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Division by Zero: the Link Between Numbers and Infinity

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Abstract **– Sanjeev Khuraijam demonstrates that ∞ 0 = 1. One of the three proofs that he presents starts from a result of Paolilli, that is 0 / 0 = 1. In this paper it is sustained that the result of Khuraijam corroborates the thesis of a successive paper of Paolilli in which it is sustained that infinities, even though having the same cardinality, are not all identical and then it should be opportune a more accurate algebraic representation of them, not defining all them with the same symbol (∞). Therefore in this paper it is sustained the thesis that the cardinality of an infinity does not describe its dimension but its typology.**

Keywords **– Infinity, Zero, Division by Zero, Multiplication of ∞ for *0*, Algebraic Representation of Infinity, Cardinality of Infinities.**

I. Introduction

Sanjeev Khuraijam (2019) asserts that $\infty\ 0 = 1$. One of the three proofs that he presents starts from a result of Paolilli (2017), that is $0 / 0 = 1$.

We sustain that the result of Khuraijam (2019) corroborates the thesis of a successive paper of Paolilli (2018) in which it is sustained that infinities, even if they have the same cardinality, are not all identical and then it should be opportune a more accurate algebraic representation of them. In other words it is more correct to write, for example, $\infty + n = \infty + n$ and not $\infty + n = \infty$.

In this frame the cardinality of an infinity describes its typology rather than its dimension.

II. From Zero to Infinity and Beyond

Khuraijam (2019) states that the multiplication $\infty\ 0$ is possible and its output is 1. One of the three proofs that he presents starts from a result of Paolilli (2017), that is 0 / 0 = 1. More generally Paolilli (2017), besides showing that 0 / 0 = 1, sustains that division by 0 is a possible operation and then he assumes as correct the thesis of Euler (1770) that $n / 0 = \infty$ and therefore $\infty\ 0 = n$, where n is any number (a quite similar opinion was expressed also by other scholars: see Bhaskara II (c. 1150); Wallis (1656); Newton (1744)).

However in a successive paper Paolilli (2018) points out that infinities, even though having the same cardinality, are not identical and then it is opportune a more accurate algebraic representation of them. For example it is not

$$\infty + n = \infty,\ \ \infty + \infty = \infty,\ \infty\,\infty = \infty\ ,\ \infty / n = \infty \tag{1}$$

but

$$\infty + n = \infty + n,\ \ \infty + \infty = 2\,\infty,\ \infty\,\infty = \infty^2,\ \infty / n = \infty / n \tag{2}$$

This algebraic representation of infinities does not change their nature of infinity, but differentiates them permitting a comparison. Obviously half an infinity, for example, is infinite, but contains only half of the elements of that infinity. So ∞^n can be associated to a n-dimensional object: for n = 1 to an half line, for n = 2 to an infinite surface and so on. This viewpoint is not totally new. Already Smullyan (1992), among others, writes that an infinite set, even though having the same cardinality of a proper subset, in a certain sense of the term "greater" can be seen as greater than this subset, because not all the elements of the set are contained in the subset. For example if *P* is the set of positive integers and *P-* is the set of positive integers greater than *1*, even if the elements of *P-* can be put in biunivocal correspondence with the elements of *P*, *P-* does not contain all the elements of *P* (Smullyan, 1992).

Reminding that assigning a measure *n* to a segment is a convention (all the segments, whatever their length, are contained infinite times in a straight line), the result of Khuraijam in the light of what pointed out by Paolilli (2018) can be seen in the following way.

Say ∞ the length of a half-line, expressed in unitary segments whose length is *1*.

$\infty\ 0 = 1$, (the result of Khursijam) is the length of the (assigned) standard segment.

To obtain a value *n* (greater than *1*) we have to add *n* times the product $\infty\ 0$, that is we have to take one unitary segment from each of *n* half-lines; in the same manner if we put in a set *n* half-lines we have, banally, *n* half-lines whose respective length is ∞ and then $n\ \infty = n\ \infty$ and it is not $n\ \infty = \infty$.

Following Paolilli (2018) "when two straight lines are added, they are not put in a string: an infinity (straight line) cannot follow another infinity (straight line) because it does not have an end" (in this paper we prefer to associate a half-line to an infinity, assuming it as the set of positive numbers). Obviously *n* can be less than *1*: for example if we consider only the segments which correspond to the odd numbers, we will have $\infty / 2$. It is however an infinity of segments, but it is a less dense infinity and then it is more correct to indicate it with $\infty / 2$.

A set can include more than a half-line and then in that set we have more than an infinity; so it is correct represent them "by the symbol ∞, but preceded by a coefficient and followed (or preceded), when required, by another arithmetical operator" (Paolilli, 2018).

III. Cantorian Cardinality of Infinities Describes their Typology Rather than their Dimension

Following Paolilli (2019), the nature of geometrical objects with a different number of dimensions is quite different: the space, for example, is not a dense set of points, even if we can individuate a dense set of points in it. According to this viewpoint a n-dimensional geometric object is not generated by the movement of a geometric object with $n - 1$ dimensions, because the latter exists only if the former exists. Also the nature of a set and a power set is seen as different and therefore a comparison between these objects is problematic.

In this frame the cardinality of an infinity does not describe its dimension, but rather its typology. For example the formula

$$\aleph_0 + \aleph_0 = \aleph_0 \quad (3)$$

is valid, in our opinion, only as a qualitative expression. It tells us that by means of the sum of two numerable infinities we obtain an infinity of the same typology of the addenda, that is another numerable infinity.

In the sum $\infty + \infty = 2\ \infty$, the term $2\ \infty$ is an infinity of the same kind of ∞, but it is the double of it. The fact that the **3** is a qualitative expression is evident if we subtract $\aleph_0$ from both the terms of **3**: we obtain $\aleph_0 = 0$, which is absurd.

According to Hilbert (1926) no one must be able to expel us out of the paradise that Cantor created for us, but it is an artificial paradise. Obviously we do not want to attack the Set Theory as a whole: its development has given huge advantages to Mathematics and even in this paper it is somehow used. We just want to warn against incorrect applications of this theory.

Finally an apostille: the result of Khuraijam gives a particular status to 1, as intermediary between 0 and ∞. This is not a surprise: just to make an example if we take into consideration the two successions $B_n = c\, a^{n}$ and $b_n = c\, a^{-n}$, 1 is the only value of c which gives values of b_n which are the reciprocal of B_n.

IV. Conclusion

In this paper we have sustained that division by zero can be the key to clarify the concept of infinity. There is a connection between 0 and ∞ and this connection can be represented by the number 1. If $\infty\ 0 = 1$, then any number n is linked to 0 and ∞ and, moreover, we can treat the infinities as we treat numbers and algebraic values.

We have also highlighted how in this frame the cardinality of an infinity does not describe its dimension, but rather its typology.

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