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Optimize the Carrying Capacity and Revenue Structure of Airport Taxi

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Abstract – **With the increasing passenger traffic of airplanes and the fact that taxis have become the main means of transportation connecting urban areas and airports, the value of optimizing the management of airport taxis is becoming increasingly important. In order to optimize the efficiency of carrying passengers and increase the profitability of taxis, the paper introduces time cost as an influencing factor for driver selection. Use queuing theory to calculate the time cost of different choices and situations, and introduce compensated temporal to balance the economic benefits of short-distance drivers and long-distance drivers. And time cost as an influencing factor for the placement of passengers at the pick-up point, constitutes the maximum flow problem to calculate the time cost, so as to obtain the optimal pick-up point placement.**

Keywords – **Non-Memory Property, Maximum Flow Problem, Gaussian Distribution, Order Statistics.**

I. INTRODUCTION

With the development of society and the improvement of people's living standards, the passenger traffic of aircraft has gradually increased. The airport is often located in the suburbs, and the main means of transportation for passengers to and from the airport to the urban area is a taxi. It would be a good plan to optimize the capacity of taxis per unit time and increase the economic benefits of taxis.

In recent years, many researchers have used queuing theory to study the management of airport taxis. In 2019, Wu [1] use mechanism analysis method and ranking theory model optimization method, the mathematical model of airport taxi queuing is obtained. In 2020, Duan [2] *et al*. using the income decision model, nonlinear regression model and queuing theory method to analyze the influencing factors of taxi drivers carrying passengers back to the city, the optimal setting plan for the “boarding point” near the airport “ride area” lane is given. In 2020, Sun [3] *et al.* use queuing theory and priority principles to optimize taxi management at airports.

It is known that taxi drivers travelling between the airport and the urban area will face the following two options after sending passengers to the airport.

(A) Taxis must first wait in line at the “storage pool” and carry passengers in the order of “come first, then arrive”. The number of queuing taxis and the number of passengers is a key factor affecting the queuing time, and taxi drivers need to pay the corresponding time cost.

(B) If the car goes straight back to the urban area to solicit passengers. Taxi drivers pay no-load fees and potential loss of passenger revenue.

What is the influencing factor for driver selection? And how to use it? How to arrange the passengers' pick-up point to improve the efficiency of carrying passengers? What measures can be taken to increase the economic benefits of drivers? The paper Taking Tibet Lhasa Gongga Airport as a background to solve this questions.

II. Model Establishment and Solution

A. *Model Assumptions*

1. Assuming that the number of passengers is unlimited, the arrival time is independent, the arrival time interval is random, and the arrival process is smooth. Model establishment and solution.
2. Both the time the customer receives the service and the time the customer arrives follow a negative exponential distribution.
3. Assuming single-line queuing, it is forbidden to quit midway during the queuing process.
4. Suppose the number of service desks is c and the service intensity is the same.
5. Taking Tibet Lhasa Gongga Airport as a background.
6. Assume that each passenger's boarding time conforms to a normal distribution.
7. Assuming that the distance of passenger destinations from the airport is normally distributed.
8. Assume the average taxi speed is the same.

B. *Symbolic Description*

Table 1. Symbolic description.

Number	**Symbol**	**Symbol Meaning**	**Number**	**Symbol**	**Symbol Meaning**
1	c	Number of service desks	10	E	service time
2	λ	Customer traffic per unit time	11	T'_n	Time interval between flight n-1 and flight n
3	μ	Average number of services completed per unit time	12	α	Time
4	ρ	Service strength	13	μ_1	the average mileage of passengers in the city from the airport
5	D_q	The expected value of the number of customers waiting in line in the system	14	δ_1	the standard deviation of mileage
6	D_s	The expected value of customers in the system	15	α	Time
7	T_q	The expected value of a customer's waiting time in the system	16	μ_1	the average mileage of passengers in the city from the airport
8	T_s	The expected value of a customer's total stay time in the system	17	δ_1	the standard deviation of mileage
9	P_n	The probability of n customers after the system reaches stability	18	T	Time required to carry passengers unilaterally

