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From Zeno to Einstein: Is there Really Not a Universal Time?

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***Abstract* – In the 5th century BC Zeno proposed his famous motion paradoxes. These paradoxes denied the reality of the motion and therefore of the time itself. In the 20th century AD the relativity theory of Einstein opened the way not only to the thesis of the relativity of time with respect to different inertial systems, but also to the thesis of the relativity of simultaneity. However the experimented *entanglement* introduces the possibility, for the subatomic particles, to be connected, and in a certain sense to communicate, instantly at any distance, thus implying an absolute synchronicity. In this paper after a discussion on two of the Zeno's motion paradoxes (the "Achilles" and the "Arrow") and after presenting two possible solutions for the two paradoxes, the issue of the relativity of synchronicity will be addressed, showing that a universal time, even if a quite complex universal time, can exist.**

***Keywords* – Achilles Paradox, Arrow Paradox, Relativity of Simultaneity, Universality of Time, Thought Experiment.**

I. Introduction

In the 5th century BC Zeno proposed his famous motion paradoxes. These paradoxes denied the reality of the motion and therefore, in our opinion, of the time itself. In the 20th century AD the relativity theory of Einstein opened the way not only to the thesis of the relativity of time with respect to different inertial systems, but also to the thesis of the relativity of simultaneity. However the experimented entanglement introduces the possibility, for the subatomic particles, to be connected, and in a certain sense to communicate, instantly at any distance, thus implying an absolute synchronicity.

In this paper after a discussion on two of the Zeno's motion paradoxes (the "Achilles" and the "Arrow") and after presenting two possible solutions for the two paradoxes, the issue of the relativity of synchronicity will be addressed, showing that a universal time, even if a quite complex universal time, can exist.

II. Zeno's Achilles and Arrow Paradoxes

2.1. *Some Considerations on the Nature of Space and Time*

Kronecker said that "God created the integers, all else is the work of man" [1]. Effectively all things seem to be representable by integers. A fraction of a man is not a man but only a set of molecules, a fraction of an atom is not an atom but a subatomic particle. It seems therefore that everything is composed by single entities which cannot be divided without losing their nature, and according to the modern Physics this could be true even for space and time (the Planck time and length).

This vision is not new: it is believed that the concept of granular time had already been introduced by Democritus. However it was certainly proposed by Isidore of Seville [2] and Bede the Venerable [3]. In 1190 Moses ben Maimon (Maimonides) [4] wrote that "Time is composed of atoms, that is to say of many parts that cannot be further subdivided, on account of their short duration" [5].

In 1927 Levi [6], studying the possibility of the existence of the atom of time, named it *chronon* and calculated for it a duration of about 4.48 10^{-24} seconds (the Planck time is instead about 5.39 10^{-44} seconds).

Obviously from a mathematical viewpoint also these entities have an extension and we could individuate infinite points in them but, according to modern Physics (and Mathematics too: see Paolilli [7]), none of these points can be recognized as space or time.

According to the concept of the granular nature of time and space the motion paradoxes could not be posed. In what follows, however, two of these paradoxes will be addressed without taking into account the hypothesis of the granularity of space and time.

2.2. *The "Achilles"*

First a note: it seems that Zeno told about a race between two athletes with different speeds and then the race between Achilles and a tortoise is only a successive representation of this paradox.

The "Achilles" paradox has been solved by means of the consideration that Achilles does not perform infinite tasks (a super-task) because he is not forced to make interruptions (about the literature on this issue, see Fano [8]).

However, already Aristotle [9] gave a possible solution noting that if the space can be divided infinite times, this is possible for a time interval too.

Here we want to underline that the Achilles Paradox could be banally solved even by means of the observation that the task of Achilles is not to reach every time the point in which the tortoise was before, at the beginning of the task. The task of Achilles is instead to overcome the tortoise and not a point where it was in the past.

