 + M\cdot(\ddot{Q}_2+\ddot{Q}_3+\ddot{Q}_4) + Q_1/C = 0$$
$$L\cdot\ddot{Q}_2 + M\cdot(\ddot{Q}_1+\ddot{Q}_3+\ddot{Q}_4) + Q_2/C = 0$$
$$L\cdot\ddot{Q}_3 + M\cdot(\ddot{Q}_1+\ddot{Q}_2+\ddot{Q}_4) + Q_3/C = 0$$
$$L\cdot\ddot{Q}_4 + M\cdot(\ddot{Q}_1+\ddot{Q}_2+\ddot{Q}_3) + Q_4/C = 0. \quad (49)$$

Using similar substitutions to obtain the normal modes as above provides:

$$q_1 = Q_1 - Q_4;\ q_2 = Q_2 - Q_4; q_3 = Q_3 - Q_1$$
$$q_4 = Q_1 + Q_2 + Q_3 + Q_4. \quad (50)$$

Thus the normal modes are given by:

$$\lambda_1 = \lambda_2 = \lambda_3 = L - M;\ \lambda_4 = L + 3M$$
$$\omega_1^2 = \omega_2^2 = \omega_3^2 = \tfrac{1}{C(L-M)};\ \omega_4^2 = \tfrac{1}{C(L+3M)}. \quad (51)$$

The Hamiltonian assumes in terms of creation– and annihilation operators the shape:

$$H = \tfrac{\hbar}{2}[\sum_{k=1}^{3}\omega_k(b_k b_k^+ + b_k^+ b_k) + \omega_4(b_4 b_4^+ + b_4^+ b_4)]. \quad (52)$$

$$[b_k, b_l^+] = \delta_{k,l}\ \ (k,l = 1,..,4). \quad (52a)$$

It has to be pointed out that SU_3 now is strictly conserved by the four mutually coupled circuits! The hypercharge now is related to the resonance oscillation $\omega_4^2 = 1/(C(L+3M))$. In principle, the formalism can be extended to higher order coupled circuits (e.g. 5^{th} order). Then the normal modes are given by the relation:

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = L - M;\ \lambda_5 = L + 4M$$
$$\omega_1^2 = \omega_2^2 = \omega_3^2 = \omega_4^2 = \tfrac{1}{C(L-M)};\ \omega_5^2 = \tfrac{1}{C(L+4M)}. \quad (53)$$

The Hamiltonian now is given by:

$$H = \tfrac{\hbar}{2}[\sum_{k=1}^{4}\omega_k\cdot(b_k b_k^+ + b_k^+ b_k) + \omega_5\cdot(b_5 b_5^+ + b_5^+ b_5)]$$
$$[b_k, b_l^+] = \delta_{k,l}\ \ (k,l = 1,..,5). \quad (54)$$

The physical content of the above equation incorporates a perturbed SU_5. Now the symmetry group SU_4 is exactly conserved. The remaining hypercharge formally increases due to the decreasing resonance frequency ω_5. The inclusion of Ohm's resistance is also straightforward due to the elaborated modifications valid for the corresponding normal modes.

Finally we consider the case of a forced voltage V_f in the circuit Fig. 1A. The Schrödinger equation of this problem reads:

$$-\tfrac{\hbar^2}{2\cdot L}\cdot\exp(-Rt/L)\cdot\tfrac{\partial^2}{\partial Q^2}\psi(Q,t)$$
$$+[\tfrac{L\cdot\omega^2}{2}\cdot Q^2 - V_f\cdot Q\cdot\exp(i\omega_f t)]\exp(R\cdot t/L)\psi(Q,t)$$
$$= i\cdot\hbar\,\tfrac{\partial}{\partial t}\psi(Q,t). \quad (55)$$

