International Journal of Innovation in Science and Mathematics
Volume 5, Issue 1, ISSN (Online): 2347–9051

A Mathematical Modelling for the Derivation of Dispersion Relation for Radio Frequency Heating in the Ion Cyclotron Frequency Range for T_i and $T_e \neq 0$

M. S. Gupta
Department of Mathematics, Govt. N.P.G. College of Science, Raipur (C.G.) 492010, Mobile No. : 094255-08074.
Corresponding author email id: ms_gupta1965@yahoo.co.in

Date of publication (dd/mm/yyyy): 17/01/2017

Abstract **– Through radio frequency heating we can launch wave from the plasma edge to the plasma with minimum losses. Radio frequency heating falls into four major frequency ranges. The wave excited at the edge may propagate into the central region and be absorbed in the range of ion cyclotron resonance frequencies. I have derived a three dimensional dispersion relation for α (electrons, ions) species. The dispersion relation exists in power of 16 in terms of n (refractive index).**

Keywords **– Dispersion Relation, Ion Cyclotron Resonance Heating, Plasma, Radio Frequency Heating, Tokamak.**

Mathematics Subject Classification **– Dispersion Relation-81U30, Mathematical Modelling-93A30, Plasma-82D10.**

I. Introduction

Since we know that enormous number of particles be contained within small volume of plasma. To describe the motion of these particles, there is need of corresponding enormous number of modes. Any mathematical consideration of such motion is based on simplified model. A relation between ω and k is known as dispersion relation. We have studied the propagation of the wave in three dimensions to the magnetic field with ion temperature (T_i) and electron temperature (T_e) not equal to zero [2].

The derivation of such dispersion relation is to see the behavior of propagation characteristics of different modes [1]. Such dispersion relations have been studied with some approximations in two dimensions for cold as well as warm plasma by T. H Stix [2]. Here a three dimensional dispersion relation is an attempt to study for a broad range for the propagation characteristics, i.e. the range not depend particularly on ion cyclotron frequency but also on higher frequencies.

In small as well as large tokamaks like “Joint European Torus”, [3], [4] the ion cyclotron frequency range of few tens of MHz is widely used. In the ion cyclotron range of frequency high power is available at reasonably low cost, conventional transmission lines can be used to carry power to the plasma, simple antenna can be used efficiently to couple the power to the plasma. In the range of ion cyclotron resonance frequencies both cold plasma electromagnetic waves i.e. slow waves and fast waves can be used to heat the plasma in which the power is deposited at the layer corresponding to the cyclotron frequency [5]-[8].

II. Theory With Derivation

The dispersion relation in two dimensions has been derived by T. H. Stix [2] using fluid theory. For obtaining more realistic results, we have derived dispersion relation using fluid theory in three dimensions. The momentum and continuity equations for α (electrons, ions) species in SI units with an assumption $\mathbf{B} = \mathbf{B}_o$ (static magnetic field) are as follows :

$$m_\alpha n_\alpha \, \partial \mathbf{V}_\alpha / \partial t = q_\alpha n_\alpha \, [\mathbf{E} + \mathbf{V}_\alpha \times \mathbf{B}_o] - \nabla p_\alpha \qquad (1)$$

$$\nabla p_\alpha = \kappa T_\alpha \, \nabla n_\alpha \qquad (2)$$

$$\partial n_\alpha / \partial t + \nabla \cdot (n_\alpha \mathbf{V}_\alpha) = 0 \qquad (3)$$

where m_α is mass, n_α is density, T_α is temperature, q_α is charge, $\mathbf{V}_\alpha$ is velocity of α - species, $\mathbf{E}$ is electric field, $\mathbf{k}$ is propagation vector, $B_o \hat{z}$ is the static magnetic field along z-axis and all the symbols have their usual meanings. For the plane wave

$$\partial/\partial t = -\iota\, \omega, \quad \nabla = \iota\, \mathbf{k} \qquad (4)$$

After perturbation, linearization and solving by Cramer’s rule the equations (1), (2), (3) becomes as follows :

$$V_{\alpha x1} = A_1 / D_1 \qquad (5)$$

$$V_{\alpha y1} = B_1 / D_1 \qquad (6)$$

$$V_{\alpha z1} = C_1 / D_1 \qquad (7)$$

where

$$A_1 = \begin{bmatrix} \iota\omega_{c\alpha} \; E_{x1}/(\omega B_o) & \{-\iota\omega_{c\alpha}/\omega - \kappa T_\alpha \, k_x k_y/(m_\alpha \omega^2)\} & \{-\kappa T_\alpha \, k_x k_z/(m_\alpha \omega^2)\} \\ \iota\omega_{c\alpha} \; E_{y1}/(\omega B_o) & \{1 - \kappa T_\alpha \, k_y^2/(m_\alpha \omega^2)\} & \{-\kappa T_\alpha \, k_y k_z/(m_\alpha \omega^2)\} \\ \iota\omega_{c\alpha} \; E_{z1}/(\omega B_o) & \{-\kappa T_\alpha \, k_y k_z/(m_\alpha \omega^2)\} & \{1 - \kappa T_\alpha \, k_z^2/(m_\alpha \omega^2)\} \end{bmatrix} \qquad (8)$$

$$B_1 = \begin{bmatrix} \{1 - \kappa T_\alpha \, k_x^2/(m_\alpha \omega^2)\} & \{\iota\omega_{c\alpha} \; E_{x1}/(\omega B_o)\} & \{-\kappa T_\alpha \, k_x k_z/(m_\alpha \omega^2)\} \\ \{\iota\omega_{c\alpha}/\omega - \kappa T_\alpha \, k_x k_y/(m_\alpha \omega^2)\} & \{\iota\omega_{c\alpha} \; E_{y1}/(\omega B_o)\} & \{-\kappa T_\alpha \, k_y k_z/(m_\alpha \omega^2)\} \\ \{-\kappa T_\alpha \, k_x k_z/(m_\alpha \omega^2)\} & \{\iota\omega_{c\alpha} \; E_{z1}/(\omega B_o)\} & \{1 - \kappa T_\alpha \, k_z^2/(m_\alpha \omega^2)\} \end{bmatrix} \qquad (9)$$

