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Numerical Solution for a Wave Equation Arising in Oscillations of Overhead Transmission Lines

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Abstract – **Numerical solution is developed for a wave equation with boundary damping arising in the study of the physical phenomena of the oscillations that occur in overhead power transmission lines. The physical model is that of a string which is fixed at one end and the other end is attached to a dashpot system, where the damping generated by the dashpot is small. The mathematical model is an initial-boundary value problem for a weekly nonlinear hyperbolic differential equation with non-classical boundary conditions. The method of the characteristics in combination with Richardson extrapolation will be used to solve the problem. The numerical method proposed takes advantage of the special mesh generated by the characteristic curves of the equations to be solved and the specialty of the initial-boundary conditions involved. Numerical results are presented in this paper.**

Keywords – **Wave Equation, Boundary Damping, Characteristics Curves, Richardson Extrapolation.**

I. Introduction

Some flexibles structures (overhead transmission lines, suspension bridges, loaded helical springs - to name a few) can be subject of oscillations due to different causes. The mathematical models that describe these oscillations can be expressed in initial-boundary value problems for wave equations as in [3], [4], [5] or for string equations as in [6], [7]. The corresponding partial differential equations can be linear or nonlinear of second or fourth order with classical or non-classical boundary conditions. Especially the nonlinear dynamics of the the suspended cable is complicated and attracts more attention in recent years, e.g., referring to the literature reviews by [8] and [9], or recent papers [10] and [11].

The following model is derived and analyzed in [1], for the vibrations of a string which is fixed at x = 0 and is attached to a dashpot system at x = π:

Find the function u(x, t) which satisfies the equation

$$u''_{tt} - u''_{xx} = \varepsilon\left(u'_t - \frac{1}{3}u'^3_t\right), 0 < x < \pi,\ t > 0, \quad (1)$$

subject to boundary conditions

$$u(0,t) = 0, \quad t \geq 0, \quad (2)$$
$$u'_t(\pi,t) = -\varepsilon\alpha u'_x(\pi,t), \quad t \geq 0, \quad (3)$$

and initial conditions

$$u(x,0) = \phi(x), \quad 0 \leq x \leq \pi, \quad (4)$$
$$u'_t(x,0) = \psi(x), \quad 0 < x < \pi. \quad (5)$$

The functions $\phi(x)$ *and* $\psi(x)$ above are the initial displacement and the initial velocity of the string, the damping parameter α is a positive constant, and ε is a small dimensionless parameter ($0 < \varepsilon \ll 1$).

Thus, in (1-5) we have an initial-boundary value problem for a weakly nonlinear partial differential equation with a non-classical right boundary condition. It can be considered as a model describing the galloping oscillations of the overhead transmission lines in a wind field. The undesired oscillations mentioned can cause material fatigue and damage to the structure, so to suppress them various types of dampers have been applied in practice (see e.g. [3], [10] and [12]). In this context, one of the objectives of this study is finding those values of the damping parameter α for which the solution $u(x,t)$ tends to zero or tends to a certain bounded function. If the dashpot device is installed to both ends of the electric lines then the left condition (2) above would be replaced by the condition (3) modified properly.

It has been assumed above that the damping generated by the dashpot is only proportional to the vertical velocity of the string in the endpoint. A similar model is derived and analyzed in [2], where the damping generated by the dashpot system is assumed to be small, and is assumed to be proportional to the vertical and the angular velocity of the string in the endpoint.

As it is shown in [1] the problem (1-5) is well-posed. The Laplace transform method is initially used there to construct analytical approximation of the solution $u(x,t)$ for the linear variant of equation (1), which is obtained after neglecting the nonlinear term $\frac{1}{3}u'^3_t$. A two-timescales perturbation method is then used for the nonlinear case, but only for simple initial conditions - referred there as the monochromatic conditions - of the form

$$\phi(x) = a_n \sin(nx) \quad \text{and} \quad \psi(x) = b_n \sin(nx).$$

Analytical solutions methods to these differential equations pose substantial difficulties. Except being very complicated, they generally are inconvenient for practical use. An indirect numerical method for the solution of the problem (1-5) is developed also in [1] by transforming the second order PDE (1) to a system of two first order PDE_s. Then, for the resulting PDE_s system it was applied a difference scheme of the first order of convergence.

