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Quantum Theory of Coupled Electromagnetic Circuits without and with Ohm's Resistance - Extensions and Transitions to the Continuum and Applications to Problems with Spin and Nuclear Physics

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***Abstract* – A generalization of a previous study has the goal to analyze the properties of four quantized, coupled electromagnetic circuits, which represent a perturbed SU_4-symmetry and SU_3 is rigorously preserved. External electromagnetic fields (vector potential) can be included in similar fashion as the Pauli or Dirac equation, but it is necessary to pass from electric charges of the circuits to charge and current densities. The transition from a chain of discrete charges to a continuum is equivalent to well-known transitions in quantum theory of solids and leads to a quantized telegrapher's equation. Applications are given in molecular physics (intersystem crossing via spin-orbit coupling) and in the domain of nuclear interactions in nuclear and particle physics.**

***Keywords* – Electromagnetic Circuits, Quantization in Charge Space, Telegrapher's Equation, Symmetry Principles of Mutually Coupled Circuits and Lie Algebras, Transitions to the Continuum, Applications to Nuclear Models.**

I. INTRODUCTION

In a previous paper [1], we have investigated problems of the quantization of coupled electromagnetic circuits without (a) and with (b) Ohm's resistance R. Thus both cases can be described by a generalized Lagrangian defined in the charge space (Q: electric charge, $\dot{Q}$: electric current also denoted by **I**, L: inductivity and C: capacitance):

$$Ł = Ł_0 \cdot \exp(R \cdot t / L)$$

$$Ł_0 = \frac{L}{2} \cdot \dot{Q}^2 - \frac{L \cdot \omega_0^{\,2}}{2} \cdot Q^2;\ \omega_0^2 = 1/(LC). \quad (1)$$

The application of the (1) to the well-known Lagrange formalisms provides the basic equation of the electromagnetic circuit, which is represented by Figure 1A:

$$\frac{d}{dt}\frac{\partial Ł}{\partial \dot{Q}} - \frac{\partial Ł}{\partial Q} = 0 \Rightarrow$$

$$L \cdot \ddot{Q} + R \cdot \dot{Q} + \frac{1}{C} \cdot Q = 0. \quad (2)$$

Thus for vanishing resistance with R = 0 $Ł$ and $Ł_0$ are identical. It has to be noted that the reduced eigen-frequency resulting from (2) reads:

$$\omega^2 = \omega_0^2 - \frac{R^2}{4 \cdot L^2}. \quad (2a)$$

The canonical quantization procedure can be obtained by standard principles from text books in quantum mechanics. The canonical momentum is given by

case (a)

$$P = \frac{\partial Ł_0}{\partial \dot{Q}} = L \cdot \dot{Q} \quad (3)$$

and in case (b) by

$$P = \frac{\partial Ł}{\partial \dot{Q}} = L \cdot \dot{Q} \cdot \exp(R \cdot t / L). \quad (3a)$$

With the help of (3a) the Hamiltonian assumes the shape:

$$H = \exp(-R \cdot t / L) \cdot \frac{P^2}{2 \cdot L} + \exp(R \cdot t / L) \cdot \frac{L}{2} \cdot \omega_0^2 \cdot Q^2. \quad (4)$$

By taking R = 0 it is evident that the Hamiltonian H of (4) agrees with the Hamiltonian resulting of (3) and denoted by H_0:

$$H_0 = \frac{P^2}{2 \cdot L} + \frac{L}{2} \cdot \omega_0^2 \cdot Q^2. \quad (4a)$$

The quantization is performed by the canonical commutation relation and has to be applied to either (4) or (4a):

$$P \cdot Q - Q \cdot P = \frac{\hbar}{i}. \quad (5)$$

There are different ways to establish (5) in the Hamiltonian H_0 or H. A very common method is the Schrödinger representation via replacing $P \rightarrow -i\hbar\partial/\partial Q$ acting on the wave-function ψ. In the case of the Hamiltonian H only the non-stationary, time-dependent Schrödinger equation can be obtained; the solutions have already been presented [1].

