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Study on the Teaching Strategy about the Issue of Moving Points in Junior High School Mathematics

Sitong Dong [1], Xinhui Zhou [2] and Qi Ge [1*]
[1, 2] Department of Mathematics College of Science, Yanbian University, Yanji, China.
*Corresponding author email id: geqi9688@163.com

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Abstract **– The moving point problem is the key question type in junior high school mathematics graduation examination, and it is also a difficult problem in junior high school mathematics. The solution of the moving point problem involves students' space concept, geometric intuition, innovative consciousness and reasoning ability, as well as the flexible mastery of mathematical thought. Based on the investigation and analysis of the current situation of junior high school students' ability to solve the moving point problem, this article puts forward the teaching strategies on the moving point problem of junior high school mathematics from four aspects: improving students' reading ability, cultivating students' mathematical thought, improving students' ability to use knowledge and paying attention to the cultivation of students' creative thinking.**

Keywords **– Mathematical Moving Point Problem, Teaching Strategy, Reading Ability, Mathematical Thought.**

I. INTRODUCTION

The moving point problem usually refers to a series of problems which are characterized by the movement changes of points, lines, or graphics under the background of geometric knowledge and graphics, and study the movement changes of basic graphics or function images from the perspective of transformation. As a comprehensive topic in junior high school learning content, the application of students' theoretical knowledge and their ability to analyze and solve problems have been comprehensively measured. The complicated and changeable "moving point problems" have appeared in different ways in all provinces and cities over the years, and they account for a large proportion of scores, which is the key investigation scope of proposers.

Learning about the moving point problems can not only cultivate students' space concept, geometric intuition, innovative consciousness, and reasoning ability but also fully embody their core qualities, which is the perfect combination of basic mathematics knowledge and basic skills. Among the 10 core concepts in the *Mathematics Curriculum Standard for Compulsory Education* issued by the Ministry of Education in 2011, the part about geometric intuition points out that "geometric intuition can help students understand mathematics intuitively and plays an important role in the whole mathematics learning process." [1].This kind of problem needs to fully grasp the relationship between quantitative relation and spatial form, understand the essence of the problem in "movement or transformation", and at the same time, it is also the clever use of mathematical ideas such as the function thought, the equation thought, the combination of number and shape thought, the classification discussion thought and so on.

However, there are often some difficulties in the teaching of the moving point problems, which are not only reflected in the students' poor mastery of the situation and confusion when they encounter problems but also reflected in the teachers' teaching methods and ideas. Therefore, the author believes that it is very necessary to study the teaching strategies of moving point problems.

II. Research Status of Teaching Strategies on Moving Point Problems

In 2017, Xiaofeng Wu thought that although the moving point problem belongs to geometric problems, in most cases, starting from life, creating a problem situation close to students' life, splitting a geometric figure into several basic geometric figures, thoroughly analyzing the basic figures, exploring the connotation of the moving point, guiding students to decompose the problem in the process of explanation, and thoroughly understanding the knowledge points of each step, so as to achieve the effect of drawing inferences from others [2].

In 2019, Chunhong Liu put forward the review strategy of junior high school mathematics "moving point problem": teachers should guide students, follow students' cognitive rules, and make it from simple to complex; Create scenes and let students think in the scenes; Stimulate students' interest in learning, and let students think about expanding and extending while mastering the foundation through communication with teachers [3].

Through the research on the literature about the moving point problem, it is found that the main problem of middle school students in solving the moving point problem is that they can't understand the nature of the topic, and it is difficult to split it into static problems for the change of graphics, and find out the basic knowledge to be actually examined. The main reason is likely to be a teacher in the teaching process lacks certain situation creation and step-by-step guidance, causes students to analyze issues is not comprehensive, they cannot see the wood for the trees and take a part for the whole. The understanding of the topic often ignores some key information, which leads to the loss of overall control of the problem.