C. *Model Establishment*

The customer source is unlimited, customers arrive independently one after another, and the number of customers arrives within a certain time is subject to poisson distribution.

$$P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad (1)$$

and the arrival $P(x = k) = \frac{\lambda^k}{k!} e^{-\lambda}$ (1) process is smooth. After reaching the system, the service is accepted according to the first-come-first-served rule. Poisson flow arrival interval follows a negative exponential distribution [4]. If the probability density function of the customer reaching the interval T is:

$$f_T\ (t) = \begin{cases} \frac{\lambda^k}{k!} e^{-\lambda}, t \geq 0 \\ 0, t < 0 \end{cases} \quad (2)$$

The service time E of a customer is equivalent to the time interval between two adjacent customers leaving the queuing system. If E follows a negative exponential distribution. It's probability density function is:

$$f_E\ (t) = \begin{cases} \mu e^{-\mu t},\ \ t \geq 0 \\ 0, t < 0 \end{cases} \quad (3)$$

It's probability distribution function is:

$$F_E\ (t) = \begin{cases} 1 - \mu e^{-\mu t},\ \ t \geq 0 \\ 0, t < 0 \end{cases} \quad (4)$$

There are c service desks here, and the system has a limit on the number of customers which allowing a maximum of N. When the number of customers in the system is $N \geq c$, the new customers are rejected by the system. Based on the previous assumption that the customer arrival rate λ and service rate μ are the same as the previous model, and let $\rho = \frac{\lambda}{c\mu}$. At this time, the corresponding state probability.

$$P_0 = \left[\sum_{k=0}^{c} \frac{(cp)^k}{k!} + \frac{c^c}{c!} . \frac{\rho(\rho^c - \rho^N)}{1-\rho}\right]^{-1}, \rho \neq 0 \quad (5)$$

$$P_n = \begin{cases} \frac{(c\rho)^n}{n!} P_0, (0 \leq n \leq c) \\ \frac{c^c}{c!} \rho^n\ \rho_0,\ (c \leq n \leq N) \end{cases} \quad (6)$$

The formula derivation results of some related indicators are as follows:

$$D_q = \frac{P_0 \rho\ (c\rho)^c}{c!\,(1-\rho)^2} [1 - \rho^{N-c} - (N - c)\rho^{N-c}(1 - \rho)] \quad (7)$$

$$D_s = D_q + c\rho(1 - P_N) \quad (8)$$

$$T_q = \frac{D_q}{\lambda(1 - P_N)} \quad (9)$$

$$T_s = T_q + \frac{1}{\mu} \quad (10)$$

Known: After arriving at the airport, the driver can directly know that the number of passengers who need to ride is N, and the number of taxis in line in front is S. Drivers can use Equation (7) to calculate the expected value of the number of people queued when the system is stable.

Situation A: when $S \geq D_q$, The driver refused to participate in this system. Need to wait for the next flight to participate in the next queuing system. The queue time required to get the next service is,

$$T = T_{s_0} + T_{s_1} + \text{---} + T'_{s_n} + T_{t_1} + \text{---}\ T_{t_n} \quad (11)$$

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Situation B: when $S < D_q$, drivers can enjoy the service. The time required is,

$$T = T'_{s_0} \tag{12}$$

Drivers can carry passengers on the way back. The driver began to carry passengers at any point in the city and showed no memory. Drivers can estimate their profitability along the way based on their experience. The direct factor that determines the driver's income is the driver's trade-off in the waiting time at the airport.