The model (1-5) is a simplified version. The real models except being more complicated, they generally involve several parameters and different physical constants. Such models usually have to be simulated and solved numerically a large number of times. Therefore, efficient numerical methods are needed to treat these models.

This paper deals with the numerical solution of the problem (1-5) and related matters. The numerical method proposed is based on the method of characteristics which is combined with the technique known as Richardson

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extrapolation. The efficiency and accuracy of the classics characteristics method is essentially improved. It is found that the conventional (empirical) local error order of the proposed numerical scheme is at least one unit more than that of the classic method of characteristics. To maintain the expected global order of accuracy special treatment of boundary conditions (2)-(3) is done. Numerical results are presented and relevant conclusions are drawn. It would be seen that formally the method could be straightforward extended for the more general problem:

$$u''_{tt} - u''_{xx} = f(t, x, u, u'_t, u'_t), \quad 0 < x < l, \ t > 0,$$

subject to boundary conditions

$$u'_t(0,t) = g_1(t, u(0,t), u'_x(0,t)), \quad t \geq 0,$$
$$u'_t(l,t) = g_2(t, u(l,t), u'_x(l,t)), \quad t \geq 0,$$

and initial conditions

$$u(x,0) = \phi(x), \quad 0 \leq x \leq l,$$
$$u'_t(x,0) = \psi(x), \quad 0 < x < l,$$

where f, g_1 and g_2 above are arbitrary nonlinear functions and l is an arbitrary constant.

II. Numerical Solution for the Wave Problem (1-5)

It is well known that the method of characteristics is applicable for the weakly nonlinear hyperbolic problem of the general form:

$$a_1 u''_{tt} + a_2 u''_{tx} + a_3 u''_{xx} = a_4, \ 0 < x < l, \ t > 0, \tag{6}$$

where a_i denotes a function in the variables x, t, u, u'_x and u'_t for i = 1, 2, 3, 4.
The initial and boundary conditions are given as

$$u(0,t) = u(l,t) \equiv 0, \quad t > 0, \qquad (7)\text{-}(8)$$
$$u(x,0) = \phi(x), \quad 0 \leq x \leq l, \tag{9}$$
$$u'_t(x,0) = \psi(x), \quad 0 \leq x \leq l. \tag{10}$$

It can be seen that differential equation (1) is a special case of the equation (6), but the initial-boundary condition (3) is different from the analogous condition (8). There is a difference also between the initial conditions (5) and (10).

Suppose P(x_1, t_1) and Q(x_2, t_2) with $t_1 = t_2 = 0$, are two points along x-axis so that $0 < x_1 < x_2 < \pi$. One can easily verify that the straight lines $t - t_1 = \pm x - x_1$ and $t - t_2 = \pm x - x_2$ are the 4 characteristic curves of the equation (1) in points P and Q respectively. Let $R(x,t)$ be the intersection of the proper lines $t - t_1 = x - x_1$ and $t - t_2 = -x - x_2$ (see Fig. 1). The standard procedure of the method of characteristics for the determination of the point $R(x,t)$ and finding the approximations for $u(x,t), u'_x(x,t)$ and $u'_t(x,t)$ at this point (hereafter known as (P,Q,R) process), is very simplified for the case of equation (1). Since $u(x,t), u'_x(x,t)$ and $u'_t(x,t)$ are known at points P and Q, the conditions under which u''_{xx}, u''_{xy} and u''_{yy} can be uniquely found, after some algebraic operations, lead to the equations:

$$u'_x(R) = \tfrac{1}{2}[u'_x(Q) + u'_x(P) + u'_t(Q) - u'_t(P)] + \tfrac{h}{8}[a(Q) - a(P)], \tag{11}$$
$$u'_t(R) = \tfrac{1}{2}[u'_x(Q) - u'_x(P) + u'_t(Q) + u'_t(P)] + \tfrac{h}{8}[a(Q) + 2a(R) + a(P)], \tag{12}$$
$$u(R) = \tfrac{1}{2}[u(P) + u(Q)] + \tfrac{h}{8}[u'_x(P) - u'_x(Q) + u'_t(P) + 2u'_t(R) + u'_t(Q)], \tag{13}$$

with step size $h = (x_2 - x_1)$, and $a = \varepsilon[u'_t - \frac{1}{3}u'^3_t]$.

It can be seen that (11) and (13) are explicit equations, while the equation (12) is implicit because of the term $a(R)$involved in it. It can be seen also that (11) – (12) for finding the approximations to $u'_x(x,t)$ and $u'_t(x,t)$ are independent and autonomous from the process (13) for finding the approximation to $u(x,t)$, because the function $u(x,t)$ is not involved explicitly in equations (11) – (12).

Suppose P(x_1, t_1), Q(x_2, t_2) and T(x_3, t_3) with $t_1 = t_2 = t_3 = 0$, are three points equally spaced along x-axis so that $0 < x_1 < x_2 < x_3 < \pi$. If the processes *(P,Q,R)*, *(Q,T,S)*, *(R,T,V)* are applied repeatedly as it is shown in Fig. 1, then approximations for $u'_x(V), u'_t(V)$ *and* $u(V)$ are received.

We denote these approximations as $u'_x(V,h), u'_t(V,h)$ and $u(V,h)$, just to express the fact that they are obtained using step size h. We denote by analogy the approximations received by the single process *(P,S,V)* with step size $2h$ as $u'_x(V,2h), u'_t(V,2h)$ and $u(V,2h)$. We adopt here the following technique, known as Richardson extrapolation: If A(h) and A($2h$) are two approximations to A with error O(h^2) then the formula

$$A = \big(4A(h) - A(2h)\big)/3 \tag{14}$$

provides an approximation to A with error O(h^4). But in the context of our application, the problem of error order is complicated because the approximation A(h) is a triple process where the last one, *(R,S,V)*, is depended on the two first *(P,Q,R)* and *(Q,T,S)*. It is also difficult to discuss for the error order of a single *(P,Q,R)* process. We have used the following problem as an example test to estimate conventionally (empirically) the above error orders:

$$u''_{tt} - u''_{xx} = 0, \ \ 0 < x < 1, \ t > 0$$
$$u(0,t) = u(1,t) = 0, \qquad t > 0$$
$$u(x,0) = \sin 2\pi x, \qquad 0 \leq x \leq 1$$
$$u'_t(x,0) = 2\pi \sin 2\pi x, \qquad 0 \leq x \leq 1$$

Exact solution: $u(x,t) = \sin 2\pi x\,[\cos(2\pi t) + \sin(2\pi t)]$

Considering some other similar problems we have concluded that the triple process *(P,Q,R)*, *(Q,S,T)*, *(R,T,V)* has almost the same conventional error order as the single process *(P,S,V)*, this was expected. Meantime, in all examples considered the formula (14) has produced approximations with the error order at least one unit higher.

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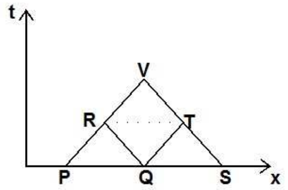