III. Investigation and Analysis of Junior High School Students' Ability to Solve Moving Point Problems

Moving point problem is an important part of the senior high school entrance examination in all provinces, and it is also a difficult point and pull-off point in the examination. It involves the comprehensive application of various knowledge, and is a changing problem based on the thought of combining numbers and shapes. It tests students' understanding of quantitative relations and spatial forms in the process of exercise. There are static in motion, dynamic in static, dynamic and static, and it contains a variety of mathematical ideas and methods. It examines students' ability to obtain mathematical information and analyze problems, and aims at the flexibility and openness of moving-point problems. Dynamic geometry involves a wide range of knowledge and is difficult [4]. It not only involves a lot of comprehensive knowledge in the topic, but also has a variety of ways to investigate the topic. It is often used as the last topic in selecting or filling in the blanks, and is often used as the finale in big questions. The difficulty of setting the problem is often increasing gradually, which fully tests students' comprehensive analysis ability and problem-solving ability.

It can be said that junior high school students' mastery of moving point problems determines their grades to a great extent. Such problems not only help to cultivate junior high school students' mathematical thinking, but also play an important role in shaping their learning ability. In order to understand the current situation of junior high school students' ability to solve moving point problems, the author selected some junior high school students from two junior high schools in Yanji City, Jilin Province as test objects, and counted the scores of students' specific problems. A total of 246 test papers were distributed and 246 were recovered, with a recovery rate of 100%.

Test question: As shown in figure 1, in $Rt\triangle ABC$, $\angle ACB = 90^{\circ}$, $AC = BC = 2$, the moving point P starts from point A at a speed of 2 units per second and moves to the end point B along the $A \to C \to B$ direction (point P does not coincide with the vertex of ΔABC). The symmetrical point of point P with respect to point C is point D, passing point P as $PQ \perp AB$ at point Q, take PD and PQ as sides as $\mathbf{Y}PDEQ$. Let the overlapping area of $\mathbf{Y}PDEQ$ and ΔABC be S, and the moving time of point P be $t(s)$.

(1) When point P moves on AC, the length of PD is expressed by algebraic expression containing t; (2 points).

(2) When the point E falls on the right-angle side of ΔABC, find the value of t; (4 points).

(3) When the overlapping figure of $\mathbf{Y}PDEQ$ and ΔABC is quadrilateral, find the functional relationship between S and t (4 points).

Solution:

(1) When the point P moves on AC, as shown in figure 2, at this time $AP = 2t$, $AC = 2$

$\therefore PD = 2PC = 2(2-2t) = -4t + 4$ $(0 < t < 1)$

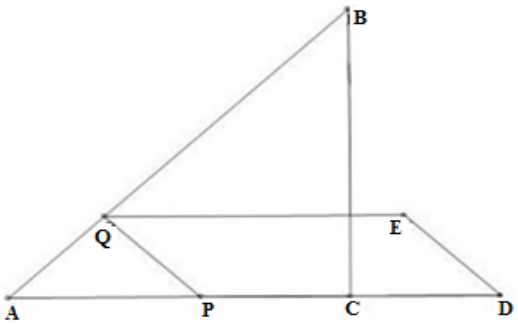

Fig. 1.

(2) When point E falls on the edge of BC, as shown in figure 3, at this time $PC = CD = 2 - 2t$

$AQ = PQ = DE$

$\therefore \frac{\sqrt{2}}{2}.2t = \sqrt{2}\,(2-2t)$, solve it $t = \frac{2}{3}$;

When point E falls on the edge of AC, as shown in figure 4, at this time, $PC = CD = 2t - 2$, $BQ = PQ = DE$

$\therefore \frac{\sqrt{2}}{2}.\,(4 - 2t) = \sqrt{2}\,(2t - 2)$, solve it $t = \frac{4}{3}$;

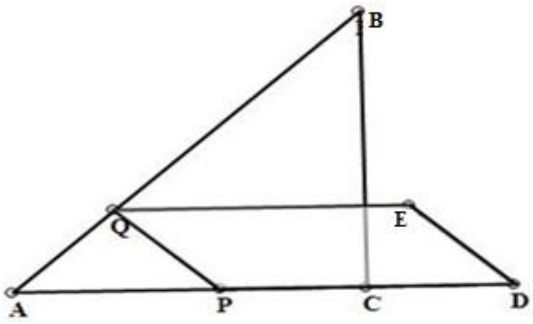

Fig. 2.