IJISM Source of Knowledge
International Journal of Innovation in Science and Mathematics
Volume 5, Issue 1, ISSN (Online): 2347–9051

$$C_1 = \begin{bmatrix} \{1 - \kappa T_\alpha k_x^2/(m_\alpha\omega^2)\} & \{-\iota\omega_{c\alpha}/\omega - \kappa T_\alpha k_x k_y/(m_\alpha\omega^2)\} & \{\iota\omega_{c\alpha} E_{x1}/(\omega B_o)\} \\ \{\iota\omega_{c\alpha}/\omega - \kappa T_\alpha k_x k_y/(m_\alpha\omega^2)\} & \{1 - \kappa T_\alpha k_y^2/(m_\alpha\omega^2)\} & \{\iota\omega_{c\alpha} E_{y1}/(\omega B_o)\} \\ \{-\kappa T_\alpha k_x k_z/(m_\alpha\omega^2)\} & \{-\kappa T_\alpha k_y k_z/(m_\alpha\omega^2)\} & \{\iota\omega_{c\alpha} E_{z1}/(\omega B_o)\} \end{bmatrix} \quad (10)$$

$$D_1 = \begin{vmatrix} \{1 - \kappa T_\alpha k_x^2/(m_\alpha\omega^2)\} & \{-\iota\omega_{c\alpha}/\omega - \kappa T_\alpha k_x k_y/(m_\alpha\omega^2)\} & \{-\kappa T_\alpha k_x k_z/(m_\alpha\omega^2)\} \\ \{\iota\omega_{c\alpha}/\omega - \kappa T_\alpha k_x k_y/(m_\alpha\omega^2)\} & \{1 - \kappa T_\alpha k_y^2/(m_\alpha\omega^2)\} & \{-\kappa T_\alpha k_y k_z/(m_\alpha\omega^2)\} \\ \{-\kappa T_\alpha k_x k_z/(m_\alpha\omega^2)\} & \{-\kappa T_\alpha k_y k_z/(m_\alpha\omega^2)\} & \{1 - \kappa T_\alpha k_z^2/(m_\alpha\omega^2)\} \end{vmatrix} \quad (11)$$

The electric displacement **D** includes the vacuum displacement and the plasma current **J** according to the relation

$$\mathbf{D} = \mathbf{K} \cdot \mathbf{E} \quad (12)$$

or

$$\mathbf{D} = \mathbf{E} + (\iota/\in_o\omega)\,\mathbf{J} \quad (13)$$

where **K** is the dielectric tensor and **J** is defined as

$$\mathbf{J} = \sum\nolimits_\alpha n_\alpha q_\alpha \mathbf{V}_\alpha \quad (14)$$

The solution of equation (13) with the perturbed values of equation (14) is as follows :

$$\left.\begin{aligned} K_{xx}\, E_{x1} &= A_{11}\, E_{x1} - A_{12}\, E_{y1} + A_{13}\, E_{z1} \\ K_{yy}\, E_{y1} &= -B_{11}\, E_{x1} + B_{12}\, E_{y1} - B_{13}\, E_{z1} \\ K_{zz}\, E_{z1} &= C_{11}\, E_{x1} - C_{12}\, E_{y1} + C_{13}\, E_{z1} \end{aligned}\right] \quad (15)$$

where

$$\left.\begin{aligned} A_{11} &= 1 - \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{1 - \kappa T_\alpha (k_y^2 + k_z^2)/(m_\alpha\omega^2)\}] \\ A_{12} &= \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{\iota\omega_{c\alpha}/\omega + \kappa T_\alpha (k_x k_y - \iota\omega_{c\alpha}\, k_z^2/\omega)/(m_\alpha\omega^2)\}] \\ A_{13} &= -\sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{\kappa T_\alpha (k_x k_z + \iota\omega_{c\alpha}\, k_y k_z/\omega)/(m_\alpha\omega^2)\}] \\ B_{11} &= \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{-\iota\omega_{c\alpha}/\omega + \kappa T_\alpha (k_x k_y + \iota\omega_{c\alpha} k_z^2/\omega)/(m_\alpha\omega^2)\}] \\ B_{12} &= 1 - \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{1 - \kappa T_\alpha (k_x^2 + k_z^2)/(m_\alpha\omega^2)\}] \\ B_{13} &= \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{\kappa T_\alpha (k_y k_z - \iota\omega_{c\alpha}\, k_x k_z/\omega)/(m_\alpha\omega^2)\}] \\ C_{11} &= -\sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{\kappa T_\alpha (-\iota\omega_{c\alpha}\, k_y k_z/\omega + k_x k_z)/(m_\alpha\omega^2)\}] \\ C_{12} &= \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{\kappa T_\alpha (k_y k_z + \iota\omega_{c\alpha}\, k_x k_z/\omega)/(m_\alpha\omega^2)\}] \\ C_{13} &= 1 - \sum\nolimits_\alpha [\{\omega_{p\alpha}^2/(\omega^2 D_1)\}\{1 - \omega_{c\alpha}^2/\omega^2 - \kappa T_\alpha (k_x^2 + k_y^2)/(m_\alpha\omega^2)\}] \end{aligned}\right] \quad (16)$$

where $\omega_{p\alpha} = [n_o q_\alpha^2/(\in_o m_\alpha)]^{1/2}$ (plasma frequency of α - species) (17)

and $\omega_{c\alpha} = q_\alpha B_o/m_\alpha$ (cyclotron frequency of α - species) (18)

Since we know that

$$\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t \quad (19)$$

$$\nabla \times \mathbf{B} = \mu_o \mathbf{J} + (1/c^2)\,\partial\mathbf{E}/\partial t \quad (20)$$

are Maxwell's equations. Taking Fourier analysis in time and space of equations (19) and (20) and after solving we get [2], [9]

$$\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \mathbf{K} \cdot \mathbf{E} = 0 \quad (21)$$

where

$$\mathbf{n} = c\,\mathbf{k}/\omega \quad (22)$$

which is a dimensionless vector and is known as refractive index.

The matrix form of equation (21) is as follows :

$$\begin{bmatrix} A_{11} - (n_y^2 + n_z^2) & -A_{12} + n_x n_y & A_{13} + n_x n_z \\ -B_{11} + n_x n_y & B_{12} - (n_z^2 + n_x^2) & -B_{13} + n_y n_z \\ C_{11} + n_x n_z & -C_{12} + n_y n_z & C_{13} - (n_x^2 + n_y^2) \end{bmatrix} \begin{bmatrix} E_{x1} \\ E_{y1} \\ E_{z1} \end{bmatrix} = 0 \quad (23)$$

The non-trivial solution of equation (23) is given by the determinant

$$\begin{vmatrix} A_{11} - (n_y^2 + n_z^2) & -A_{12} + n_x n_y & A_{13} + n_x n_z \\ -B_{11} + n_x n_y & B_{12} - (n_z^2 + n_x^2) & -B_{13} + n_y n_z \\ C_{11} + n_x n_z & -C_{12} + n_y n_z & C_{13} - (n_x^2 + n_y^2) \end{vmatrix} = 0 \quad (24)$$

International Journal of Innovation in Science and Mathematics
Volume 5, Issue 1, ISSN (Online): 2347–9051