Fig. 1. The processes *(P,Q,R)*, *(Q,T,S)*, and *(R,T,V)*

The formula (14), as described above, is used to provide new approximations for $u_x'(V), u_t'(V)$ and $u(V)$, so the accuracy of the classic method of characteristics is expected to be essentially improved. Denote by *(P,Q,S,V)* the process of finding the node V and the approximations for $u_x'(V), u_t'(V)$ and $u(V)$ as above described. A mesh G_0 is obtained by discretizing the interval $0 \leq x \leq \pi$ into m subintervals, each of width $h = \pi/m$. It is assumed that $\phi'(x)$ exists so that $u_x'(x,0) = \phi'(x)$, whenever $0 \leq x \leq \pi$. Consequently, $u(x,t), u_x'(x,t)$ and $u_t'(x,t)$ will be known functions whenever $0 \leq x \leq \pi$ and t = 0. Meanwhile, following the conditions (2 – 5), one can easily obtain:

$$u(0,0) = 0,\ \ u_x'(0,0) = \phi'(0),\ u_t'(0,0) = 0, \qquad (15)$$

$$u(\pi,0) = \phi(\pi), u_x'(\pi,0) = \phi'(\pi),$$

$$u_t'(\pi,0) = -\varepsilon\alpha u_x'(\pi,0). \qquad (16)$$

The notation G_0 will be used hereafter to denote the (m+1) points of G. The process *(P,Q,S,V)* is applied for each of three successive nodes of G_0 and so (m-1) points are obtained where the function u and its partial derivatives can be approximated. Denote by G_1 the mesh received from these (m-1) points and the endpoints A and Z (Fig. 2).

It will be shown now how the values of $u(x,t), u_x'(x,t)$ and $u_t'(x,t)$ will be approximated at endpoints A and Z. By the boundary left condition (2) it is easily found that $u(A) = 0$ and $u_t'(A) = 0$. An interpolation process can be used to evaluate $u(Z)$ with sufficient accuracy using the values of u in the left neighbor of Z. In the implementation below, polynomial interpolation of degree two and Matlab utilities *polyfit* and *polyval* are used properly for the abovementioned interpolation. The choice of degree two is motivated by the need to maintain the global accuracy provided for the interior nodes. Other interpolation strategies such as Hermit interpolation may be more efficient. The derivative $u_x'(A)$ is estimated using a three-point forward difference formula, whereas a three-point backward formula is used for the derivative $u_x'(Z)$. The choice of these formulas, as well as the choice of degree two above, is governed by the need to maintain the same accuracy at point Z, which is attained at this point for u. Finally $u_t'(Z)$ can be computed by the boundary right condition (3).

It can be seen that based on the mesh G_1, a mesh G_2 may be constructed in the same way, and then a mesh G_3 and so on, in order to move farther up the time axis. So a uniform and square mesh of points G is obtained where the function $u(x,t)$ and its partial derivatives are approximated.

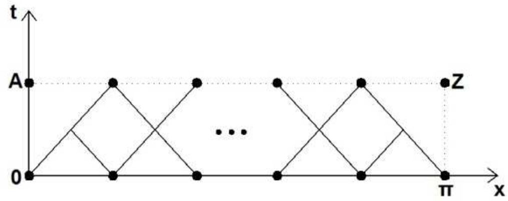


Fig. 2. The characteristic curves of the equation (1) and the meshes G_0 and G_1

The numerical method produced and described in this section takes advantage of the particularities of the problems to be solved, namely the special configuration of the mesh of points generated by characteristics curves of the equation and the specialty of the initial-boundary conditions involved. The method is implemented in Matlab. The main code, built as function file, f*main.m*, is presented in appendix of this paper.

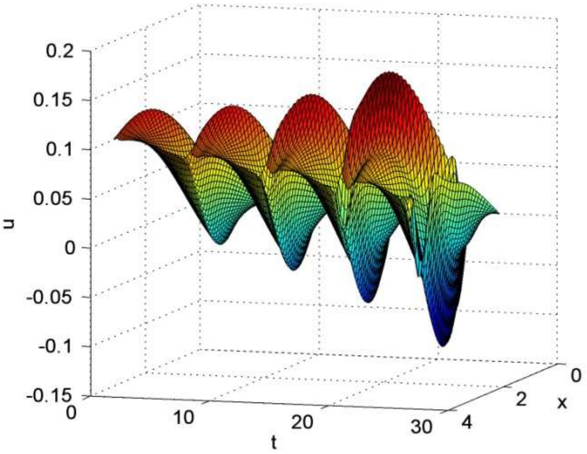
III. Numerical Results

In a series of numerical experiments, following [1], the problem (1-5) is solved for different values of damping parameter α, and for various initial conditions that could not be treated analytically at [1]. The most typical results are presented graphically in the Figs. 3-6 and can be compared with those of [1]. It can be seen that for values of the damping parameter α larger than or equal to π/2 all solutions will tend to zero as time t tends to infinity. For $0 < x < \pi/2$ the solutions are bounded and the string-system oscillates. So the damping α can be used effectively to suppress the oscillation-amplitudes.