(3) When $0 < t \leq \frac{2}{3}$, as shown in figure 2, the figure of overlapping part is trapezoidal *PQNC*

$\therefore S = S_{TrapezoidalPQNC} = \frac{1}{2}[2(2-2t)+t].t = -\frac{3}{2}t^2 + 2t$

When $\frac{4}{3} \leq t < 2$, as shown in figure 5, the figure of overlapping part is trapezoidal *PQNC*

$\therefore S = S_{TrapezoidalPQNC} = \frac{1}{2}[2(2t-2)+\frac{1}{2}(4-2t)].\frac{1}{2}(4-2t) = -\frac{3}{2}t^2 - 4t - 2$

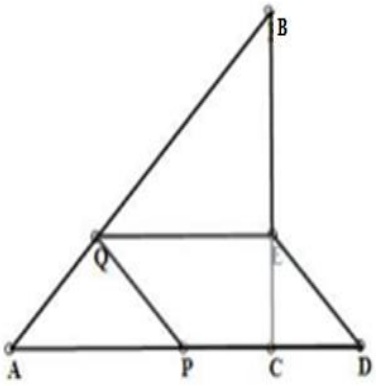

Fig. 3.

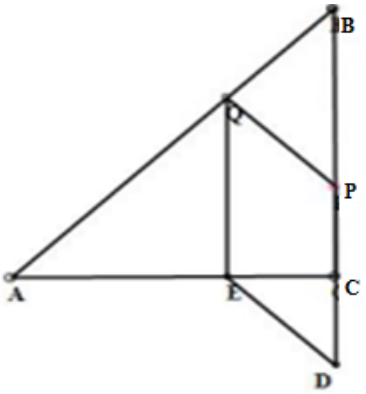

Fig. 4.

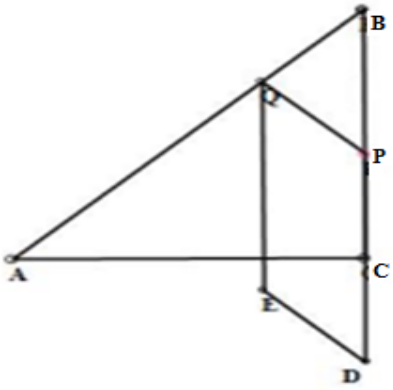

Fig. 5.

Take this question as an example: after reading the title, students first associate the symmetrical knowledge they have learned, and point *P* and point *D* are symmetrical points about point *C*, which can obtain the quantitative relationship between *PD* and *PC* and further simplify it; When the point *E* falls on the right side of Δ*ABC*, there will be two cases, so it is necessary to classify and discuss. It is necessary to consider that the point *E* can fall on the right-angle sides *AC* and *BC*, draw the corresponding graphics, and then use the properties of isosceles triangles and parallelograms to list the corresponding linear functions, and summarize the piecewise functions; The whole movement process should be considered when the figure of overlapping part of **Y***PDEQ* and Δ*ABC* is quadrilateral, including the overlapping situation of different time periods, find out the time period when the figure of the overlapping part is quadrilateral, draw the figure, and solve the area of quadrilateral or trapezoid.

Test results are as follows:

Table 1. Statistical analysis table of test scores.

Number of Cases	**246**
Average	3.98
Median	4.00
Mode	2.00
Standard Deviation	2.48
Variance	6.16