The solution of equations (24) is given below :

$$[(n_y^2 + n_z^2)\{(n_x^2 + n_y^2) B_{12} + (n_z^2 + n_x^2) C_{13}\} - (n_y^2 + n_z^2)(C_{12} n_y n_z + B_{13} n_y n_z) - n_x^2 n_y^2 C_{13} - n_x n_y (n_x^2 + n_y^2) B_{11} - n_x^2 n_y n_z B_{13} + n_x n_y^2 n_z C_{11} - n_x^2 n_y n_z C_{12} - n_x n_y n_z^2 B_{11} - n_x^2 n_z^2 B_{12} + n_x n_z (n_z^2 + n_x^2) C_{11} - A_{11} n_y^2 n_z^2 + A_{11} (n_z^2 + n_x^2)(n_x^2 + n_y^2) - A_{12} n_x n_y (n_x^2 + n_y^2) - A_{12} n_x n_y n_z^2 + A_{13} n_x n_y^2 n_z + A_{13} n_x n_z (n_z^2 + n_x^2)] + [A_{11}\{-(n_x^2 + n_y^2) B_{12} - (n_z^2 + n_x^2) C_{13} + C_{12} n_y n_z + B_{13} n_y n_z\} - (n_y^2 + n_z^2) B_{12} C_{13} + (n_y^2 + n_z^2) B_{13} C_{12} + A_{12}\{n_x n_y C_{13} + B_{11}(n_x^2 + n_y^2) + n_x n_z B_{13} - C_{11} n_y n_z\} - n_x n_y (-B_{11} C_{13} + C_{11} B_{13}) + A_{13}\{-C_{12} n_x n_y - B_{11} n_y n_z - n_x n_z B_{12} + C_{11}(n_z^2 + n_x^2)\} + n_x n_z (B_{11} C_{12} - C_{11} B_{12})] + [A_{11}\{B_{12} C_{13} - B_{13} C_{12}\} + A_{12}\{-B_{11} C_{13} + C_{11} B_{13}\} + A_{13}\{B_{11} C_{12} - C_{11} B_{12}\}] = 0 \quad (25)$$

Let $n_x^2 + n_y^2 = \alpha = n^2 \alpha_1$ where $\alpha_1 = (\sin^2\theta \sin^2\phi + \cos^2\theta)$

$n_y^2 + n_z^2 = \beta = n^2 \beta_1$ where $\beta_1 = (\sin^2\theta \cos^2\phi + \cos^2\theta)$

$n_z^2 + n_x^2 = \gamma = n^2 \gamma_1$ where $\gamma_1 = \sin^2\theta$

and $n_x n_y = n^2 Q_1$ where $Q_1 = \sin\theta \cos\theta \sin\phi$

$n_y n_z = n^2 Q_2$ where $Q_2 = \sin\theta \cos\theta \cos\phi$

$n_x n_z = n^2 Q_3$ where $Q_3 = \sin^2\theta \sin\phi \cos\phi$

$n_z n_z = n^2 Q_4$ where $Q_4 = \sin^2\theta \cos^2\phi$

where θ = angle between y-axis and propagation vector **k,**

ϕ = angle between z-axis and the projection of the propagation vector in x-z plane.

Substituting all these values in equation (25) we get,

$$n^4 [-A_{11}(Q_2^2 - \alpha_1 \gamma_1) - A_{12}(Q_1 \alpha_1 + Q_2 Q_3) + A_{13}(Q_1 Q_2 + Q_3 \gamma_1) - B_{11}(\alpha_1 Q_1 + Q_2 Q_3) + B_{12}(\alpha_1 \beta_1 - Q_3^2) - B_{13}(\beta_1 Q_2 + Q_1 Q_3) + C_{11}(Q_1 Q_2 + \gamma_1 Q_3) - C_{12}(\beta_1 Q_2 + Q_1 Q_3) + C_{13}(\beta_1 \gamma_1 - Q_1^2)] + n^2 [A_{11}(-\alpha_1 B_{12} - \gamma_1 C_{13} + C_{12} Q_2 + B_{13} Q_2) + (B_{13} C_{12} - C_{13} B_{12}) \beta_1 + A_{12}(Q_1 C_{13} + B_{11} \alpha_1 + Q_3 B_{13} - Q_2 C_{11}) + (B_{11} C_{13} - C_{11} B_{13}) Q_1 + A_{13}(-Q_1 C_{12} - Q_2 B_{11} - Q_3 B_{12} + C_{11} \gamma_1) + (B_{11} C_{12} - C_{11} B_{12}) Q_3] + [A_{11}(B_{12} C_{13} - B_{13} C_{12}) + A_{12}(B_{13} C_{11} - B_{11} C_{13}) + A_{13}(B_{11} C_{12} - C_{11} B_{12})] = 0 \ [10]. \quad (26)$$

Consider the term

$$\sum_\alpha [\omega_{p\alpha}^2 / (\omega^2 D_1)] = \sum_\alpha [\omega_{p\alpha}^2 / \{\omega^2 \{1 - \omega_{c\alpha}^2/\omega^2 - \kappa T_\alpha (k^2 - \omega_{c\alpha}^2 k_z^2/\omega^2)/(m_\alpha \omega^2)\}\}]$$

$$= \omega_{pi}^2 / (T_1 - n^2 T_2) + \omega_{pe}^2 / (P_1 - n^2 P_2) \ [11]. \quad (27)$$

where symbols indication ‘ $_i$ ’ for ions and ‘ $_e$’ for electrons,

$T_1 = (\omega^2 - \omega_{ci}^2)$, $P_1 = (\omega^2 - \omega_{ce}^2)$,

$T_2 = (b_1 / c^2)(\omega^2 - \omega_{ci}^2 Z_1^2 Z_4^2)$, $P_2 = (b / c^2)(\omega^2 - \omega_{ce}^2 Z_1^2 Z_4^2)$

$b_1 = \kappa T_i / m_i$, $b = \kappa T_e / m_e$;

Substituting all these values in A_{11} to C_{13} and solving we get,

$A_{11} = (n^4 Y_1 + n^2 Y_2 + Y_3) / S$,

$A_{12} = (n^4 Y_4 + n^2 Y_5 + Y_6) / S$

$A_{13} = (n^4 Y_7 + n^2 Y_8) / S$,

$B_{11} = (n^4 Y_9 + n^2 Y_{10} + Y_{11}) / S$

$B_{12} = (n^4 Y_{12} + n^2 Y_{13} + Y_{14}) / S$,

$B_{13} = (n^4 Y_{15} + n^2 Y_{16}) / S$

$C_{11} = (n^4 Y_{17} + n^2 Y_{18}) / S$,

$C_{12} = (n^4 Y_{19} + n^2 Y_{20}) / S$

$C_{13} = (n^4 Y_{21} + n^2 Y_{22} + Y_{23}) / S$

where

$Y_1 = [T_2 P_2 - \omega_{pi}^2 T_3 P_2 - \omega_{pe}^2 T_2 P_3]$

$Y_2 = [-(T_1 P_2 + T_2 P_1) + \omega_{pi}^2 (T_3 P_1 + P_2) + \omega_{pe}^2 (P_3 T_1 + T_2)]$

$Y_3 = [T_1 P_1 - \omega_{pi}^2 P_1 - \omega_{pe}^2 T_1]$

$Y_4 = [-(\omega_{pi}^2 T_4 P_2 - \omega_{pe}^2 P_4 T_2)]$

$Y_5 = [\omega_{pi}^2 (T_4 P_1 - \iota R_1 P_2) + \omega_{pe}^2 (P_4 T_1 - \iota R_2 T_2)]$

