
Massive Gravity from a Fractal-Cantorian Spacetime Perspective

Mohamed S. El Naschie

Department of Physics, Faculty of Science, University of Alexandria, Egypt.

Corresponding author email id: Chaossf@aol.com

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Abstract – A physico mathematical comparison is undertaken between the remarkable theory of massive gravity and the fractal Cantorian spacetime methodology of integrating all the fundamental forces with gravity. We conclude that at a minimum the two theories are complimentary from the deep understanding viewpoint. However from certain other perspectives, E-infinity theory may be simpler in several number theoretical and physical respects as we expose here at some length. Finally and in addition to the above, we give an exceedingly simple mathematical rationale for the mystery of quantum vacuum fluctuation.

Keywords – De Rham Massive Gravity, Ord-Nottale-El Naschie Fractal Cantorian Spacetime, Unification of All Fundamental Forces, Golden Mean Number System, Quantum Gravity, Graviton with Non-Zero Mass, Quantum Vacuum Fluctuation.

I. INTRODUCTION

Einstein's relativity, both special and general, was conceived some time before the inception of the quantum and its quantum relativity theory [1-25]. Consequently Einstein did not ponder the existence or non-existence of massive gravity because there were no quantum particles named graviton around to consider it massless or massive to solve the mystery and contradiction of dark matter energy, which were, also discovered even much later than quantum mechanics and our present classical form of the standard model [1-2], [10-12]. None the less, the theory of massive gravity which proposed a non-zero mass to the hypothetical spin 2 quantum particle, i.e. the graviton is a profound, radical theory which did not appear out of nothing and surely deserves careful consideration [25-30]. On the other hand we attempted in our own E-infinity Cantorian spacetime theory to explain dark matter energy without involving anything more than classical topology and classical and chaotic mechanics [30-58] and in the course of doing that, explain the accelerated expansion of the cosmos [16-25]. However the massive gravity theory has roots going back in time as a fleeting look at the literature, particularly the powerful publication of Prof. Claudia de Rham [26-28] and her school of thoughts clearly reveal [28]. None the less, coming from a completely different direction, there is the said other way of looking at the same thing but through a radically different spacetime setting [5-25], [35-60], namely that of fractal geometry and the E-infinity theory of fractal Cantorian spacetime [8-25]. The remarkable and in fact more than radical theoretical findings of this E-infinity theory is that the standard model as it is already contains all what it needs and is complete when seen through fractal eyes using fractal logic which implies fractal counting [29-30]. The 14 particles counted using our ordinary integer counting include not only the Higgs boson but also the graviton. It is not only any graviton but a pre-quantum graviton with a fractal weight equal to $k = \phi^3 (1 - \phi^3)$ to the power of two, i.e. the square of 'tHooft's renormalon k^2 and ϕ^3 is the inverse of the scale invariant Einstein fractal spacetime dimension $4 + \phi^3$ where $\phi = (\sqrt{5}-1)/2$ is the golden mean [10-20]. This is what we christened fractal graviton while the corresponding fractal Higgs is given by $k_o = (\phi^5) (1-\phi^5)$ where ϕ^5 is the famous Hardy's probability of quantum entanglement [31]. Incredible as it may sound, counted using fractal logic, the fourteen

are only $\sqrt{a_0} = 11.7082$, i.e. almost 12 [29], [60]. This situation eliminates the need to talk about this quasi graviton particle as massive or not. Seen in this way it is really good news not to contemplate the massiveness of the graviton since mass is by its very nature not a deep concept [32-34] and the relation between mass and energy is also open to questions that constitute a large part of the achievements of E-infinity Cantorian spacetime theory in elucidating the deep meaning of $E = mc^2$ of special relativity [29-34]. In the present work we will reconsider all the preceding questions in depth and we will draw on some of the fundamental concepts of Gross' Heterotic superstring theory [35-37] as well as the Gravi weak theory [38-40] to show how we can uphold the dark matter theory and arrive at accelerated cosmic expansion while keeping maximal simplicity without the need to invoke the massiveness of the graviton [26-28] and loose the consistency of the present framework of the standard model of high energy particle physics [34-37]. On balance it may be possible that a theory of massive graviton without the need for dark matter energy and a theory without a massive graviton but with the Heterotic superstring structure with ordinary energy, dark matter energy and pure dark energy are two different sides of the same theoretical coin [33-40]. We always strive for understanding although if we are really honest with ourselves, we must admit we do not know exactly what we mean with this word and that we do not fully understand the structure and functioning of our own brain [9]. At the end analysis only painstaking refined experiments would be able to discriminate between the two theories although it is only fair to admit that the present author, as probably one expects, tends to prefer the idea of keeping the dark matter energy theory in place and follow the path chartered by E-infinity Cantorian spacetime which we will explain in some detail next [5-60] and in this context we will give a very simple mathematical proof for quantum vacuum fluctuation [1-10].