In symbols, named A Achilles and V_A his speed, x the tortoise and V_x its speed, L_0 the vantage of the tortoise, that is the initial distance between the tortoise and Achilles, T_i , where $i = 1 \dots \infty$, the tasks of Achilles, and t_i the time intervals required for Achilles to carry out his tasks, we have the following situation.

$$T_1 = L_0 \qquad (1)$$

The time required is

$$t_1 = L_0 / V_A \qquad (2)$$

but in t_1 the tortoise travels L_1 (that is $V_x t_1$) which is the new task of Achilles:

$$T_2 = L_1 \qquad (3)$$

Achilles will do this task in the period:

$$t_2 = L_1 / V_A \qquad (4)$$

and so on. The segments L_0, L_1, L_2… of space traveled by Achilles are shorter and shorter but also the time intervals t_1, t_2, t_3,.. are shorter and shorter, as already noted by Aristotle, while the time, in a given inertial system, does not slow down and does not stop.

According to this approach only apparently the task of Achilles is to reach the tortoise: this is the declared task, but actually each time he is required to travel a stretch already traveled by the tortoise, so reaching a point that the tortoise has already reached in the past, at the beginning of the Achilles' task. The Achilles paradox, voluntarily or not, is therefore a mystification. It is a task designed so that it can not be done: to reach and overcome the tortoise, but only after an infinite number of activities.

Moreover the task of Achilles is not to reach infinite points, but it is to cover a line. According to Paolilli [7] a line is not composed by points, even if we can individuate infinite points (positions) in it. In the same manner Achilles makes this task in a period of time: also in the time axis we can individuate infinite points whose length (duration) is *0*. However, even if Achilles has to surpass infinite points in the space, he has a time in which there are infinite points to do this but, as we have told, this is not his task.

Note that after the overtaking the problem can be reversed: the task of the tortoise could become reaching all the points reached by Achilles. In this case the tortoise will never reach Achilles, but it will reach, in an increasingly long time, any point reached by Achilles, thus prefiguring a situation similar to that of Tristram Shandy, a fictional character who takes a year to keep his diary for each day lived [10].

2.3. *The Arrow Paradox*

There is no general consensus about a solution for the Zeno's arrow paradox. In this case the paradox is caused by the assumption that space is a dense set of points.

In the light of the redefinition of the geometrical objects genesis [7], according to which a n-dimensional geometrical object can not be generated by a geometrical object with less than *n* dimensions, the space is not composed by points. Also the time, representable on a line as a spatial distance, if we assume that the duration of an instant is *0*, is not composed by instants. Instants, so defined, are in time axis as points are in lines and these are in planes and so on, but lines are not composed by points whose length is *0* and time is not composed by a-dimensional instants. A line is a distance or a boundary. In the same manner time represents a duration while an a-dimensional instant is only a position in the line which describes the time duration. Therefore the arrow is never stopped in time because time is not composed by a-dimensional instants. Time has always an extension, and so even if we do not take into account the hypothesis of its granularity. We can in fact identify infinite positions of the arrow in its trajectory, but these positions are detectable only if we assume that $\Delta t = 0$, and we know that in the universe, at least out of a black hole, it never happens that $\Delta t = 0$.

III. Relativity or Absoluteness of Simultaneity

It is well known that two events which appear simultaneous to one observer may appear non-simultaneous to another observer in motion with respect to the first observer.

In 1916 Einstein proposed a thought experiment in his popular book [11] but reiterating a concept already expressed in 1905 [12] (the experiment has been proposed, among others, also by Gardner [13]). In the experiment an observer O_1 stands on a platform in a railway station while a train passes very quickly. On the train there is an observer O_2. When O_2 is exactly in front of O_1 two bolts of lighting fall at two points (*A* and *B*) on the railway, opposite to the two observers but at the same distance from them.

Case 1: O_1 at rest, O_2 moving towards *B*. To the observer O_1, standing on the platform, the two events appear simultaneous because the light of the two bolts of lighting must travel the same distance from *A* and *B* to O_1. The observer O_2, being in motion towards *B*, will first see the bolt of lighting fallen in *B*. However he knows to be in motion towards *B* and then he can calculate that the two events are simultaneous. The graph a of Figure 1 shows the thought experiment exposed above.