Thus in many domains of applied physics damped circuits with forced oscillations are taken account for. The basis solution is the case V_f = 0. We consider this task based on time-dependent perturbation theory. The conventional perturbation expansions start with a time-independent Hamiltonian H_0 and a complete set of eigen-functions $\psi_n(x)\cdot\exp(-i\cdot E_n\cdot t/\hbar)$. In our case the set of wave-functions is already time-dependent. The matrix elements of $H_I = V_f\cdot Q\cdot\exp(i\cdot\omega_f\cdot t)\cdot\exp(R\cdot t/L)$ result from the following integrations:

$$H_{I,nm}(t) = \int\psi_n(Q,t)\cdot V_f\cdot Q\cdot\exp(i\cdot\omega_f\cdot t)\cdot$$
$$\cdot\exp(R\cdot t/L)\cdot\psi_m(Q,t)dQ. \quad (56)$$

The expansion coefficient $c_m(t)$ is determined by:

$$i\cdot\hbar\,\tfrac{d}{dt}c_m(t) = \sum_{n=1}^{\infty}H_{I,nm}(t)\cdot c_n(t)\cdot$$
$$\cdot\exp(i(\omega(m-n))\cdot t). \quad (57)$$

Due to the orthogonal properties of (34) with regard to the variable ξ we rewrite (55) in the form:

$$H_{I,nm}(t) = V_f\cdot\sqrt{\tfrac{\hbar}{L\cdot\omega}}\cdot\exp(-\tfrac{(m+n+1)\cdot R\cdot t}{2L})\cdot\exp(i\cdot\omega_f\cdot t)$$
$$\cdot\int_{-\infty}^{\infty}\exp(-\xi^2)\cdot\xi\cdot H_m(\xi)\cdot H_n(\xi)d\xi. \quad (58)$$

The essential task is to evaluate the above integral via partial integration:

$$\int_{-\infty}^{\infty}\exp(-\xi^2)\cdot\xi\cdot H_m(\xi)\cdot H_n(\xi)d\xi = -\tfrac{1}{2}\delta_{nm}\cdot n!\cdot 2^n\cdot\sqrt{\pi} +$$
$$\tfrac{1}{2}\int_{-\infty}^{\infty}\exp(-\xi^2)\cdot[H_m'(\xi)\cdot H_n(\xi) + H_m(\xi)\cdot H_n'(\xi)]d\xi. \quad (59)$$

With the help of the relation $H_n'(\xi) = 2\cdot n\cdot H_{n-1}(\xi)$ all terms provide:

$$\tfrac{1}{2}\int_{-\infty}^{\infty}\exp(-\xi^2)\cdot[H_m'(\xi)\cdot H_n(\xi)+H_m(\xi)\cdot H_n'(\xi)]d\xi =$$
$$\tfrac{1}{2}\delta_{nm}\cdot n!\cdot 2^n\cdot\sqrt{\pi}+\tfrac{1}{2}2\cdot m\cdot[\delta_{m-1,n}\cdot 2^{m-1}\cdot(m-1)!\cdot\sqrt{\pi}]$$
$$+\tfrac{1}{2}\cdot 2\cdot n\cdot[\delta_{m,n-1}\cdot 2^{n-1}\cdot(n-1)!\cdot\sqrt{\pi}]. \qquad (60)$$

Evaluation of (60) leads to the matrix elements of H_I:

$$H_{I,nm}=\sqrt{\pi}\cdot V_f\cdot\sqrt{\tfrac{\hbar}{L\cdot\omega}}\cdot\exp(-\tfrac{(m+n+1)\cdot R\cdot t}{2L})\cdot$$
$$\exp(i\cdot\omega_f\cdot t)\cdot$$
$$\cdot\{m!\cdot 2^{m-1}{}_{(if:n=m-1)}+n!\cdot 2^{n-1}{}_{(if:m=n-1)}-n!\cdot 2^{n-1}{}_{(if:n=m)}\}. \quad (61)$$

Inserting (61) into (57) provides:

$$i\cdot\hbar\tfrac{d}{dt}c_m(t)=\sqrt{\pi}\cdot V_f\cdot\sqrt{\tfrac{\hbar}{L\cdot\omega}}\cdot\sum_{n=0}^{\infty}c_n(t)\cdot$$
$$\cdot\exp(-\tfrac{(m+n+1)\cdot R\cdot t}{2L})\cdot\cdot\exp(i(\omega_f+\omega(m-n))\cdot t)\cdot$$
$$\cdot\{m!\cdot 2^{m-1}\delta_{n,m-1}+n!\cdot 2^{n-1}\delta_{n,m+1}-n!\cdot 2^{n-1}\delta_{n,m}\}. \quad (62)$$