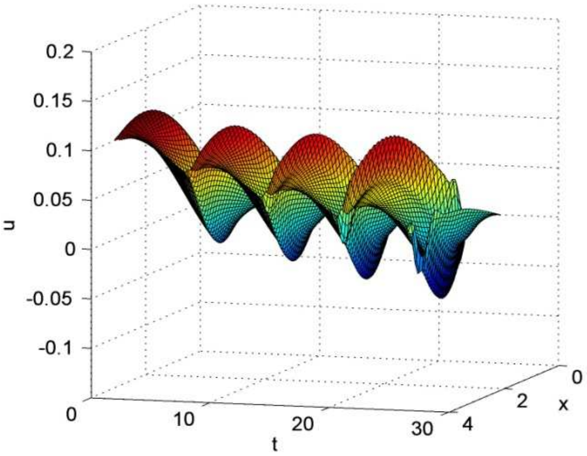
The numerical method proposed is stable and its efficiency and accuracy are demonstrated in all the experiments carried out. It is verified that (1-5) can be integrated by economic computational effort for an interval of time t considerably larger than that presented in Figs. 3-6. The elapsed time per each running of the code f*main.m* has been about 10 seconds, so the method can be effectively used when (1-5) is to be simulated and solved a large number of times.

IV. Conclusions

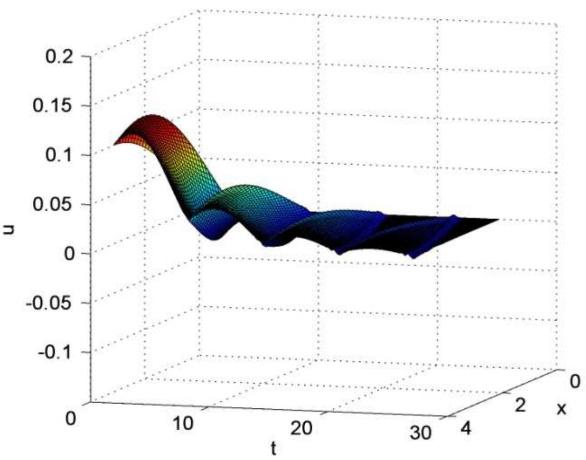
A computationally efficient numerical method is presented and implemented in Matlab to analyze galloping, which is characterized by large amplitude vibrations of electrical transmission lines. The galloping solutions can always be damped to zero with imposition of an appropriate damping device at one end of the line. The proposed method can be adopted to analyze similar models appearing in literature. Moreover the numerical solution could be obtained for initial-boundary value problems with initial values which could not be treated analytically in the literature. The work done can be extended and applied in practice when a dashpot device is to be installed at the pole(s) to an electric line. Formally the method can be straightforward extended for the more general problem described in the introduction. The code f*main.m* can be easily adopted for this case.