A algebraic method, which is well-known in the case of the Hamiltonian H_0, works by introducing annihilation - and creation operators:

$$b'(t) = \alpha \cdot Q \cdot \exp(\frac{R \cdot t}{2 \cdot L}) + i\beta \cdot P \cdot \exp(-\frac{R \cdot t}{2 \cdot L})$$

$$b^{+\prime}(t) = \alpha \cdot Q \cdot \exp(\frac{R \cdot t}{2 \cdot L}) - i\beta \cdot P \cdot \exp(-\frac{R \cdot t}{2 \cdot L}). \quad (6)$$

The parameters α and β are given by:

$$\alpha^2 = \frac{L \cdot \omega}{2 \cdot \hbar},\ \beta^2 = \frac{1}{2 \cdot \hbar \cdot L \cdot \omega},$$

$$\omega^2 = \frac{1}{C \cdot L} - \frac{R^2}{4 \cdot L^2}. \quad (6a)$$

By using (5), (6) and (6a) the commutation relation results:

$$b'(t)b^{+\prime}(t) - b^{+\prime}(t)b'(t) = 1. \quad (7)$$

In spite of each operator b' and $b^{+\prime}$ is time-depending, it is identical with well-known relations of the harmonic oscillator, which also results from (7) by taking R = 0:

$$bb^+ - b^+b = 1. \quad (7a)$$

However, the state space related to (7) is a time-depending Schrödinger equation $\psi(b', b^{+\prime})$ associated with the Hamiltonian:

$$i\hbar \frac{\partial}{\partial t}\psi = H \cdot \psi$$

$$H = \frac{\hbar \cdot \omega}{2} \cdot [b'(t)b^{+\prime}(t) + b^{+\prime}(t)b'(t)]. \quad (8)$$

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Thus the handling of (8) with the help of the operator algebra (7) is a little more difficult to calculate excited states and transitions, since the time-dependent version has to be accounted for each excited state $\psi_n(b_k'(t),b_k^{+\prime}(t),t)$ and the procedure is equivalent to the previously presented one based on solution functions of the Schrödinger equation via Hermite polynomials [1].

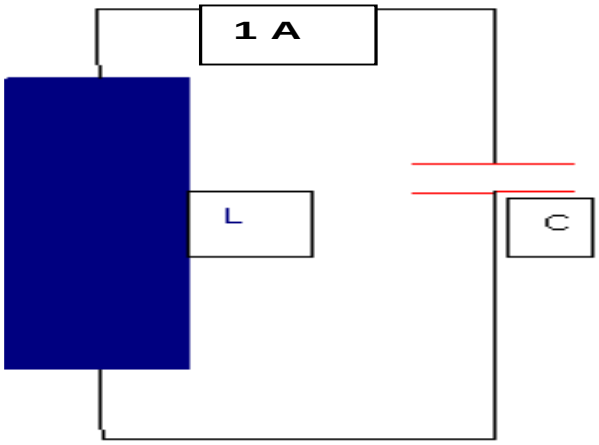

Fig.1A. One single electric oscillator (basis for further investigations in the paper).

It should be mentioned that the mechanical analogue of (1) results by the substitutions $Q \rightarrow$ q (position coordinate), $\dot{Q} \rightarrow \dot{q}$ (velocity), L → m (mass of a particle, $C^{-1} \rightarrow$ f (force constant) and R → γ (friction parameter of a mechanical system). This analogue has been discussed in detail during the past decades, and even a logarithmic nonlinear Schrödinger equation for the description of friction has been accounted for [1 - 10]. Some fundamental problems of the linear (and nonlinear) Schrödinger equation containing friction has been discussed in detail [7], since time-reversal is violated due the irreversible heat production, and the implementation of the uncertainty relation is a further crucial aspect due to the property of friction to lead every motion of a particle finally to the total rest. Therefore it is evident that a mechanical system containing the phenomenological parameter friction is not adequate to describe the motion of single electrons or protons. One should be aware of that quantized circuits containing Ohm's resistance R yield similar problems as discussed in the mechanical case. Therefore one has to be careful by formally taking the time $t \rightarrow \infty$.