In reality, there is no taxi desk at Gongga Airport, and there are often taxi drivers waiting in line to wait for guests. Therefore, there are many taxis arriving per unit time, assuming Customer traffic per unit time is $\lambda = 11$. Combined with the local geographical environment and economic development, assuming the average number of services completed per unit time is $\mu = 3$. Assuming that the number of service desks is $c = 4$ to calculate the service intensity

$$\rho = \frac{c}{\lambda\mu} \tag{13}$$

Running the code through MATLAB got the following chart.

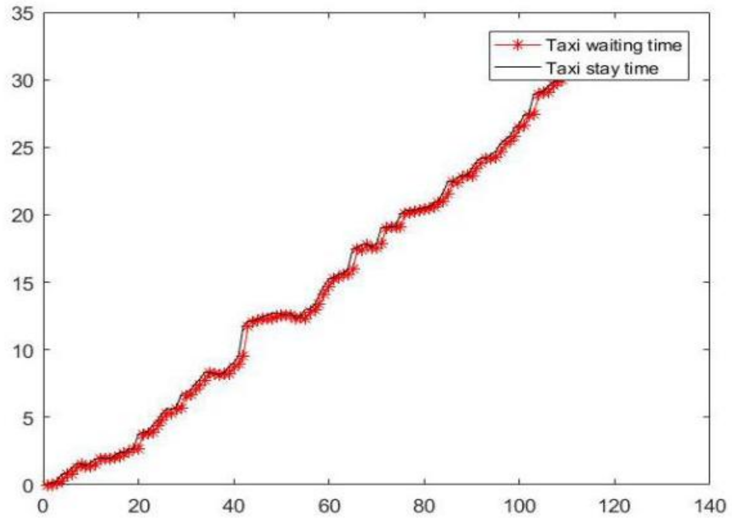

Fig. 1. Diagram of the relationship between waiting time and stay time.

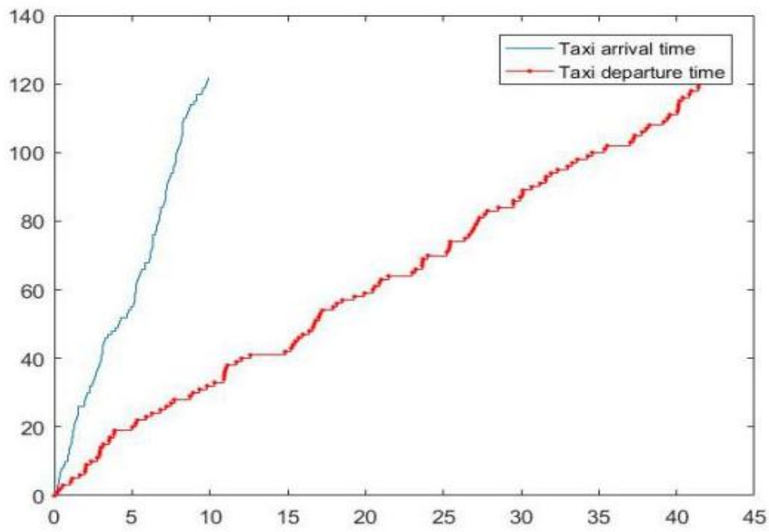

Fig. 2. Diagram of the relationship between arrival time and departure time.

When the dependent variable is fixed, the difference ΔT on the abscissa is equal to the time spent by the driver at the airport.

Drivers can judge the economic benefits that can be obtained outside the airport within ΔT time based on experience and make trade-off choices.

This is a parallel lane, with “boarding points” on both sides. Because the time for passengers to get on the car is normally distributed, the degree of mutual influence between cars and their probability also change due to the existence of probability. We can solve this system optimally by the maximum flow method. In real life, the queuing “boarding point” of the station is almost always on the side, so we assume that the “boarding point” is all on the side is the best solution.