(a) α=π/4

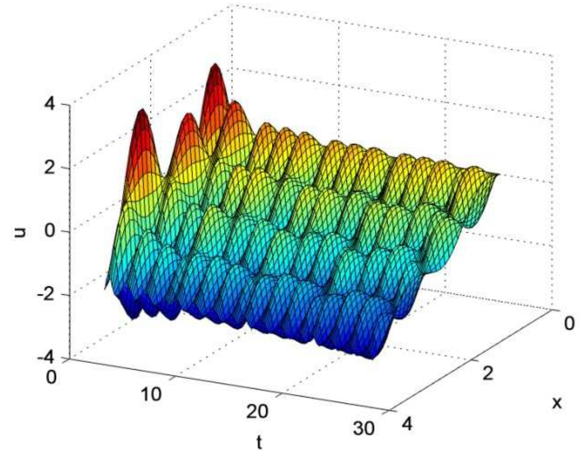


(b) α=π/2

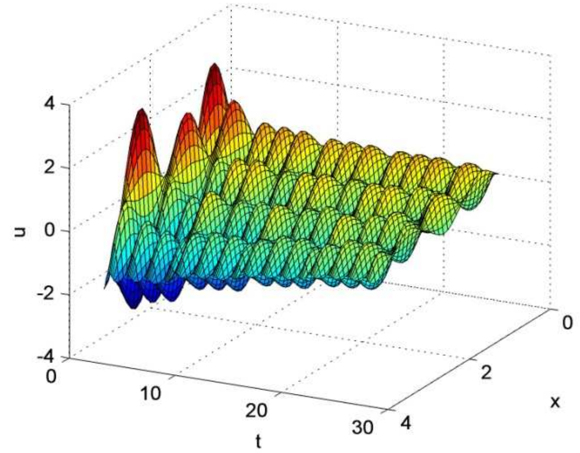


(c) α=3π/2

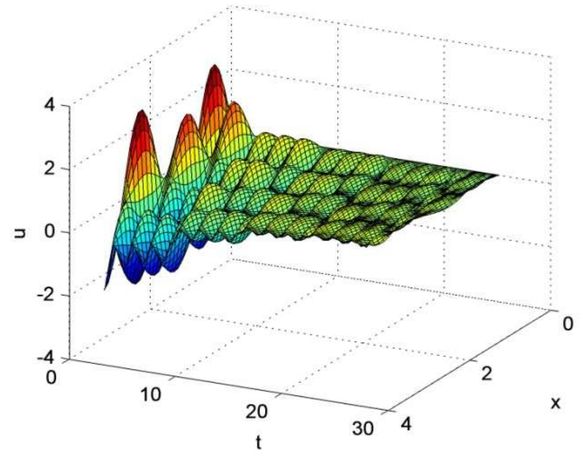
Fig.3. Displacement u vs. x and t for $\varphi(x)=0.1\sin(0.5x)$, $\psi(x)=0.05\sin(0.5x)$ and $\epsilon=0.1$.