The first-order approach for the calculation of $c_{m,1}$ is obtained by the assumption that all terms of zero order are 1, then the integration provides:

$$c_{m,1}=\tfrac{1}{i\cdot\hbar}\cdot V_f\cdot\sqrt{\tfrac{\hbar\cdot\pi}{L\cdot\omega}}\cdot\sum_{n=0}^{\infty}\exp(-\tfrac{(m+n+1)\cdot R\cdot t}{2\cdot L})\cdot$$
$$[1-\tfrac{2\cdot L}{(m+n+1)\cdot R}]\cdot[\exp(i\cdot(\omega_f+\omega(m-n))\cdot t)\cdot$$
$$(\tfrac{1}{i\cdot(\omega_f+\omega(m-n))}-1))]\cdot$$
$$\{m!\cdot 2^{m-1}\delta_{m-1,n}+n!\cdot 2^{n-1}\delta_{n-1,m}-n!\cdot 2^{n-1}{}_{\delta m,n}\}. \quad (63)$$

This equation is usually solved iteratively by suitable boundary conditions, i.e. if t = 0 the exponential functions assume the value 1. The calculation of the second order approach is straightforward by inserting $c_{m,1}(t)$ into the right-hand side of (63). Thus the most outstanding effect is that all terms of $c_{m,2}(t)$ are now proportional to the term V_f^2 according to (64):

$$c_{m,2}=-V_f^2\cdot\tfrac{\pi}{\hbar\cdot L\cdot\omega}\cdot\sum_{n=0}^{\infty}\exp(-\tfrac{(m+n+1)\cdot R\cdot t}{2\cdot L})\cdot$$
$$[1-\tfrac{2\cdot L}{(m+n+1)\cdot R}]^2\cdot$$
$$[\exp(i\cdot(\omega_f+\omega(m-n))\cdot t)\cdot(\tfrac{1}{i\cdot(\omega_f+\omega(m-n))}-1))]^2\cdot$$
$$\cdot\{m!\cdot 2^{m-1}\delta_{m-1,n}+n!\cdot 2^{n-1}\delta_{n-1,m}-n!\cdot 2^{n-1}{}_{\delta m,n}\}. \qquad (64)$$

In the next we shall consider two application cases with the forced oscillator in radiation physics. The resonance conditions of the quantum mechanical harmonic oscillator again can be verified from (63) and (64).

V. Some Applications to Molecular and Radiation Physics

Excited States of H_3PO_4

With respect to the problem of cyclotron resonance in a constant magnetic field with friction in a medium considered at the end of section 2, we have already given an example with wide applications. Thus we can identify the charge q_0 of (21 – 25) with an electron charge and friction with Ohm's resistance in a semiconductor, i.e. $D_{Friction} \approx R_{Ohm}$, then the reduced frequency provides the connection to the quantum Hall-effect. In section 3 the quantization of coupled electromagnetic circuits with damping has been presented. The coupled circuits implying the symmetry groups SU_2, SU_3, SU_4, etc. may have applications in nuclear physics and/or particle physics, if the parameters L, M and C are appropriately interpreted as distributions of baryonic charges and currents induced by mesons and gluons. In the domain of molecular physics, electric circuit oscillator models have first been introduced many years ago [10] - [12], but only in the past decade they have obtained increasing importance. As an example we consider the excited states of phosphoric acid, which has already been calculated previously by two rather different methods, i.e. self-interacting field [10] and CNDOS-CI method [13]. However, the conformation of this acid depends on pH and solvent. Thus in a neutral medium (pH ≈ 7) this molecule is present by one double bond P = O (3d-electron), i.e. the summation formula is H_3PO_4. The configuration of this case is given by a T_d structure. The lowest excited states of the perturbed SU_4 can readily associated with the discrete group T_d. The UV-absorptions starts at E = 4.8 eV. This fact also indicates that in this domain a double bond P = O is mainly involved in the excitation process. The remaining single P-OH bonds show absorption bands between 5.5 eV and 5.8 eV. Referring to the free H_3PO_4 (without interaction with a solvent) we have to account for the property that the double bond resulting from 3d electrons of phosphorus does not favor one of the four oxygen atoms. Thus from previous results [10] we have calculated $\hbar\cdot\omega_0$ = 5.31 eV and the energy shift due to the term MC. The calculated, perturbed SU_4 with regard to the four resonances amounts to 4.79 eV and 5.59 eV (3fold degeneracy). The interaction with the neutral solvent provides an excitation band between 4.76 eV and 4.84 eV and 5.52 eV and 5.77 eV. Therefore it is interesting to note that the damping influence, which is now mediated by dipole – dipole interactions of H_3PO_4 with the solvent and molecular vibrations, is accounted for the shifts given the corrected resonances:

$$\omega^2 \Rightarrow \omega^2 - R^2/4\lambda^2$$
$$\lambda = L+3M \text{ or } \lambda = L-M. \qquad (65)$$

The resistance R realized by a solvent interaction and molecular vibrations can adopt the same role as the damping constant D in macroscopic physics.

Klystron

The quantum circuit model of a multi-cavity klystron can be treated according to Fig. 1B, where the right-hand circuit additionally contains a forced oscillator $V_f\cdot\exp(i\cdot\omega_f\cdot t)$, but the left-hand circuit does not and emits waves with a much higher frequency. The condensers incorporate cylindrical cavities. The forced oscillator circuit (right-hand side) now is characterized by $\lambda_1 = L + M$; the left-hand condenser by $\lambda_2 = L - M$. Therefore, the inductivity L appearing in (55 - 65) have to be replaced by $\lambda_1 = L + M$. The action of the forced oscillator has a longer duration t = τ, i.e. τ >> T_f and T_f results from ω_f =

$2\cdot\pi/T_f$. With regard to the mutual inductivity M it is useful that the solenoids of both circuits have a very narrow linkage, i.e. a chain-linking of the solenoids is realized. Assume: $M = 0.9\cdot L$ provides two resonance frequencies via $\lambda_1 = 1.9\cdot L$ and $\lambda_2 = 0.1\cdot L$. The ratio between the both amounts to (if R is negligible):

$$\frac{\omega_2}{\omega_1} = 19. \qquad (66)$$

An extremely strong coupling $M = 0.99\cdot L$ provides the ratio:

$$\frac{\omega_2}{\omega_1} = 199. \qquad (67)$$

By that, the energy $Q\cdot V_f\cdot\exp(i\cdot\omega_f\cdot t)$ undergoes pumping to the second resonator, which can emit waves with a much higher frequency. If we add, from the left-hand side, a further oscillator circuit with a very low inductivity L (but M should assume values very close to L) and capacitance C, then we can reach a further drastic amplification of the frequency. I.e. the GHz domain will be reached, when e.g. via the relation $\omega_3/\omega_2 = 200$ the initial frequency of the forced oscillator was assumed to be $\nu_f = 200$ Hz. The wave impedance $\rho_{I,k}$ of the oscillator k (k = 1,.., 3) is given by:

$$\rho_{I,k} = \frac{\lambda_k}{\sqrt{\lambda_k\cdot C - R^2\cdot C^2/4}}. \qquad (68)$$

By that, the total impedance ρ_I of the model klystron consisting of 3 coupled oscillators presented above has the form:

$$\rho_I = \sqrt{\sum_{k=1}^{3} {\rho_{I,k}}^2}. \qquad (69)$$

We should also point out that (68) and (69) can certainly be optimized, when the LC-terms are taken different for each oscillator. This fact indicates that wave-guides can be designed by quantized, forced circuits, which are mutually coupled.