$Y_6 = \iota [\omega_{pi}^2 R_1 P_1 + \omega_{pe}^2 R_2 T_1]$

$Y_7 = -[T_5 P_2 + T_2 P_5]$

$Y_8 = [T_5 P_1 + T_1 P_5]$

$Y_9 = -[\omega_{pi}^2 T_6 P_2 + \omega_{pe}^2 T_2 P_6]$

$Y_{10} = [\omega_{pi}^2 (T_6 P_1 + \iota R_1 P_2) + \omega_{pe}^2 (P_6 T_1 + \iota R_2 T_2)]$

$Y_{11} = -\iota [\omega_{pi}^2 R_1 P_1 + \omega_{pe}^2 R_2 T_1]$

$Y_{12} = [T_2 P_2 - \omega_{pi}^2 T_7 P_2 - \omega_{pe}^2 T_2 P_7]$

$Y_{13} = [-(T_2 P_1 + T_1 P_2) + \omega_{pi}^2 (T_7 P_1 + P_2) + \omega_{pe}^2 (P_7 T_1 + T_2)]$

$Y_{14} = [T_1 P_1 - \omega_{pi}^2 P_1 - \omega_{pe}^2 T_1]$

$Y_{15} = -[T_8 P_2 + T_2 P_8]$

$Y_{16} = [T_8 P_1 + T_1 P_8]$

$Y_{17} = -[T_9 P_2 + T_2 P_9]$

$Y_{18} = [T_9 P_1 + T_1 P_9]$

$Y_{19} = -[T_{10} P_2 + T_2 P_{10}]$

$Y_{20} = [T_{10} P_1 + T_1 P_{10}]$

$Y_{21} = [T_2 P_2 - \omega_{pi}^2 T_{11} P_2 - \omega_{pe}^2 T_2 P_{11}]$

$Y_{22} = [-(T_2 P_1 + T_1 P_2) + \omega_{pi}^2 \{T_{11} P_1 + (1 - R_1^2) P_2)\} + \omega_{pe}^2 \{T_1 P_{11} + (1 - R_2^2) T_2)\}]$

$Y_{23} = [T_1 P_1 - \omega_{pi}^2 (1 - R_1^2) P_1 + \omega_{pe}^2 (1 - R_2^2) T_1\}]$

$T_3 = (b_1 \beta_1) / c^2$, $P_3 = (b \beta_1) / c^2$,

$T_4 = b_1 (QQ_1 - \iota R_1 Q_4) / c^2$, $P_4 = b (QQ_1 - \iota R_2 Q_4) / c^2$,

$T_5 = -\omega_{pi}^2 b_1 (QQ_3 + \iota R_1 Q_2) / c^2$, $P_5 = -\omega_{pe}^2 b (QQ_3 + \iota R_2 Q_2) / c^2$,

$T_6 = b_1 (QQ_1 + \iota R_1 Q_4) / c^2$, $P_6 = b (QQ_1 + \iota R_2 Q_4) / c^2$,

$T_7 = b_1 \gamma_1 / c^2$, $P_7 = b \gamma_1 / c^2$,

$T_8 = \omega_{pi}^2 b_1 (QQ_2 - \iota R_1 Q_3) / c^2$, $P_8 = \omega_{pe}^2 b (QQ_2 - \iota R_2 Q_3) / c^2$,

$T_9 = -\omega_{pi}^2 b_1 (QQ_3 - \iota R_1 Q_2) / c^2$, $P_9 = -\omega_{pe}^2 b (QQ_3 - \iota R_2 Q_2) / c^2$,

$T_{10} = \omega_{pi}^2 b_1 (QQ_2 + \iota R_1 Q_3) / c^2$, $P_{10} = \omega_{pe}^2 b (QQ_2 + \iota R_2 Q_3) / c^2$,

$T_{11} = b_1 \alpha_1 / c^2$, $P_{11} = b \alpha_1 / c^2$

$R_1 = \omega_{ci} / \omega$, $R_2 = \omega_{ce} / \omega$,

QQ_1 = complex of (Q_1), QQ_2 = complex of (Q_2),

QQ_3 = complex of (Q_3), QQ_4 = complex of (Q_4),

$S = \{(T_1 - n^2 T_2)(P_1 - n^2 P_2)\}$

Let

$W_1 = \alpha_1 \beta_1 - Q_3^2$, $W_2 = \beta_1 \gamma_1 - Q_1^2$, $W_3 = \beta_1 Q_2 + Q_1 Q_3$,

$W_4 = \beta_1 Q_2 + Q_1 Q_3$, $W_5 = \alpha_1 Q_1 + Q_2 Q_3$, $W_6 = Q_1 Q_2 + \gamma_1 Q_3$,

$W_7 = Q_2^2 - \alpha_1 \gamma_1$, $W_8 = \alpha_1 Q_1 + Q_2 Q_3$, $W_9 = Q_1 Q_2 + Q_3 \gamma_1$

Substituting W_1 to W_9 and A_{11} to C_{13} in equation (26) and solving for refractive index we find the a polynomial,

$$n^{16} G_1 + n^{14} G_2 + n^{12} G_3 + n^{10} G_4 + n^8 G_5 + n^6 G_6 + n^4 G_7 + n^2 G_8 + G_9 = 0 \quad (28)$$