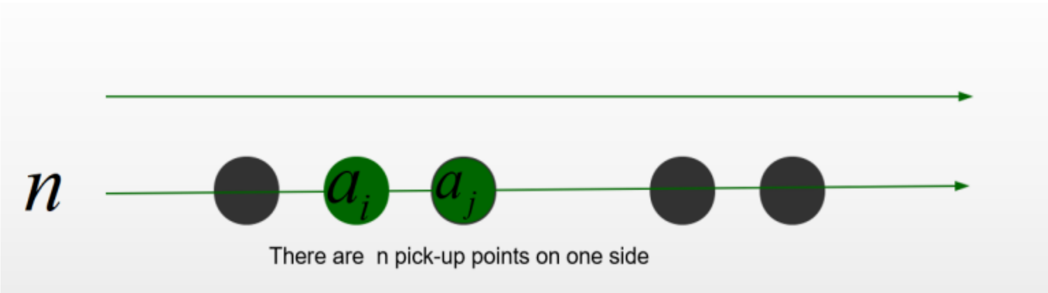

Fig. 3. “Boarding point” is all on the side.

Proof the “boarding point” is all on the side is the best solution. Each passenger's boarding time is different, according to the order statistics $[a_0, a_n]$.

Proof:

First assume that the side lane where no "boarding point" is placed can be used as a sparse flow. We have:

$$T_{n_1} = \sum a_i \tag{14}$$

Take one of the pick-up points and install it on the other side of the road,

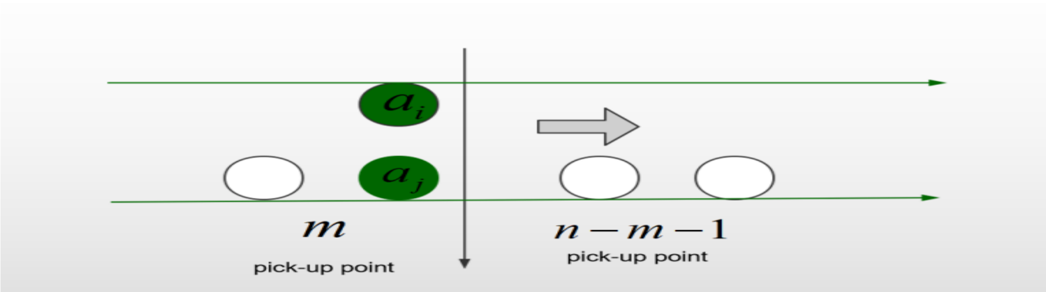

Fig. 4. “Boarding point” is on both sides.

Suppose there are m cars after the a_j pick-up point,

We have:

$$T_{n_2} = T_{(n-1-m)} - m.a_i \tag{15}$$

Further:

$$T_{n_1} - T_{n_2} = T_{(m+1)} - m.a_i \tag{16}$$

According to Theorem of large numbers, we have:

$$E(T_{n_2} - T_{n_1}) = a_i \quad (17)$$

It can be deduced that $T_{n_X} > T_{n_1}$ when X pick-up points are assigned to the other side.

According to the previous derivation, we can know that putting the "boarding point" on the same side is the best choice.

Assuming that the driver's mileage from the airport is consistent with L and has a normal distribution [5]:

$$L \sim N(\mu_1, \delta^2{}_1) \quad (18)$$

μ_1 represents the average mileage of passengers in the city from the airport, and $\delta_1$represents the standard deviation of mileage.

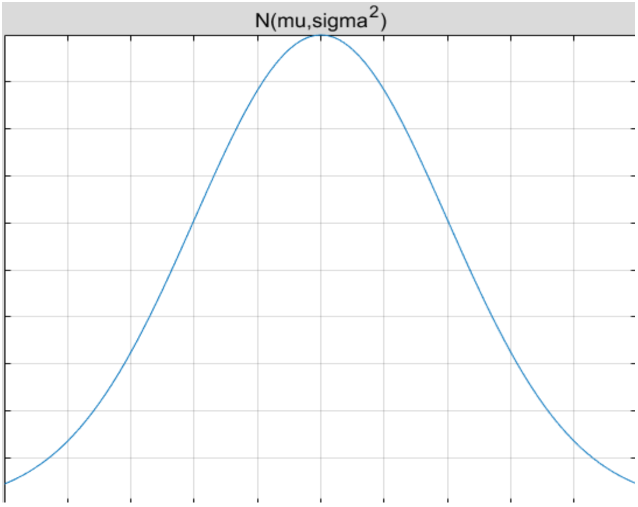

Fig. 5. Normal distribution density map.