Bremsstrahlung

Typical atomic properties in terms of circuits have been presented in some publications [10] – [16], and further references can be found in the quotations. The creation of 'bremsstrahlung' is a particular quantum theoretical problem, which finds various important applications in rather wide fields, e.g. radiation oncology [17], if the kinetic energy $E_{kinetic}$ of the impinging electron satisfies $E_{kinetic} >> m_0c^2$ (0.52 MeV). Therefore the 'bremsstrahlung' creation of electrons in metals ($E_{kinetic} << m_0c^2$) being proportional to the nuclear charge Z is not the topic here. The theoretical base of this phenomenon has first been given [18]. However, the present study prefers the restriction given by viewpoint of Feynman [19], since we restrict ourselves here to principal method.

Fig. 4A and Fig. 4B show an impinging electron moving in direction to the nucleus (A) and passing it with a deflection angle, which implies an increased virtual orbital. The connection of an LC-circuit to atomic properties have been previously been studied [14 – 16]. The motion of an electron, which represents a current, around the nucleus is connected to the inductivity *L* and the charge distribution to the capacitance *C*. In similar fashion as previously carried out we have to fix the atomic parameters with regard to the virtual orbital according to Fig. 4A. Since this presentation is preferably model consideration we assume that for the velocity **v** of electrons with $E_{kinetic} >> 1$ MeV to be c (velocity of light). The creation of 'bremsstrahlung' of a single electron is a very fast process, but the behavior of the electron thereafter will be associated with a current and the resistance $R \neq 0$.

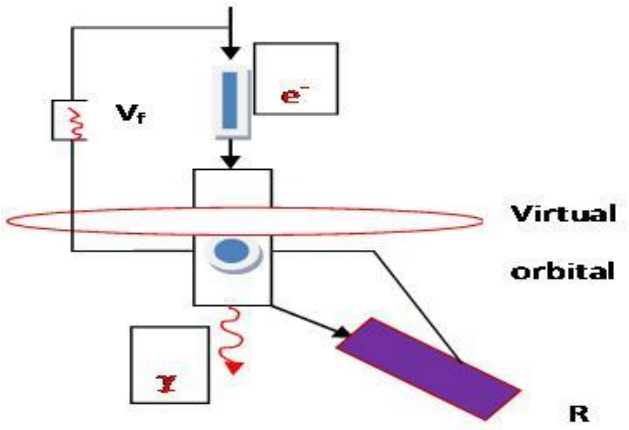

Fig.4A. Circuits for the creation of 'bremsstrahlung' (smallest possible virtual orbital) – Impinging electron immediately hits the nucleus.

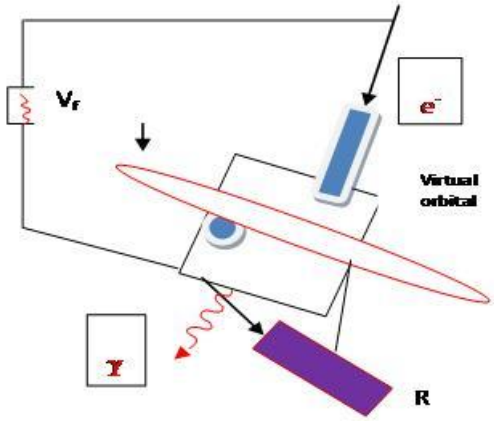

Fig.4B. Oblique impinging electron (the virtual orbital perpendicular to the impinging plane is increased). The energy loss due to the residual electron energy is denoted by the resistance R.

With the help of the propagator perturbation method [19] the photon creation with energy $\hbar\cdot\omega$ by an electron in the Coulomb field of a nucleus the differential cross-section reads:

$$\frac{d\sigma}{d\omega} = \frac{1}{2\pi}\cdot Z^2 e_0^6\cdot\frac{1}{\omega}\cdot\frac{p_f}{p_i}\sin\theta_2\sin\theta_1 d\theta_2\, d\theta_1\, d\phi\cdot$$
$$\cdot F(E_i, E_f, p_i, p_f, \omega, \theta_1, \theta_2, \phi). \qquad (70)$$

The function ***F*** detailed stated in [19] incorporates the dependence of the initial electron energy E_i, final energy E_f, and related momentums p_i, p_f. The angles θ_1, θ_2 (electron) and ϕ photon refer to the behavior after the central hit with the nucleus Z. Thus (70) corresponds to Fig. 4A with the general assumption that the outgoing electron and photon possess certain spherical angles

different from 0. In Fig. 4A we have already tacitly used that for $E_i >> m_0 \cdot c^2$ (0.52 MeV) these angles are approximately 0, i.e. the created photon has the direction of the impinging electron. Fig. 4B represents a more complex situation [18].