II. MINIMALISTIC BACKGROUND FOR E-INFINITY DARK ENERGY SET THEORETICAL MATHEMATICS

In a nutshell E-infinity Cantorian spacetime theory postulates that the zero set and the empty set are identical to the pre-quantum particle [10] and the pre-quantum wave respectively [10], [12], [13]. In both cases they are described with the bi-dimension $D(0) \equiv (0, \phi)$ and $D(-1) \equiv (-1, \phi^2)$ respectively where zero and minus one are the topological dimension while ϕ and ϕ^2 are the corresponding Hausdorff dimension respectively. This follows straightforwardly from A. Connes' dimensional function [10-13]

$$D = a + b \phi \tag{1}$$

Where $a, b \in Z$ and $\phi = (\sqrt{5}-1)/2$ is describing Penrose fractal tiling universe which is modelling the holographic boundary of our cosmos according to our E-infinity Cantorian spacetime theory [8-14]. After reasoning that our universe is Kaluza-Klein like and possesses five topological dimensions, it is easily concluded and computed that the ordinary energy density of our universe is given by [8-19], [60]

$$\begin{aligned} \gamma(0) &= \phi^5 / 2 \\ &\approx 4.5\% \end{aligned} \tag{2}$$

while the dark energy density is given by [8-14], [17-25], [60]

$$\begin{aligned} \gamma(D) &= 5\phi^2 / 2 \\ &\approx 95.5\% \end{aligned} \tag{3}$$

On the other hand this dark energy, which cannot be measured directly consists of two parts. The first is dark matter energy amounting to 22.2% of the total maximal energy [13-14], [17-25] and the rest of about 73.3% is pure dark energy with negative (repulsive) pressure [17-25] to which the accelerated cosmic expansion, which is not to be confused with the ordinary Hubble expansion is attributed. This accelerated expansion was solved quantitatively but not completely using D. Gross et al via the ingenious theory of Heterotic superstring theory [35-37]. It is the purpose of the present work to complete and refine the Heterotic derived accelerated expansion of the universe to show how it constitutes a rather simple explanation without the need to attribute a mass to the hypothetical graviton of an extended standard model of high energy particle physics as required by the equally brilliant theory of massive gravity of Prof. Claudia de Rham [26-28].

To understand the above strategy we need one more additional information gained from E-infinity theory, namely that from a number theoretical viewpoint, the 12 messenger particles of the standard model, namely $|SU(3) SU(2) U(1)| = 12$ are in fact 14 messenger particles within the fuzzy fractal counting, i.e. dimensional number counting weight of $\sqrt{\bar{a}_o} = 11.7082$ when applying the fractal logic of counting in a Cantorian-fractal spacetime where $\bar{a}_o = 137 + k_o$ is the inverse electromagnetic coupling constant, $k_o = \phi^5 (1-\phi^5)$ and ϕ^5 is Hardy's quantum probability of two particles quantum entanglement [29-31]. In other words, 11.70803398 already includes a pre-graviton described by k^2 where $k = \phi^3 (1-\phi^3)$ is 'tHooft's renormalon as explained above and in numerous previous publications [16-25].

III. HETEROTIC STRINGS PLUS GRAVI-WEAK LEADS TO FRACTAL PURE DARK ENERGY ACCELERATED EXPANSION VIA E-INFINITY FRACTAL CANTORIAN SPACETIME THEORY