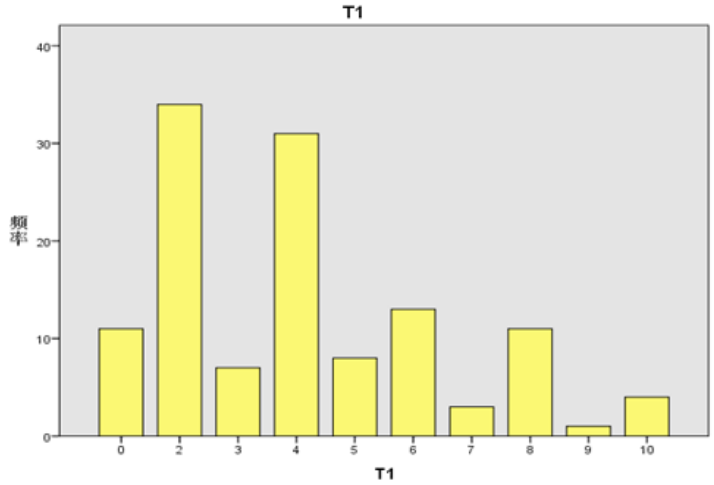

Fig. 6. Score of test questions.

The first question is 2 points, the second question is 4 points, and the third question is 4 points, totaling 10 points. According to the statistical results, 27.6% of the students only got 2 points for the first question.

The second question needs to be discussed in different categories. The point e to be considered may fall on two right-angled sides *AC, BC* of Δ*ABC,* Some students only consider the situation that point *E* can fall on the right angle side *AC,* while some students make mistakes in the calculation of one-dimensional linear equation.

The third question needs to use the thinking ability in images to analyze graphics. Secondly, it uses intuitive thinking ability to segment time, which runs through the application of logical thinking ability in the whole process and is a comprehensive embodiment of the application of mathematical thinking ability. Many students can't draw the correct figure, which leads to the failure to analyze the correct method to solve the problem, thus failing to score, and some students make mistakes in finding the critical point.

Through the test, it is found that students' ability to solve key problems is not strong. Mainly manifested in: A. It is difficult to find a breakthrough to solve the problem because of unclear examination, poor reading ability, inability to understand the nature of the topic and insufficient understanding of the conditions; B. The ability of spatial imagination is not strong, and the lack of geometric intuitive literacy, that is, when the movement of moving point is more complex, the image after the movement can not be clearly drawn. The most important idea in the moving point problem is "combination of number and shape". If there is no subjective representation of shape, it is difficult to grasp the whole problem; C. The main problems in the use of knowledge are that we can not flexibly use the knowledge and make adaptations to the knowledge. The specific embodiment is that there are some calculation errors in the problem making, the situation of classification will be missed or segmentation error, and do not know how to list the relationship.

In view of the actual problems in the above survey, in order to improve the junior high school students' ability to solve the moving point problem, the author puts forward some teaching strategies to solve the moving point problem.

IV. Teaching Strategies on the Moving Point Problems of Junior High School Mathematics

A. *Improve Students' Reading Ability*

For any kind of problem-solving, the understanding of the problem is always put in the first place, because the moving point problem involves a wide range of knowledge and is difficult to synthesize. Usually, the problem stem is set long, and students need to obtain a large amount of information when reading the topic, which often makes them feel helpless. The first thing teachers should do is to guide students to read the questions carefully and pay attention to them without fear of difficulties in the process of reading questions, make clear the conditions and problems, and mark some important information in the topic in the diagram, so as to avoid wasting time by repeatedly searching for the stem of the question. Determine the background figure, as well as the number of moving points, the relationship between the trajectory and the moving point, and have an overall grasp of the variables and invariants.

To understand mathematical problems, the most important thing is to read the questions with mathematical thoughts, and to establish a knowledge chain in your mind for the conclusions that can be derived from each known condition, that is, to associate the corresponding theorems or definitions with known conditions, and to obtain those conclusions from conditions, which are the basis for solving the problem of moving points. The most important thing is to truly understand the implicit conditions in the topic through repeated understanding of the whole topic and the experience of key words. These implicit conditions can be scrutinized according to existing conditions.

B. *Cultivate Students' Mathematical Thoughts*

Moving point problem is not an intuitive presentation of geometric problems, but often combines a series of graphic changes to understand the essence of the problem in "movement or change". Therefore, if students want to solve problems, they must master some mathematical ideas and form a certain thinking process.