For brevity we now consider the case 4A; the initial electron energy may be of the order between 6 MeV and 20 MeV, i.e. this order corresponds to linear accelerators (usual abbreviation: linacs) applied in radiotherapy. The transition between virtual orbital state and ground state producing the 'bremsstrahlung' photon $\hbar \cdot \omega$ is determined according to the perturbation equations (59 – 66) and the associated basis (55). The impinging electron is characterized by its Compton wave length and forcing frequency:

$$e_0 \cdot V_{ex} = \hbar \cdot \omega_{ex}; \quad \omega_f = \frac{2 \cdot \pi \cdot M \cdot c^2}{\hbar}. \qquad (71)$$

In similar fashion the equations of the next section have to be interpreted.

The calculation of the circuit associated to $L_{virtual}$ and $C_{virtual}$ of the virtual orbital *partially* follows elaborated standard methods, since the impinging electron incorporates a current and $C_{virtual}$ is associated with the Bohr radius of the hydrogen divided by Z (e.g. tungsten Z = 74). One should recall that the Bohr radius r_B is equivalent to the ground state solution (n = 1) of the Schrödinger equation for the H atom. Thus by a transfer to the quantized circuit model we obtain the following results:

$$r_B = \frac{\lambda_0}{2\pi \cdot \alpha} \; with: \; \alpha = \frac{\hbar \cdot c}{e_0^2}; \; \lambda_0 = \frac{\hbar}{m_0 \cdot c}$$

$$S_B = 4\pi \cdot r_B^2; L_B = \frac{\mu_0 \cdot S_B}{\lambda_0}; \; C_B = \frac{\varepsilon_0 \cdot S_B}{\lambda_0}. \qquad (72)$$

Please note that parameters are constants in atomic physics and electrodynamics; S_B is the Bohr surface, α is the fine structure constant and λ_0 the Compton wave length of the electron. If we pass from the H atom with Z = 1 to a nucleus with Z > 1 we have to replace S_B according to (72) by:

$$S_B(Z) = 4\pi \cdot r_B^2 / Z^2. \qquad (73)$$

By that, the values for $L_{B,Z}$ and $C_{B,Z}$ will be changed correspondingly. Thus for Z = 74 (tungsten) the ionization energy is changed from 13.55 eV (hydrogen) to 74 keV. However, we are interested in the energy domain of Mega electron Volt (MeV) with reference to bremsstrahlung creation and not in the keV region with discrete transitions of tungsten. The spherical model for the determination of $L_{B,Z}$ and $C_{B,Z}$ is no longer adequate here. The magnetic flux and capacitance induced by the fast electron in the immediate environment of the nucleus rather show a cylinder form; the Compton wavelength λ_0 has to be replaced by the de Broglie wavelength λ_{deBr}:

$$\vec{p} = \hbar \cdot \vec{k}; \; \lambda_{deBr} = \frac{2\pi \cdot \hbar}{|\vec{p}|}$$

$$E^2 = \vec{p}^{\,2} \cdot c^2 + m_0^2 \cdot c^4. \qquad (74)$$

These both equations (74) are also able to account for direction of the momentum of the impinging electron, and $S_B(Z)$ now reads:

$$S_B(Z) \rightarrow S_{deBr}(Z) = \pi \cdot \lambda_{deBr}^2 / Z^2. \qquad (75)$$

The energy of the electron in the final state is expressed by the resistance R. Thus we obtain in every order of perturbation theory the condition:

$$E_{photon} = \hbar \cdot \omega = \hbar \cdot \omega_0 - R^2 / (4 \cdot L_{deBr}{}^2)$$

$$\hbar \cdot \omega_0 = \hbar \cdot \frac{1}{\sqrt{L_{virtual} C_{virtual}}} \qquad (76).$$

It follows from (76) that the total kinetic energy of the electron resulting from (75) is transferred to the photon, if the resistance R becomes 0. This condition is rather good satisfied by a central hit of a high energy electron, and a consequence is that for $E_{kinetic} >> m_0 \cdot c^2$ the outgoing photon assumes the direction of the impinging electron.