where

$G_1 = F_{66}$, $G_2 = F_{67}$, $G_3 = F_{68}$, $G_4 = F_{69}$, $G_5 = F_{70}$,

$G_6 = F_{71}$, $G_7 = F_{72}$, $G_8 = F_{73}$, $G_9 = F_{60}$

where

$F_1 = W_1 Y_{12} + W_2 Y_{21} - W_3 Y_{19} - W_4 Y_{15} - W_5 Y_9 + W_6 Y_{17} - W_7 Y_1 - W_8 Y_4 + W_9 Y_7$

$F_2 = W_1 Y_{13} + W_2 Y_{22} - W_3 Y_{20} - W_4 Y_{16} - W_5 Y_{10} + W_6 Y_{18} - W_7 Y_2 - W_8 Y_5 + W_9 Y_8$

$F_3 = W_1 Y_{14} + W_2 Y_{23} - W_5 Y_{11} - W_7 Y_3 - W_8 Y_6$

$F_4 = - \alpha_1 Y_{12} - \gamma_1 Y_{21} + Q_2 Y_{19} + Q_2 Y_{15}$

$F_5 = - \alpha_1 Y_{13} - \gamma_1 Y_{22} + Q_2 Y_{20} + Q_2 Y_{16}$

$F_6 = - \alpha_1 Y_{14} - \gamma_1 Y_{23}$

$F_7 = Y_{15} Y_{19} - Y_{21} Y_{12}$

$F_8 = Y_{15} Y_{20} + Y_{16} Y_{19} - Y_{21} Y_{13} - Y_{22} Y_{12}$

$F_9 = Y_{16} Y_{20} - Y_{21} Y_{14} - Y_{23} Y_{12} - Y_{22} Y_{13}$

$F_{10} = - Y_{22} Y_{14} - Y_{23} Y_{13}$

$F_{11} = - Y_{23} Y_{14}$

$F_{12} = Q_1 Y_{21} + \alpha_1 Y_9 + Q_3 Y_{15} - Q_2 Y_{17}$

$F_{13} = Q_1 Y_{22} + \alpha_1 Y_{10} + Q_3 Y_{16} - Q_2 Y_{18}$

$F_{14} = Q_1 Y_{23} + \alpha_1 Y_{11}$

$F_{15} = Y_9 Y_{21} - Y_{17} Y_{15}$

$F_{16} = Y_9 Y_{22} + Y_{10} Y_{21} - Y_{16} Y_{17} - Y_{15} Y_{18}$

$F_{17} = Y_{10} Y_{22} + Y_9 Y_{23} + Y_{11} Y_{21} - Y_{18} Y_{16}$

$F_{18} = Y_{10} Y_{23} + Y_{11} Y_{22}$

$F_{19} = Y_{11} Y_{23}$

$F_{20} = - Q_1 Y_{19} - Q_2 Y_9 - Q_3 Y_{12} + \gamma_1 Y_{17}$

$F_{21} = - Q_1 Y_{20} - Q_2 Y_{10} - Q_3 Y_{13} + \gamma_1 Y_{18}$

$F_{22} = - Q_2 Y_{11} - Q_3 Y_{14}$

$F_{23} = Y_9 Y_{19} - Y_{17} Y_{12}$

$F_{24} = Y_9 Y_{20} + Y_{10} Y_{19} - Y_{17} Y_{13} - Y_{18} Y_{12}$

$F_{25} = Y_{10} Y_{20} + Y_{11} Y_{19} - Y_{17} Y_{14} - Y_{18} Y_{13}$

$F_{261} = Y_{11} Y_{20} - Y_{18} Y_{14}$

$F_{26} = Y_{12} Y_{21} - Y_{15} Y_{19}$

$F_{27} = Y_{12} Y_{22} + Y_{13} Y_{21} - Y_{15} Y_{20} - Y_{16} Y_{19}$

$F_{28} = Y_{12} Y_{23} + Y_{14} Y_{21} - Y_{16} Y_{20} + Y_{13} Y_{22}$

$F_{29} = Y_{13} Y_{23} + Y_{14} Y_{22}$

$F_{30} = Y_{14} Y_{23}$

$F_{31} = Y_{15} Y_{17} - Y_9 Y_{21}$

$F_{32} = Y_{15} Y_{18} + Y_{16} Y_{17} - Y_9 Y_{22} - Y_{10} Y_{21}$

$F_{33} = Y_{16} Y_{18} - Y_9 Y_{23} - Y_{11} Y_{21} - Y_{10} Y_{22}$

$F_{34} = - Y_{10} Y_{23} - Y_{11} Y_{22}$

$F_{35} = - Y_{11} Y_{23}$

$F_{36} = Y_9 Y_{19} - Y_{17} Y_{12}$

$F_{37} = Y_9 Y_{20} + Y_{10} Y_{19} - Y_{17} Y_{13} - Y_{18} Y_{12}$

$F_{38} = Y_{10} Y_{20} + Y_{11} Y_{19} - Y_{17} Y_{14} - Y_{18} Y_{13}$

$F_{39} = Y_{11} Y_{20} - Y_{18} Y_{14}$

$F_{40} = Y_1 F_4 + F_7 \beta_1$

$F_{41} = Y_1 F_5 + Y_2 F_4 + F_8 \beta_1$

$F_{42} = Y_1 F_6 + Y_2 F_5 + Y_3 F_4 + F_9 \beta_1$

$F_{43} = Y_2 F_6 + Y_3 F_5 + F_{10} \beta_1$

$F_{44} = Y_3 F_6 + F_{11} \beta_1$

$F_{45} = Y_4 F_{12} + F_{15} Q_1$

$F_{46} = Y_4 F_{13} + Y_5 F_{12} + F_{16} Q_1$

$F_{47} = Y_4 F_{14} + Y_5 F_{13} + Y_6 F_{12} + F_{17} Q_1$

$F_{48} = Y_5 F_{14} + Y_6 F_{13} + F_{18} Q_1$

$F_{49} = Y_6 F_{14} + F_{19} Q_1$

$F_{50} = Y_7 F_{20} + F_{23} Q_3$

$F_{51} = Y_7 F_{21} + Y_8 F_{20} + F_{24} Q_3$

$F_{52} = Y_7 F_{22} + Y_8 F_{21} + F_{25} Q_3$

$F_{53} = Y_8 F_{22} + F_{261} Q_3$

$F_{54} = Y_1 F_{26} + Y_4 F_{31} + Y_7 F_{36}$

$F_{55} = Y_1 F_{27} + Y_2 F_{26} + Y_4 F_{32} + Y_5 F_{31} + Y_7 F_{37} + Y_8 F_{36}$

$F_{56} = Y_1 F_{28} + Y_2 F_{27} + Y_3 F_{26} + Y_4 F_{33} + Y_5 F_{32} + Y_6 F_{31} + Y_7 F_{38} + Y_8 F_{37}$

$F_{57} = Y_1 F_{29} + Y_2 F_{28} + Y_3 F_{27} + Y_4 F_{34} + Y_5 F_{33} + Y_6 F_{32} + Y_7 F_{39} + Y_8 F_{38}$

$F_{58} = Y_1 F_{30} + Y_2 F_{29} + Y_3 F_{28} + Y_4 F_{35} + Y_5 F_{34} + Y_6 F_{33} + Y_8 F_{39}$

$F_{59} = Y_2 F_{30} + Y_3 F_{29} + Y_5 F_{35} + Y_6 F_{34}$

$F_{60} = Y_3 F_{30} + Y_6 F_{35}$

$F_{61} = F_{40} + F_{45} + F_{50}$

$F_{62} = F_{41} + F_{46} + F_{51}$

$F_{63} = F_{42} + F_{47} + F_{52}$

$F_{64} = F_{43} + F_{48} + F_{53}$

$F_{65} = F_{44} + F_{49}$

$F_{66} = E_2 F_1$

$F_{67} = E_2 F_2 + E_3 F_1 + T_2 P_2 F_{61}$

$F_{68} = E_2 F_3 + E_3 F_2 + E_4 F_1 - E_1 F_{61} + F_{54} + T_2 P_2 F_{62}$

$F_{69} = E_3 F_3 + E_4 F_2 + E_5 F_1 + T_2 P_2 F_{63} - E_1 F_{62} + T_1 P_1 F_{61} + F_{55}$

$F_{70} = E_4 F_3 + E_5 F_2 + E_6 F_1 + T_2 P_2 F_{64} - E_1 F_{63} + T_1 P_1 F_{62} + F_{56}$

$F_{71} = E_5 F_3 + E_6 F_2 + T_2 P_2 F_{65} - E_1 F_{64} + T_1 P_1 F_{63} + F_{57}$

$F_{72} = E_6 F_3 - E_1 F_{65} + T_1 P_1 F_{64} + F_{58}$

$F_{73} = T_1 P_1 F_{65} + F_{59}$

$E_1 = T_1 P_2 + T_2 P_1$

$E_2 = T_2^2 P_2^2$

$E_3 = -2 T_2 P_2 E_1$

$E_4 = E_1^2 + 2 T_1 T_2 P_1 P_2$

$E_5 = -2 T_1 P_1 E_1$

$E_6 = T_1^2 P_1^2$

The polynomial given by equation (28) having degree 16 in n i.e. degree 8 in n^2. For verifying the result I have taken following cases:

III. Limiting Cases For Verification

Case 1: Cold plasma dispersion relation in 2D from the above analysis

Let the propagation in x-z plane, then $\theta = 90^0$,
and for cold plasma $T_i = 0$, $T_e = 0$, deuterium percentage = 0 [12], [13] (29)

taking an approximation $k_x = 0$ (30)

and using equations (29), (30), the equation (27) reduces as

$$\sum_\alpha [\omega_{p\alpha}^2/(\omega^2 D_1)] = \omega_{pi}^2/(\omega^2 - \omega_{ci}^2) + \omega_{pe}^2/(\omega^2 - \omega_{ce}^2) \quad (31)$$

using equations (29), (30), (31), the equation (16) reduces as

$$\left.\begin{array}{lll} A_{11} = S, & A_{12} = iD, & A_{13} = 0 \\ B_{11} = -iD, & B_{12} = S, & B_{13} = 0 \\ C_{11} = 0, & C_{12} = 0, & C_{13} = P \end{array}\right] \quad (32)$$

using equations (29), (30), (31), (32), the equation (24) reduces as

$$\begin{vmatrix} S - n^2\cos^2\phi & -iD & n^2 \sin\phi\cos\phi \\ iD & S - n^2 & 0 \\ n^2 \sin\phi\cos\phi & 0 & P - n^2\sin^2\phi \end{vmatrix} = 0 \quad (33)$$

International Journal of Innovation in Science and Mathematics
Volume 5, Issue 1, ISSN (Online): 2347–9051

The solution of equation (33) becomes

$$A\,n^4 - B\,n^2 + C = 0 \quad (34)$$

where

$A = S \sin^2\phi + P \cos^2\phi$

$B = RL \sin^2\phi + PS\,(1+\cos^2\phi)$

$C = PRL$

S, D, R, L, P be Stix's [2] standard notations and equation (34) be Stix's [2] standard dispersion relation for cold plasma in 2 dimensions.

Case 2: Warm plasma dispersion relation in 2D from the above analysis

Let the propagation in x-z plane, then $\theta = 90^0$,
and let $T_i = 0$, deuterium percentage = 0 (35)

taking an approximation $k_x = 0$ (36)

and using equations (35), (36), the equation (27) reduces as

$$\sum_\alpha [\omega_{p\alpha}^2/(\omega^2 D_1)] = \sum_\alpha [\omega_{p\alpha}^2/\{(\omega^2 - \omega_{c\alpha}^2)(1 - \kappa T_\alpha k_z^2/(m_\alpha \omega^2))\}] \quad (37)$$

using equations (35), (36), (37), the equation (16) reduces as

$$\left.\begin{array}{lll} A_{11} = S, & A_{12} = iD, & A_{13} = 0 \\ B_{11} = -iD, & B_{12} = S, & B_{13} = 0 \\ C_{11} = 0, & C_{12} = 0, & C_{13} = P^{**} \end{array}\right] \quad (38)$$

using equations (35), (36), (37) and (38), the equation (24) reduces as

$$\begin{vmatrix} S - n^2\cos^2\phi & -iD & n^2 \sin\phi\cos\phi \\ iD & S - n^2 & 0 \\ n^2\sin\phi\cos\phi & 0 & P^{**} - n^2\sin^2\phi \end{vmatrix} = 0 \quad (39)$$

where

$P^{**} = P_i + (P^* / (n^2 \cos^2\phi))$

$P_i = 1 - \omega_{pi}^2/\omega^2$

$P^* = c^2/(\lambda_D^2\,\omega^2)$

$\lambda_D = [\in_o \kappa T_e / (n_e q_e^2)]^{1/2}$ (Debye length)

The solution of equation (39) becomes

$$a\,n^6 - b\,n^4 + c\,n^2 - d = 0 \quad (40)$$

where

$a = S \sin^2\phi \cos^2\phi + P_i \cos^4\phi$

$b = RL \sin^2\phi\cos^2\phi + P_i S\,(1+\cos^2\phi)\cos^2\phi - P^*\cos^2\phi$

$c = P_i RL \cos^2\phi - P^* S\,(1+\cos^2\phi)$

$d = -P^* RL$

S, D, R, L, P be Stix's [2] standard notations and equation (40) be Stix's [2] standard dispersion relation for warm plasma in 2 dimensions.

IV. Conclusions

Through the derivation we have seen that 8 roots occur in n^2. The above polynomial which is given by equation (28) becomes complicated due to oblique propagation with magnetic field in x-y-z plane and unlimited range of frequencies. For verifying our result we have used the limiting cases:

Case 1: Limiting case for cold plasma: The derivation converted into the dispersion relation as derived by [2] for cold plasma. When propagation parallel to the magnetic field, the roots ($n^2 = R$) and ($n^2 = L$) [14] be transverse right circularly polarized and transverse left circularly polarized respectively. Similarly when propagation perpendicular to the magnetic field, the roots ($n^2 = P$) and ($n^2 = RL/S$) be ordinary and extra ordinary respectively.

Case 2: Limiting case for warm plasma: We get the same dispersion relation as derived by [2] for warm plasma, which having cold as well as electrostatics modes.

Thus we have seen that by using both the limiting cases the polynomial given by equation (28) reduces same as the polynomial derived by [2]. The multiplicative factor S = $\{(T_1 - n^2 T_2)(P_1 - n^2 P_2)\}$ provides the repeated roots. For separation of the other roots of the polynomial one can develop a computer code. And for avoiding the mixing of the roots they can insert many existing dispersion relations in the computer code and compare their data's. Without doing so it is very difficult to distinguish the roots of the polynomial with specific name.

References

[1] T. H. Stix, Waves in Plasma: Some Historical Highlights, PPPL - 2139, August 1984.
[2] T. H. Stix, Theory of Plasma Waves, Mc-Graw Hill, New York, (1962).
[3] https://en.wikipedia.org/wiki/Joint_European_Torus.
[4] http://www.ccfe.ac.uk/tokamak.aspx.
[5] W. N.-C. Sy, T. Amano, R. Ando, A. Fukuyma, T. Watari, Nuclear Fusion, Vol. 25, No.7, (1985), pp.795.
[6] V. L. Granatstein, P.L. Colestock, Wave Heating and Current Drive in Plasmas, Gordon and Breach Science Publishers, New York, (1985).
[7] W. N.-C. Sy, M. Cosaftis, Plasma Phys. 21 (1979), pp. 985.
[8] A. Fukuyma, S. Nishiyama, T. Watari, K. Itoh, S.-I. Itoh, Nuclear Fusion 23 (1983) 1005.
[9] M. S. Gupta, Reflections des ERA-JPS (ISSN 0973-8576), Vol.2, Issues 1-4 2008 pp. 155-188 No. B (Science), 2006, pp 33-51.
[10] M. S. Gupta, Journal of Ravishankar University, ISSN 0970-5910, Vol.19, No. B (Science), 2006, pp 33-51.
[11] M. S. Gupta, Journal of Ravishankar University, ISSN 0970-5910, Vol.14, No. B (Science), 2001, pp 64-79.
[12] D. C. Swanson, Mode Conversion and Absorption in a Deuterium Plasma with a Hydrogen Impurity, Proceeding of Third Topical Conference on Radio Frequency Heating, Pasadena, California, Jan. 11-13, 1978.
[13] D. C. Swanson, Plasma Waves, Harcourt Brace Javanovich Publishers, 1989.
[14] M. S. Gupta, Journal of Ravishankar University, ISSN 0970-5910, Vol.15-16, No. B (Science), 2002-03, pp 39-47.