The thought of combination of number and shape is an important mathematical thinking method to solve the problem of moving point, and it is also the embodiment of students' spatial imagination ability. On the premise of understanding the conditions in the stem of the problem, it combines the quantitative relationship with intuitive geometric figures, concretizes the abstract problem, accurately understands the process of graphic change in the process of doing the problem, and understands it as a static problem. In the teaching process, teachers should consciously use the relationship between numbers and shapes to help students establish geometric intuitive ability, let students transform the idea of combining numbers and shapes into their own ability, use graphics to demonstrate the problem-solving process, and fully display the changes of points, lines or graphics in the whole movement process with the help of geometric sketchpad or PPT with some dynamic effects, and cultivate students' spatial imagination intentionally in teaching, so that students can see dynamic pictures. Only in this way can students understand the problem dynamically when they think about it, and at the same time, they can make the moving point static at the proper position, segment the movement process, and skillfully use mathematical ideas such as classified d

-iscussion and combination of numbers and shapes, so as to find the key to solve the problem.

C. *Improve Students' Ability to Use Knowledge*

When solving the problem of moving points, sometimes even if students have the correct ideas, problems such as unlisted relational expressions or calculation errors often occur in solving the quantitative relationship of static figures after segmentation, which is related to students' ability to solve problems by using knowledge comprehensively. Therefore, teachers pay attention to improving students' ability to use knowledge points in the teaching process.

The listing of relational expressions is the premise of solving the problem of moving points, which mainly involves the properties of planar geometric figures or functions, and most of planar figures involve their own properties. Combined with triangle similarity, congruence, parallel line property theorem, etc., through some given quantitative relations, by listing the quantitative relations of sum, difference, product and quotient, the equations are listed, thus being solved. Sometimes, problems about functions need to be solved by the properties of functions, such as symmetry, maximum value, and so on, which is also a method to list the relational expressions under special circumstances.

In addition, we should pay attention to cultivating students' computing ability, which is the foundation of mathematics, so we should emphasize the basic algorithms and formulas repeatedly in teaching to deepen students' understanding.

D. *Pay Attention to the Cultivation of Students' Creative Thinking*

Mathematical creative thinking is a kind of creative thinking, that is, creative presupposition and rational judgment in the process of mathematics learning, which is the synthesis of logical thinking and non-logical thinking, and also the dialectical unity of divergent thinking and convergent thinking [5]. In solving the problem of moving point, teachers should make a certain way of thinking according to various types of moving point problems. The teaching method of forming a mathematical concept is the activity of creative thinking in mathematics. In the process of learning, students should make pre-set judgment and thinking, and carry out reasonable verification under thinking. Based on the results, it is often a very successful thought to solve the problem of moving point road, this requires the cultivation of students' innovative consciousness in teaching, not only stick to boring teaching, but also guide students to find problems by themselves, form problem-solving strategies through their own thinking process, and establish students' self-confidence.

V. CONCLUSION

To sum up, with the rapid development of modern information technology, mathematics is more and more widely used in all aspects of social life and daily life. Mathematics curriculum in compulsory education stage is the foundation of cultivating mathematical thinking and core literacy, and "moving point problem" is an important comprehensive problem in junior high school mathematics, and it is the key to cultivate students' space concept and geometric consciousness, for teachers, it is also an important aspect of students' ability training. It is hoped that the teaching strategy of moving point problem proposed in this paper can give junior high school mathematics teacher certain reference. Junior high school mathematics teachers only continue to explore the teaching strategy of moving point problems, using the problem to cultivate students' mathematical

thinking, in order to continuously improve students' thinking ability and problem-solving skills, the knowledge involved in moving point problems can be integrated, and learning will achieve twice the result with half the effort.

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AUTHOR'S PROFILE

First Author
Sitong Dong, female, Jilin Province, China, born in July 2000, Studying at Yanbian University, as an undergraduate in mathematics education. email id: 394018165@qq.com

Second Author
Xinhui Zhou, female, Jilin Province, China, born in Jauary 1997, studying at Yanbian University, as a graduate student of Subject teaching (mathematics). email id: 2230106910@qq.com

Third Author
Qi Ge, female, Yanji City, Jilin Province, China, born in September 1975, master of science, teaching at Yanbian University, as associate professor, and master tutor. Research direction: Theory of mathematics teaching.