The calculation of the transition probability, which proportional to the square of the result of (63 - 64), delivers the results of the Feynman method, if additional information like the direction of the momentums is introduced in connection to the impinging current. The usual restriction to electromagnetic circuits would ignore some space-depending properties such as current and charge density (**j,** ρ) and focus the complete problem to energy and time.

Extension of the theory to synchrotron radiation

The case according to Fig. 4A and its generalization given by Fig. 4B can be best treated by an extension of the Hamiltonian (55):

$$\frac{1}{2L} \cdot \exp(-R \cdot t / L) \cdot ((\frac{\hbar}{i} \frac{\partial}{\partial Q} - f(t))^2 \cdot \psi(Q,t) +$$

$$[\frac{L \cdot \omega^2}{2} \cdot Q^2 - V_f \cdot Q \cdot \exp(i \omega_f t)] \cdot$$

$$\cdot \exp(R \cdot t / L) \cdot \psi(Q,t) = i \cdot \hbar \frac{\partial}{\partial t} \cdot \psi(Q,t), (77)$$

Thus function *f* incorporates an additional coupling to the magnetic flux, which provides the properties of the outgoing photon. The introduction of the function *f* is consistent with the canonical commutation relation P →Φ (magnetic flux) – *f*, which is expressed by [P, Q] = -i·ħ. However, the classical equation of motion including a forcing voltage and damping now reads:

$$Ł = \exp(R \cdot t / L) \cdot$$

$$\{\frac{L}{2} \cdot \dot{Q}^2 + L \cdot f(t) \cdot \dot{Q} + Q \cdot V_f(t) - \frac{1}{2C} \cdot Q^2\}. \quad (78)$$

This equation incorporates the absorption of energy by $V_f(t)$ due to the virtual circuit and stimulation of the emission of radiation ('bremsstrahlung'). Thus the case 4B increases the area of the capacitance C and, by that, the resonance frequency ω_0 and related photon energy is decreased. With the help of (78) the generalized Langevin equation reads:

$$L \cdot \ddot{Q} + L \cdot \dot{f}(t) + R \cdot f + R \cdot \dot{Q} + \frac{1}{C} \cdot Q = V_f(t). \; (79)$$

The 2nd term of (79) describes a further source of loss, namely the diminishment of the magnetic flux independent of the collision of electrons with lattice vibrations. If we put the time-dependence of the forced oscillator $V_f(t) = V_0 \cdot \exp(i \cdot \omega_f \cdot t)$, then a compensation is possible (R = 0):

$$f(t) = \frac{V_0}{L} \cdot \int_0^t \exp(i \cdot \omega_f \cdot t') dt' =$$

$$\frac{V_0}{i \cdot L \cdot \omega_f} \cdot [\exp(i \cdot \omega_f \cdot t) - 1]. \qquad (80)$$

If R ≠ 0 we have to modify (80) slightly, and with the help of the Green's function technique we obtain:

$$f(t)=\frac{V_0}{R+i\cdot\omega_f\cdot L}\cdot[\exp(i\cdot\omega_f\cdot t)-\exp(-R\cdot t/L)]. \quad (81)$$

By this way, the handling of (77 - 81) becomes very convenient: The term related to $Q\cdot V_f(t)$ describes the behavior of the accelerated electron subjected to creation of 'bremsstrahlung' and the term containing *f(t)* the outgoing photon (synchrotron radiation).