Case 2: O_2 at rest, O_1 moving towards *A*. On the other hand, according to the relativity theory we can assume that O_2 is at rest while O_1 and the platform are moving towards *A*. In this case Gardner [13], similarly to Einstein [11], notes that even if O_1 sees the two bolts of lighting in the same moment this does not signify that they are simultaneous due to the fact that O_1, in this scenario, is moving towards *A*. In this scenario the event *A* happens after the event *B*: a single event, therefore, can be considered as previous, simultaneous or subsequent against another according to the reference system that we assume as at rest. The graph b of Figure 1 shows the Case 2 (the coordinates of the graph b are calculated taking into account the Lorentz's transformations factor (y) and for the sake of simplicity all the axes are orthogonal).

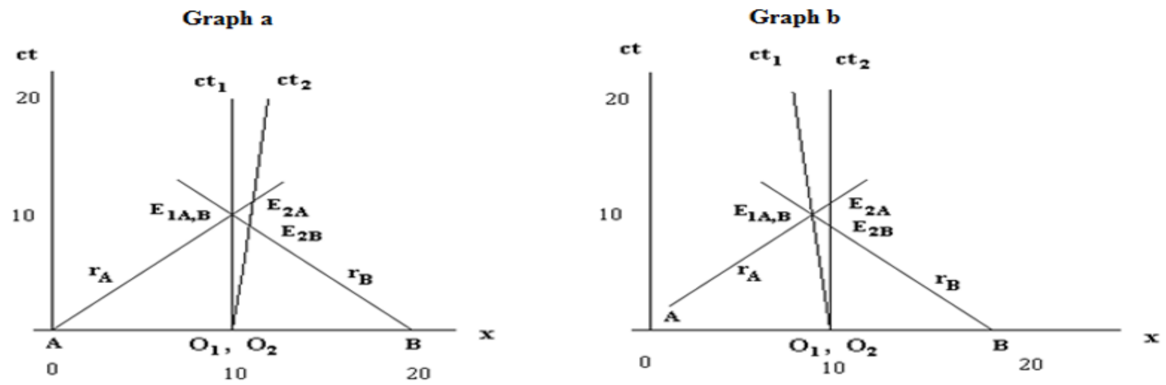

Fig. 1. In Graph a the observer O_1 is at rest while O_2 is moving towards B. In Graph b O_2 believes to be at rest while O_1 is moving towards A. The coordinates of Graph b are calculated taking into account the Lorentz transformation factor (y) and for the sake of simplicity all the axes are orthogonal. $v = 0.1\ c$, $y = 1.005$, where v is the speed of the moving observer and c is the light speed.

To this thesis the following objections can be presented.

First objection. It is possible to assume that O_2 is at rest and O_1 is moving towards A, but in this case the whole Earth is moving towards A and then also the atmosphere, the bolts of lighting and therefore even the points A and B: therefore the distance between A and O_1 is constant and so the distance between B and O_1 . In other words:

3.1 *If two Observers are Moving Relative to Each other along a Line in Which One or More Light Source (s) is (are) Fixed, Only one of them Can be at Rest with Respect to the Light Source(s).*

Second objection. Moreover, even if we do not consider the first objection but O_1 and O_2 know the frequency of the light emitted by the sources of light (which is also reasonably isotropic), they will be able to know the real sequence of the events on the basis of the Doppler effect. About this issue Wayne [14] presents a similar thought experiment (in which two lamps are mounted on the front and on the back of a railroad car and then with the observer on the train at rest with respect to the lamps and the observer on the platform moving towards the lamp mounted on the back of the railway car). The scholar stresses that "Even if the lamps on the front and back of a train are identical and emit light with a wavelength of λ_{source} , as a result of the Doppler effect, the wavelength of the light emitted by the lamp at the back of the railroad car would appear to the observer on the platform to be shorter than the light emitted by the lamp at the front of the railroad car".

In conclusion the graph b of Figure 1 shows an impossible situation: taking into account the objection 3.1 the only correct representation is that of the Graph a of Figure 1.

In conclusion time is of course always relative in the sense that it flows at different speeds according to the relative motions, but it is instead always possible to relate a specific moment for an observer to a specific moment for another observer, at least in our middle world, out of black holes and behind the subatomic greatness's.