Let us go first to the rudimentary integer form of applying Heterotic string theory to the issue of ordinary and dark energy density of the universe [24], [35-37]. To do this we recall the basic ingenious idea devised by D. Gross et al to join the old bosonic string theory with its 26 dimensions and the ten dimensional superstring theory into one theory via the deceptively trivial equation [24], [29], [35], [37], [60]

$$26 - 16 = 10 \tag{4}$$

where an additional 16 bosonic dimensions are 'running' into an opposite direction to the rest. In what follows we will reason that the negative pressure responsible for the accelerated expansion of the universe is linked to the pure dark energy and these 16 extra bosonic dimension, as well as a subtle link between these ideas and the theory of Gravi-weak [38-40]. Now following previous expositions, the total maximal energy may be written using Gross et al Heterotic superstring theory slicing as [24], [29], [35-37], [60]

$$\begin{aligned} E &= (22/22)mc^2 \\ &= \left(\frac{1+5+16}{22}\right)mc^2 \\ &= \left(\frac{1}{22}\right)mc^2 + \left(\frac{5}{22}\right)mc^2 + \left(\frac{16}{22}\right)mc^2 \\ &= (\gamma(O))mc^2 + (\gamma(DM))mc^2 + (\gamma(PD))mc^2 \\ &= E(\text{Einstein}) \\ &= mc^2 \end{aligned} \tag{5}$$

Here m is the mass and c is the speed of light while $(\gamma(O))$, $(\gamma(DM))$ and $(\gamma(PD))$ are the ordinary, the dark m-

-atter and the pure dark energy densities respectively [24], [29], [35], [37]. In the above it was of course tacitly implied that we are dealing with modulus of γ irrespective of the sign so that our 100 core space dimensions as given by the fundamental equation [29-31], 35-37].

$$\begin{aligned} \sum_{i=1}^4 \bar{a}_i &= \bar{a}_1 + \bar{a}_2 + \bar{a}_3 + \bar{a}_4 \\ &= 60 + 30 + 9 + 1 \\ &= 100 \end{aligned} \tag{6}$$

are effectively subdivided into [10-19], [17-25]

$$\begin{aligned} \sum_{i=1}^4 \bar{a}_i &= 4 + 22 + 74 \\ &= (\gamma(0)) + \gamma(DM) + \gamma(PD) \end{aligned} \tag{7}$$

In other words, we have assigned four dimensions for ordinary energy, $26 - 4 = 22$ compactified bosonic dimensions for dark matter energy and the remaining 74 dimensions for the pure dark energy sector [34-37]. However from another viewpoint things are not additive in view of the 16 dimensions running in the opposite direction as devised by Gross et al's ingenious insight. Consequently we really have an auxiliary equation saying de facto that [13-14], [16-25]

$$|22| + |74| = 96\% \tag{8}$$

in accordance with Dvoretzky measure concentration theorem [13-25]. This situation also leads to including the minus sign of the negative 16 extra dimensions which are the source of 74 percent of pure dark energy as per the value of $16/22$ expressed in a percentage of the 100% total of equations (7) and (8). Consequently it is obvious that we are justified in writing [41-44]

$$22 - 74 = -52 \tag{9}$$

This result is now consistent with $|F_4| = \dim F_4 = 52$ of the exceptional Lie groups that we invoke in the unification of the weak force with gravity, which is well known as the Gravi-weak unification [26-28], [41-44]. The weak force on the other hand is responsible for the radioactive decay that is balanced by the strong force in all stable forms of matter that we encounter in nature. In other words, the negative sign of Gross et al's 16 extra bosons is directly linked to the $|F_4| = 52$ and the repulsive force of the weak interaction [39-44] and hence the negative pressure of pure dark energy, which we and others consider responsible for the accelerated cosmic expansion [20-25]. This subtle mathematical explanation seems to us as compelling and simple as any but at a minimum, it is equivalent and as a convincing an argument as assigning mass to the hypothetical graviton in the massive gravity theory [38-40]. At present this is an attractive explanation when remembering that we have not experimentally found the graviton with or without mass until this moment.

Finally for additional coverage of subtle intricate points of the above, the reader may be advised to consult some of the very recent publications on E-infinity and the platonic quantum set Cantorian spacetime theory [45-60].

IV. FROM TOLSTOY TO VACUUM FLUCTUATION VIA NONLINEAR DYNAMICS AND NONCOMMUTATIVE E-INFINITY FRACTAL GEOMETRY - A BIRD'S EYE VIEW AND GENERAL DISCUSSION [15], [55-60]

Writing about history is in general a difficult and intricate undertaking as is well known from literary works such as War and Peace by Leo Tolstoy [59]. However writing about history of science, let alone the inception of our own scientific ideas and their source and when it first came to mind, is more than a slippery slope with countless pitfalls which could be more misleading than helpful.