Author's Profile

First A. Author

Place: Banda (Utter Pradesh); Date of Birth: 10th June 1965
Author's Educational Background: M.SC., Ph.D., M.C.A.

Degree	Field	Institution	City	State	Country	Year of Degree	Major field of Study
B.Sc.	PCM	University of Allahabad	Allahabad	Utter Pradesh	India	1983	PCM
M.Sc.	Mathematics	University of Allahabad	Allahabad	Utter Pradesh	India	1985	PCM
Ph.D.	Mathematics	University of Ujjain	Ujjain	Madhya Predesh	India	2000	EM Theory, Plasma, Simulation
M.C.A.	As prescribed Subjects	Bhoj Open University	Raipur	Madhya Predesh	India	2006	As prescribed Subjects

Second paragraph : He
Job Title: ASSISTANT PROFESSOR, RAIPUR (CHHATTISGARH)

Book	Title	City	State	Publisher Name	ISBN
	Vector Calculus and Geometry (Hindi Medium)	Raipur	Chattisgarh	Yugboth Publication	ISBN-81-7924-040-1

A. Summary of Publications:
- **A1. Research papers in Journals** : **Eleven** published.
- **A2. Research papers in Proceedings** : **Three** published.
- **A3. Papers / articles presented in the Conferences / Seminars / Workshops** : mentioned below

B. Book Published : **One** - for U.G. standard
Ten - for Higher Secondary Standard
C. Articles in magazines : **Nine** published.
D. Project : **One** being prepared.
E. Resource Person : Invited in different sessions in many institutions.
F. Other :
(i) Invited Talk / Guest Lecture:

- A talk in the following topic *"Partition of Energy Flux and the Propagation Characteristics of ion Bernstein Waves",* has been delivered by me in the Institute for Plasma Research, Bhat, Gandhinagar - 382 428, India. (15th January 1998).
- Invited as a Chief Speaker in the 'Mathematical Society', Government P. G. College, Dhamtari, (Chhattisgarh) (13th October 2001). Delivered a talk in the following topic, "An Introduction About Tokamak".
- Invited as a Chief Speaker in the 'Mathematical Society', Government P. G. College, Dhamtari, (Chhattisgarh) (12th October 2002). Deliverd a talk in the following topic "A Lecture on PCTeX32 in the reference of LaTeX".
- Delivered guest lecture on 15th Oct. 2009 in the Dept. of Mathematics, Govt. G. N. A. P. G. College, Bhatapara Dist. Raipur (C.G.).
- Delivered a talk on 2nd Nov. 2009 during **Regional Mathematics Olympiads Coordination Camp**, organized by Department of Mathematics, Govt. J. Y. Chhatisgarh College, Raipur, sponsored by CGCOST, Raipur (C.G.).
- Delivered guest lecture on 28 & 29 January, 2011, in the Dept. of Mathematics, Govt. K.P.G. College, Jagdalpur (C.G.).
- Delivered a talk on 24th Nov. 2011, during **Mathematics Olympiads Workshop**, organized by Department of Mathematics, Adarsh Govt. Multi Purpose H.S.S., Janjgeer-Chanpa (C.G.).
- AICON'12 (All India Conference) Address the Keynote Address and Chair the session under the theme of Recent Advance in Applied and Pure Mathematics, organized by Department of Engineering Mathematics, held at CSIT, Durg, (C.G.), (20-21 January, 2012). (Delivered a Key Note address).
- Delivered a Keynote Address on Engineering Mathematics, at AICON'13 (All India Conference-13), organized by Department of Engineering Mathematics, held at CSIT, Durg, (C.G.), (12th -13th April, 2013).

- Delivered a talk on 19th January 2013 during "National Workshop on Fuzzy Sets & Its Application & General Mathematics", organized by Department of Mathematics, Govt. J.Y.Chhattisgarh College, Raipur (C.G.) Sponsored by CCOST, Raipur, (January, 17-19, 2013), Topic : **"Contour Integration".**
- Delivered a talk on 25th November 2013 during "State Level Workshop on Mathematics Teachers / Student and RMO District Coordinators", organized by Department of Mathematics, Govt. J. Y. Chhattisgarh College, Raipur (C.G.) Sponsored by CCOST, Raipur, (25 November 2013). "How to remove fear from Mathematics".
- Talk delivered as a visiting faculty in SoS in Computer Science, Pt RSU, Raipur.

- Delivered a talk on 8th November 2014 as a resource person during, "National Workshop on Graph Theory and its Applications", organized by Department of Mathematics, Dr.K.C.B.Govt. PG College, Bhilai-3 Dist. Durg (C.G.) Sponsored by CCOST, Raipur, "Analysis of shortest path algorithms with reference to mobile adhoc network routing protocols".
- Delivered a talk on 22nd Dec 2015 as a resource person during, on "Ramanujan and his contribution", organized by Department of Mathematics, Govt. P. G. College Dhamtari (C.G.).
- Delivered a talk on 7th April 2016 as a resource person during, on "DBMS and its Applicationsc", organized by Department of Mathematics, Govt. D.B. Girls P.G. College, Raipur.

(ii) Audio-Visual UGC National Programme: Participated actively in an audio-visual educational programme (released on national U.G.C. telecast by DD-1 on 29th October, 1996 at 1.00 P.M.) based on Orientation Course at Academic Staff College, Dr. Hari Singh Gaur University, Sagar (M.P.) India.

(iii) Conduct Interview: Conduct interview for the teacher on contract basis on Jawahar Navodaya Vidyalaya, Mana-Camp, Raipur, on 7th May, 2013.

Third Paragraph : Dr. Gupta

Life Membership/ Membership:

(i) Life Member: Bharata Ganita Parisad (LM-262) Department of Mathematics & Astronomy, Lucknow University, Lucknow-226 007, (U.P.), India
(ii) Life Member: Plasma Science Society of India (LM-165) Institute for Plasma Research, High Way, Bhat, Gandhinagar-382 428, India
(iii) Life Member: International Academy of Physical Sciences, Ramanand Nagar, Bhardwajpuram, Allahabad – 211006, (India)
(iv) Life Member: Chhattishgarh Vigyan Bharti, Raipur (C.G.)
(v) Member: Forum of Scientists and Technologists (Membership No. 116) A – 58 Surya Apartment, Katora Talab, Raipur (C.G.) 492 001 -(Membership had been taken for the one session)
(vi) Associate Member: Astronomical Society of India (A /917) Department of Astronomy, Osmania University, Hyaderabad-500 007 (A. P.) (Associate membership had been taken for the two sessions)

Award :

I have been awarded "Second Prize" for paper presentation in XIII Gujarat Science Congress, Organized by Gujarat Science Academy, hosted by: North Gujarat University, University Road, Patan-384 265, [N.G], (November 8-9, 1997).