(a) α=π/4

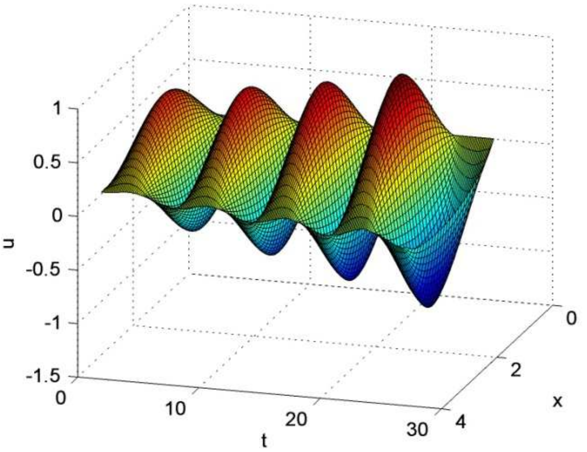


(b) α=π/2

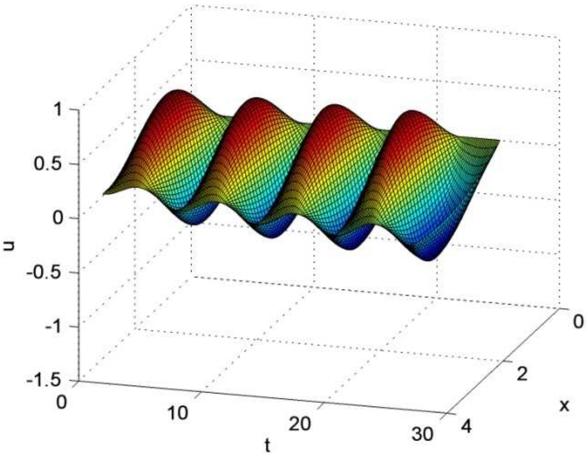


(c) α=3π/2

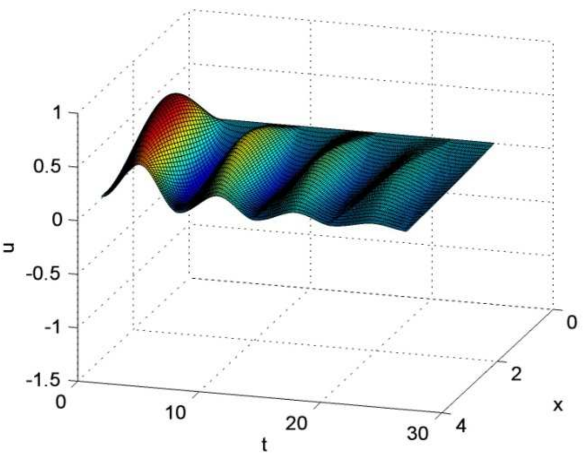
Fig. 4. Displacement u vs. x and t for $\varphi(x)=2.5\sin(3.5x)$, $\psi(x)=0.05\sin(3.5x)$ and $\epsilon=0.1$.



(a) α=π/4

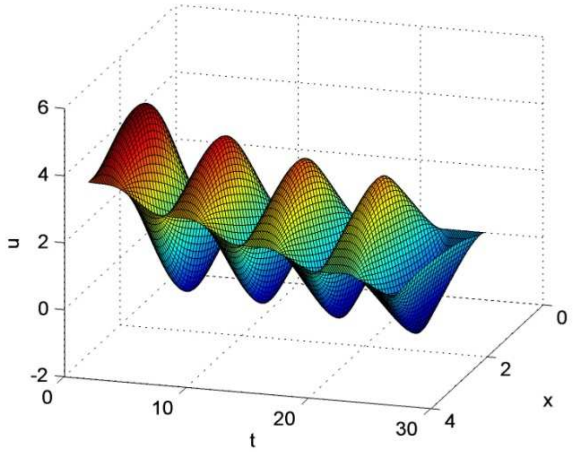


(b) α=π/2

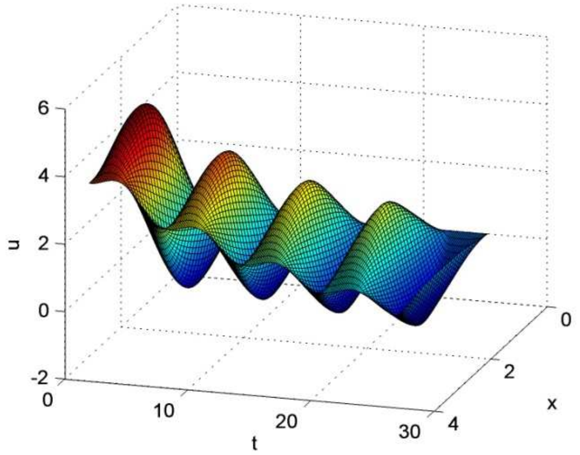


(c) α=3π/2

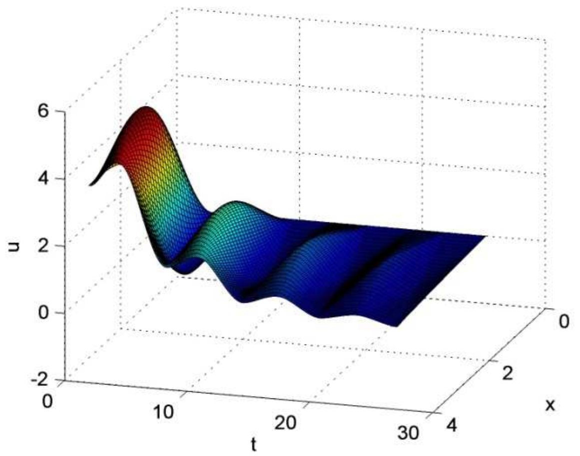
Fig. 5. Displacement u vs. x and t for $\varphi(x)=0.01x^3e^{-3x/\pi}$, $\psi(x)=0.2x(\pi-x)$ and $\epsilon=0.1$.