To make clearer this thesis we present Figure 2. Figure 2 shows a situation similar to that of graph b of Figure 1, but with different values of the coordinates. Here O_2 is really at rest with respect to the two sources of light A and B, while O_1 is moving towards A. The speed of O_1 with respect to O_2 is $0.8\ c$ (c is the light speed). Initially the observers are at the same distance from A and B. O_1 can receive the two signals at the same time only in $E_{1A,B}$, but in this way both the signals come from the right and then he can not believe to be at rest with respect to the two sources of light.

This always happens for $v > 0.5\ c$ (absolute value). In fact, named a the distance between A and the initial position of O_1 (and O_2) and $2a$ the distance between A and B, the interval of time necessary for the ray r_B to reach A will be $2a / c$, while the interval of time necessary for the observer O_1 to reach A will be a / v, and then O_1 and r_B will reach A in the same moment if $v = c / 2$. For $v \geq c / 2$ the source of light A must emit its signal when it is reached by r_B so that O_1 will receive the two signals simultaneously.

Both the observers (as it has already been noted, also with the aid of the examination of the Doppler effect) can reckon if the two events are simultaneous or not.

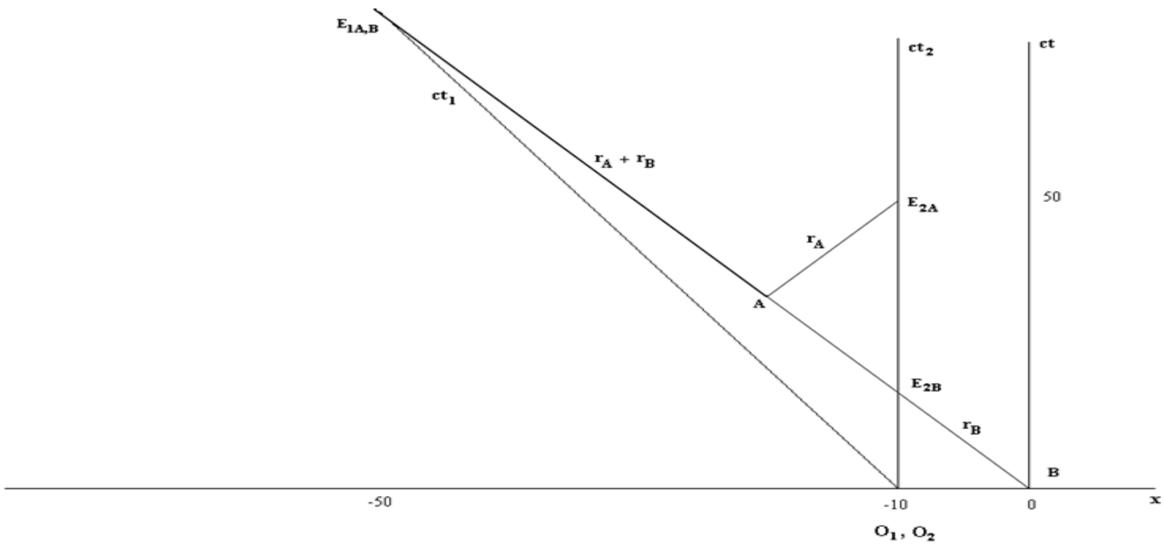

Fig. 2. O_1 The speed of O_1 is $-0.8\ c$. He receives the two signals simultaneously but both the rays arrive to him from the right. The ray r_A starts when the ray r_B reaches A. The overlapping rays are highlighted by a thicker line.

IV. Conclusion

In this paper it has been presented a brief discussion about the Zeno's motion paradoxes and about some simple solutions of two of these paradoxes (Achilles and Arrow). Particularly it has been shown that, while for the Achilles paradox a solution, alternative to the super-task approach, can be given by means of a simple redefinition of the problem, for the Arrow paradox it is necessary and sufficient a recently proposed redefinition of the geometrical objects genesis. Above all it has been stressed that the discussion about the "reality" of time, introduced by the Eleatic school, continues today, particularly with the debate on its relativity. In the Special Theory of Relativity a specific space-time is associated with each inertial system. This association has often been interpreted in a way that implies a concept of simultaneity ultimately subjective. In this paper, by means of a simple observation on the priority of the position and motion of the light sources with respect to the observers, confirmed also by studies on the Doppler effect, it has been shown that the various times of different inertial systems are however correlated, thus making always possible, at least in theory, to establish a before and after.

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