Liabilities in other activities :

i. Worked as a Head of the Department of Mathematics, in Govt. college, Chachaura-Binaganj, (Guna), since 18th Feb, 1995 to 7th Dec. 2000.
ii. N.S.S. Programme Officer, in Govt. college, Chachaura-Binaganj, (Guna), since 1st July, 1995 to 7th Dec. 2000.
iii. N.C.C. Incharge Officer, in Govt. College of Science, Raipur (C.G.), since 7th Oct. 2003 to continue and get commissioned N.C.C. Officer as a Lieutenant on 19th Feb, 2005 and completed part I refresher course for Capt. on 29/04/2015.
iv. Assistant Coordinator for the **Regional Mathematics Olympiad (NBHM)**, since 21/07/2008.
v. I/c H.O.D. Information Technology, since 10th June 2012 to 13th Oct. 2015 in Govt., N. P. G. College of Science, Raipur.
vi. Honorary Member of Advisory Committee : **Indian Journal of Sciences**.
vii. Organizing Secretary : National Workshop on "Mathematical Modelling of Stochastic and Fuzzy Systems", organized by department of Mathematics on 6-7th September 2013.
viii. Convener : "National Symposium on Distributed Computing & Network Communication", organized by department of IT and Computer Science on 30/01/2015.
ix. Expert : "XIIIth Chhattisgarh Young Scientists Congress – 2015", organized by Agriculture University, Raipur on (28/02/2015).

A. List of Publications :

1(a). Papers published in refereed journals :

i. Gupta, M. S., Kulkarni, S. V., *" Effect of Poloidal Magnetic Field on the Propagation Characteristics of Ion Bernstein waves in the Ion Cyclotron Resonance Frequency Range for Tokamak".*
Paper published in the journal **"Indian Journal of Physics 74B** (3), 213-216 (2000), (**'IJP' B - an international journal**)".

ii. Gupta, M. S., *"Dispersion behavior of radio frequency waves in ion cyclotron frequency range of Aditya Tokamak Plasma".* Paper published in the journal **"Journal of Ravishankar University Vol. 14 No. B (Science) 2001 pp 64-79." ISSN 0970 - 5910.** (corrigendum published in the **Journal of Ravishankar University Vol. 15-16 Number B (Science) 2002 - 03 (ISSN 0970 - 5910)".**

iii. Gupta, M. S., *"The propagation characteristics of slow wave to ion Bernstein wave affected by poloidal magnetic field for JIPPT- IIU Tokamak"* Paper published in the **"Journal of Ravishankar University Vol. 15-16 Number B (Science) 2002 - 03 pp 39-47 (ISSN 0970 - 5910)".**

iv. Gupta, M. S. "Generalised dispersion relation for radio wave in ion cyclotron frequency range of fusion plasma". Paper published in the journal **"Journal of Ravishankar University Vol. 19 No. B (Science) 2006 pp 33-51", (ISSN 0970 – 5910).**

v. Fifth Conference of the "International Academy of Physical Sciences" CONIAPS-V Organized by Department of Mathematical Sciences and Computer Applications, Bundelkhand University, Jhansi (U.P.) (April 07 - 09, 2002). (Attended and presented the paper) Gupta, M. S., *"Antenna Plasma Coupling Characteristics in Presence of Poloidal Magnetic Field for Aditya Tokamak".*
Paper accepted for publication in the ***"Proceeding of the Fifth Conference of the International Academy of Physical Sciences"****. But due to some circumstances the paper has been published in the following Journal,* ***"Reflections des ERA- JPS (ISSN 0973 – 8576), Vol. 2. Issues 1-4 2008, Page 155 of 155-188".***

vi. "National Conference on Advances in Physics", organized by Department of Physics and Computer Science, Govt. N.P.G. College of Sciences, Raipur (C.G.), (15-16 March, 2012). (Participated and present the paper).
M. S. Gupta, R. A. Siddiqui, Aparna Borkar, *"Antenna-Plasma Coupling in the Range of Ion Cyclotron Resonance Frequency for Adtiya Tokamak".* Paper published in the Journal **"Journal of Pure and Applied Industrial Physics Vol. 2 (3A), 378-380 (2012), ISSN 2229-7596".**

vii. "International Conference in Mathematical Sciences (ICMS-2013)", organized by Department of Mathematics, Government Degree College Haripur (Manali), Himanchal Pradesh, India-175136. (8-9 March, 2013), (Paper Presented by the Co-author).
Aparna Borkar, M. S. Gupta *"Behavior of Antenna Spectral Form Factor for Wave Mode for Plasma Heating"*. Paper published in the Journal **"Research Journal of Science and Technology, 5 (1), Jan.-Mar. 2013, ISSN-0975-4393, p-220-223.**

viii. Gupta, M. S.,*" Contribution of Genius Indian Mathematician S. Ramanujan with reference to Magic Square",* Paper published in the journal **"Indian Journal of Sciences Vol. 3, June 2013, ISSN 2249-9393, p-21-30.**

ix. "International Conference on Mathematics-2013", "International Conference on Mathematics, Statistics & Computer Engineering -2013 (ICMSCE-2013)", would be organized by International Multidisciplinary Research Foundation, Promoted by Ratna Prasad Multidisciplinary Research & Educational Society (Regd.), August 09 & 10, 2013, Kochi, Kerala, India. (Paper accepted for Publication).
Prof. Aparna Borkar, Dr. M. S. Gupta, *"Behavior of Antenna Input Spectral Impedance for Reactance in Aditya Tokamak Geometry"*. Paper published in the Journal - **Mathematical Sciences International Research Journal Volume 2 Issue 2 (2013), ISSN 2278-8697, p-141-144.**

x. Gupta, M. S., Aamir Hasan*"* **A Code for Matrix in C++ with reference to Genius Indian Mathematician S. Ramanujan",** Paper published in the journal **"Indian Journal of Sciences Vol. 4, June 2014, ISSN 2249-9393, p-15-29.**

xi. Dr. M. S. Gupta, Aamir Hasan, "Analysis of shortest path algorithms with reference to mobile adhoc network routing protocols", Paper published in the journal **"Indian Journal of Sciences Vol. 5, June 2016, ISSN 2249-9393.**

G. Other Experiences

1.	**Teaching, Academic Administration & Research**	**:**	Approx. 23 years
2.	**Ph.D. Scholar**	**:**	One (work in progress)
3.	**Others**	**:**	Get a commissioned NCC officer as a Caption.
4.	**Administration**	**:**	2 Years 2 months as a deputy director in the Govt. of Higher Education, Raipur (C.G.)
5.	**Research**	**:**	2 Years 2 months as a research scholar regular in the Institute For Plasma Research, Bhat, Gandhinagar, Gujarat