Because the taxi passenger carrying revenue at the airport is related to the mileage of passenger, so we have:

$$\varpi \sim N(\mu_1, \delta^2{}_1) \quad (19)$$

(Where ϖ is passenger revenue)

When $x < x_1$, the service means short-distance transportation. When $x > x_2$, the service means long-distance transportation. According to the nature of normal distribution, we have:

$$x_2 = (\mu_1 - x_1).\ 2 - x_1 \quad (20)$$

Let the average taxi speed be v . So, Then the profit time for short-haul passengers:

$$T_{short} = \frac{x_1}{v} \quad (21)$$

Benefit time for long-distance passengers:

$$T_{long} = \frac{(\mu_1 - x_1).2 - x_1}{v} \quad (22)$$

So the difference in working hours is ΔT,

$$\Delta T = T_{long} - T_{short} = \frac{(\mu_1 - x_1).2}{v} \quad (23)$$

Let the average queue waiting time for taxis at the airport be α . Priority time (compensated temporal) should be given to short-haul passenger taxis.

$$\alpha_p = \alpha - \Delta T \tag{24}$$

When the short-distance passenger car returns to the airport, the airport staff should lead the taxi to jump in line for giving compensated temporal.

The calculation method of the specific cut-in position is as follows.

$$T_q = \frac{D_q}{\lambda(1-P_N)} = \alpha \tag{25}$$

$$P_n = \begin{cases} \frac{(c\rho)^n}{n!} P_0, \ (0 \le n \le c) \\ \frac{c^c}{c!} \rho^n P_0, (c \le n \le N) \end{cases} \tag{26}$$

Assuming that short-distance drivers do not jump in line, the time required is

$$\alpha = \frac{D_q}{\lambda(1-P_{N_1})} \tag{27}$$

Assuming that the short-distance driver jumps in line *j* positions forward, the time required is

$$\alpha' = \frac{D_q}{\lambda(1-P_{N_{1-j}})} \tag{28}$$

Let

$$\alpha_p = \alpha - \alpha' \tag{29}$$

We can get the value of *j* therefore, the airport staff needs to arrange short-distance passenger drivers to insert *j* positions forward to get compensation time.

III. Conclusion

This paper Taking Tibet Lhasa Gongga Airport as a background. We use cast time as the key to solve those questions. Under a series of reasonable assumptions, we found the cost of time to complete the service through the mathematical knowledge of order statistics. We use MATLAB visualize formulas and data. It is convenient for the driver or relevant management personnel to decide to go or stay according to the result. We can know that putting the “boarding point” on the same side is the best choice by previous derivation. Through the adjustment of the position of the short-distance taxi by the airport staff, short-distance passenger drivers obtain the compensation time for balancing the economic benefits between the drivers.

But there have many factors need to be considered in real life.

(1) Economic development and structure, demographic factors and their composition.

(2) Traffic condition.

(3) Airport layout and functional positioning.

(4) The total number of passengers on holidays increased sharply, and the total number of passengers on working days contract.

(5) Weather and climate.

The Model Established in this Paper Have Advantages:

A large number of ideas of probability theory and mathematical statistics are used. The relevant theoretical knowledge of operations research and graph theory is appropriately added to quantify the real-life passenger transportation difficulties with mathematical knowledge. To a certain extent, the effective use of resources is greatly improved, saving precious time.

The Model Established in this Paper also have Disadvantages:

Ignore many secondary factors to create a more ideal environment system. The biggest difference between ideal and reality is the large error.

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