VI. Conclusion and Outlook

We have considered the aspects of friction and resistance in the frame work of quantum mechanics. Thus friction may only have a quantum mechanical importance, if it can be restricted to typical atomic/molecular considerations such as dipole-dipole interactions with solvents or collisions with vibrations. Ohm's resistance principally appears in connection with lattice vibrations. The common feature is that in the microscopic domain irreversibility is introduced, i.e. energy is transferred to the environment. The quantization of circuits and its connection to self-interacting nonlinear fields with internal structure has at first been studied some decades ago [10]. However, in the meantime quantized electromagnetic circuits have been considered [14] – [16]. A main topic was the importance of these considerations in the rather novel field of quantum information processes. The inclusion of 'friction' in the form of typical influences of Ohm's resistance leading again to a reduced characteristic frequency incorporates the essential extension of the present study. Finally it should be pointed out that the equidistant harmonic oscillator levels of circuits may often reduce the applicability to molecular processes, which generally do not show these properties resulting from interacting many-body systems. The application of forced quantized circuits to problems, e.g. wave-guide due to a Klystron or creation of 'bremsstrahlung', indicates that the present study opens the door for problems of quantum electronics and even quantum electrodynamics.

The application of forced quantized circuits to problems like optimization of Klystrons indicates that the present study opens the door for problems of quantum electronics and even quantum electrodynamics. It should also pointed out that the mutual coupling M between three or four oscillators can readily extended to interaction problems of nuclear physics [8] – [10] with the help of introduction of meson currents between nucleons. The external force can be realized by the interaction of a charged particle with a nucleus. Let assume the charge of this external particle to be q_0, then the forcing perturbation oscillator assumes the form:

$$q_{ex}\cdot V_{ex}=\hbar\cdot\omega_{ex}. \quad (82)$$

The frequency ω_f appearing in the exponential function $\exp(i\cdot\omega_f\cdot t)$ of the forcing oscillator is different from ω_{ex} and results from the time-factor of plane waves, i.e.:

$$\omega_f=\frac{2\cdot\pi\cdot M\cdot c^2}{\hbar}. \quad (83)$$

Thus the model of four coupled oscillators appears to be an attractive viewpoint, since due to the mutual interaction of these oscillators SU_3 is conserved and the total symmetry group is described by $\mathbf{SU_3}\ x\ \mathbf{U(1)}$. The generators of SU_3 can be previously verified [10] and references therein. The present model of a quantized theory of coupled electromagnetic circuits with inclusion of damping processes can readily be transferred to problems of nuclear physics, such as nuclear reactions in order to describe reaction channels like:

$$proton+nucleus\Rightarrow outgoing\ particles + nuclear\ fragments. \quad (84)$$

Thus an example of (84) would be the conversion of an impinging proton with rather low energy (e.g. E < 20 MeV) to an outgoing neutron due to an exchange interaction, where the Pauli principle plays a dominant role due to its generalization to the isospin characterization. The charge number of the target nucleus is increased by one, but the mass number remains constant. This nuclear reaction mechanism can be made apparent by the proton - oxygen reaction, which is very important in radiotherapy with protons:

$$p+O_8^{16}\Rightarrow n+F_9^{16}. \quad (85)$$

The isotope F_9^{16} undergoes a ß$^+$ - decay and yields again O_6^{16} by emission of e^+ (positron) and a γ-quant.

An extension of the presented theory to nuclear physics is apparent, since the currents and mutual couplings via the parameter M has not to be restricted to the electromagnetic case: The exchange interactions between nucleons result from currents mediated by mesons. Thus the nuclear reaction processes analyzed previously [8] can be much easier handled by the charge quantization theory considered in this study than by a many-particle Schrödinger equation in the position space based on suitable nuclear potentials and couplings between the nucleons. Research in this direction is in progress.

This final example makes evident that systems of quantized circuits may have applications in rather wide-spread fields such as nuclear excitations and reactions, but also with regard to quantum information problems [14] - [16]. The latter aspect of possible applications is very outstanding, in particular, with respect to resonance circuit problems in biophysics [11], but nevertheless it deserves a separate and very thorough investigation, which will be the subject of a further communication.

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