In the present paper we emphasized one direction from where our ideas came, namely the work of Ord, Nottale, Gross and in no minor measure, the theory of continuous pointless spacetime geometry of von Neumann and its extension and perfection to the theory of noncommutative geometry of Connes. By doing this we now feel that we may have underplayed the pre-eminent direction from which we realized the central role that the golden mean and the golden mean number system came to play in shaping the Cantorian E-infinity theory and the transfinite set theory based quantum mechanics. At that time, all these realizations came from the very fashionable and vigorous modern theory of chaos, fractals and nonlinear dynamics with R. Thom VAK and Kolmogorov-Arnold-Moser KAM theorem and the golden mean at its centre. In fact for a man initially trained as an engineer who went on to become a professor of engineering before studying physics or pure mathematics, it is very doubtful that he would have taken number theory and the golden mean as well as the associated Cantor sets seriously if it were not for the impressive application which came from the pioneers of deterministic chaos, such as Ruelle, Feigenbaum, Procaccia, Mandelbrot, and Rossler [15] to mention only a few of many dedicated and courageous scientist and engineers who had the courage to break with orthodoxy and discover a nova terra connecting determinism with indeterminism and coining what on the surface must appear as a totally contradictory name for a new science, i.e. deterministic chaos which was what the applied mathematician J. York proposed [15]. Thus these critical after thoughts must be followed by some additional references documenting some of the most important early papers which carried the seeds for the fully-fledged expansion and development of the present Author's E-infinity theory [10-20]. In this respect one must mention the role that the founding editor of a completely unorthodox and unusual Springer Journal played in teaching the E-infinity infant how to walk and we mean here the late Prof. Alan McKay and his journal Speculation in Science and Technology [55], [58]. Speaking of Leo Tolstoy [59], it was indeed Alan McKay who drew my attention first to the unexpected relevance of the great Leo Tolstoy to the present Author's work so with the kind permission of the patient reader, the writer and without much more ado would like to add the following references as an additional source of relevant information [55-59]. It is in more than one respect important reading to the subject of the present paper. We must in particular stress the paper of Allen D. Allen, which A. McKay drew the attention of the author to [58]. This paper, which was published in his remarkable speculation journal, was to my mind one of the first to draw strong attention to the role of fractals in quantum mechanics [55-59]. Returning to the great Leo Tolstoy [59], the Author admits that he strongly believes that he combined the two opposites, i.e. reality and fiction in the same way the heroes of deterministic chaos [15] reconciled chance and disorder with determinism in unheard of ways [56]. In conclusion of this section it is of the utmost importance to realize how the pre-quantum particle zero set and the pre-quantum wave empty set as given by ϕ and ϕ^2 respectively based on the dimensional function of noncommutative geometry (see equation (1) and Ref. [10]) give an exceedingly

simple mathematical proof for the otherwise seemingly perplex phenomena of quantum vacuum fluctuation, i.e. pair creation and annihilation [1-10]. The mathematical rationale of quantum vacuum fluctuation is disarmingly simple since ϕ is effectively the particles quantum probability and ϕ^2 is the waves quantum probability and since the particle is essentially regarded in an experiment as a zero set matter while the wave is regarded also in experiments as an empty set space, then it follows by virtue of $\phi \otimes \phi \rightarrow \phi^2$ and the reverse of fusion, i.e. the reverse of intersection $\phi^2 \rightarrow \phi, \phi$ that pair creation and annihilation are almost a trivial inevitable result of E-infinity Cantorian spacetime and thus nature [10-20]

V. CONCLUSION

Looking at the subject of the present work as objectively as humanly possible, we do not have any experimentally conclusive watertight proof that the theory of massive gravity is more real or convincing than the E-infinity Cantorian spacetime theory of dark matter and dark energy or do we? What is simpler and more natural? It seems fair to say that one is permitted to ask oneself what could be better from the following two alternatives:

- (a) To assume that dark energy is real and that it is the energy of the empty set quantum wave while ordinary energy is the energy of the zero set quantum particle or
- (b) To assume that the hitherto hypothetical massless graviton, i.e. the messenger particle of the gravitational field, against our initial supposition, does have a mass after all.

In this respect it is hard for the present author to be really, totally objective so our tentative most possible objective conclusion is that the situation is almost equal or almost identical with the scale may be tilting slightly towards (a), i.e. our E-infinity resolution of the mystery of dark energy which we do not in all events think is a real mystery when we apply our physico-number theoretical machinery of transfinite set theory based quantum mechanics to it.

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AUTHOR'S PROFILE



Professor M.S. El Naschie was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree in Engineering plus being a professional chartered engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt. Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work in cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems. Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 81 and his i-10 index is 793 and total citations are 36006 according to Google Scholar Citation.