(a) α=π/4



(b) α=π/2



(c) α=3π/2

Fig. 6. Displacement u vs. x and t for $\varphi(x)=x+\sin(x)$, $\psi(x)= x(\pi-x)$ and $\epsilon=0.1$.

Appendix

The main Matlab code: *fmain.m*

```
function F=fmain(fi,dux,dut,alfa)
global pp qq kkk PP QQ KKK
epsi=0.1; kk=epsi*alfa;
m=30; N=8*m;
h=pi/m; kkk=epsi*h/8;
H=2*h;  KKK=epsi*H/8;
u=zeros(N+1,m+1); ut=u; ux=u;
x=linspace(0,pi,m+1);
u(1,1:m+1)=feval(fi,x);
ux(1,1:m+1)=feval(dux,x);
xint=x; xint(1)=[]; xint(end)=[];
ut(1,2:m)=feval(dut,xint);
ut(1,m+1)=-kk*ux(1,m+1);
for i=1:N
for k=1:(m+1)
a(k)=ut(i,k)-1/3*ut(i,k).^3;
end
for j=1:m
bux(j)=(ux(i,j)+ux(i,j+1)+ut(i,j+1)-
ut(i,j))/2+kkk*(a(j+1)-a(j));
pp=(-
ux(i,j)+ux(i,j+1)+ut(i,j+1)+ut(i,j))/2;
qq=a(j+1)+a(j);
but(j)=fzero('impl',ut(i,j));
bu(j)=(u(i,j)+u(i,j+1))/2+h/8*(ux(i,j)-
ux(i,j+1)+ut(i,j)+2*but(j)+ut(i,j+1));
end
for k=1:m
b(k)=but(k)-1/3*but(k).^3;
end
for j=2:m
ux(i+1,j)=(bux(j-1)+bux(j)+but(j)-
but(j-1))/2+kkk*(b(j)-b(j-1));
pp=(-bux(j-1)+bux(j)+but(j)+but(j-
1))/2;
qq=b(j)+b(j-1);
ut(i+1,j)=fzero('impl',but(j-1));
u(i+1,j)=(bu(j-1)+bu(j))/2+h/8*(bux(j-
1)-bux(j)+but(j-1)+2*ut(i+1,j)+but(j));
end
for j=1:(m-1)
cux(j)=(ux(i,j)+ux(i,j+2)+ut(i,j+2)-
ut(i,j))/2+KKK*(a(j+2)-a(j));
PP=(-
ux(i,j)+ux(i,j+2)+ut(i,j+2)+ut(i,j))/2;
QQ=a(j+2)+a(j);
cut(j)=fzero('impl2',ut(i,j));
cu(j)=(u(i,j)+u(i,j+2))/2+H/8*(ux(i,j)-
ux(i,j+2)+ut(i,j)+2*cut(j)+ut(i,j+2));
end
u(i+1,2:m)=(4*u(i+1,2:m)-cu)./3;
ux(i+1,2:m)=(4*ux(i+1,2:m)-cux)./3;
ut(i+1,2:m)=(4*ut(i+1,2:m)-cut)./3;
aa=polyfit(x(m-2:m), u(i+1,m-2:m),2);
u(i+1,m+1)=polyval(aa,x(m+1));
ux(i+1,1)=(-u(i+1,3)+4*u(i+1,2)-
3*u(i+1,1))/(2*h);
ux(i+1,m+1)=(u(i+1,m-1)-
4*u(i+1,m)+3*u(i+1,m+1))/(2*h);
ut(i+1,m+1)=-kk*ux(i+1,m+1);
end
t=0:h:N*h;
[X,T]=meshgrid(x,t);
surf(X,T,u);
```

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