According to the previous study [1] we have to determine the normal modes in order to make use of creation/annihilation operators according to (6 - 8). The easiest case are to coupled circuits (Figure 1B) and the normal modes are given by (R = 0):

$$L\cdot\ddot{Q}_1 + M\cdot\ddot{Q}_2 + Q_1/C = 0$$
$$L\cdot\ddot{Q}_2 + M\cdot\ddot{Q}_1 + Q_2/C = 0$$
$$\tilde{Q}_1 = Q_1 - Q_2;\quad \tilde{Q}_2 = Q_1 + Q_2$$
$$\lambda_1 = L - M;\ \lambda_2 = L + M;\quad \omega_{1,2}{}^2 = \frac{1}{C\cdot\lambda_{1,2}}. \quad (9)$$

The determination of the annihilation/creation operators is straightforward, which now assume the shape:

$$b_k = \alpha_k\cdot\tilde{Q}_k + i\beta_k P_k$$
$$b_k^{+} = \alpha_k\cdot\tilde{Q}_k - i\beta_k P_k$$
$$\alpha_k{}^2 = \frac{\lambda_k\cdot\omega_k}{2\cdot\hbar},\ \beta_k{}^2 = \frac{1}{2\cdot\hbar\cdot\lambda_k\cdot\omega_k},\ (k=1,2). \quad (10)$$

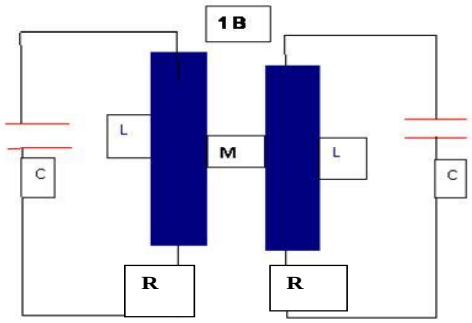

Fig.1B. Two identical oscillators with mutual magnetic coupling M between the currents.

These operators stated by (10) satisfy the well-known algebraic relation, i.e. i.e. $[b_y, b_y^{+}] = 1$, (k = 1, 2), and the Hamiltonian now reads:

$$H_0 = \hbar\cdot\omega_1\cdot(n+1/2)\cdot b_1^{+}\cdot b_1$$
$$+\hbar\cdot\omega_2\cdot(m+1/2)\cdot b_2^{+}\cdot b_2,(n,m=0,1,2...). \quad (11)$$

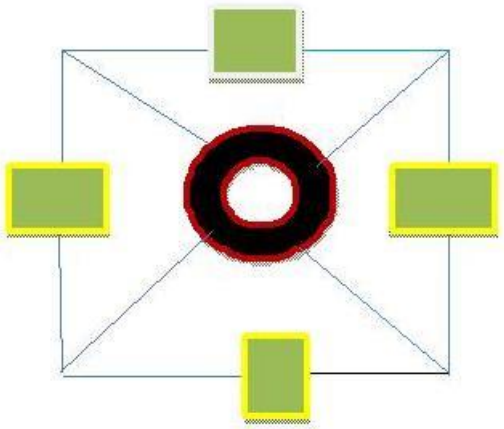

Fig.2. Schematic model of 4 coupled electromagnetic circuits referring to (13). The inductivity L of each circuit is placed in the center, and each oscillator is coupled to all other oscillators via M. The magnetic properties denoted by L and M are represented by the circle. The rectangles represent the capacity C of each circuit. The model may formally be extended to higher order associations of mutually coupled electromagnetic circuits according to (13).

The inclusion of friction is also straightforward, and we have to perform the following substitutions:

$$b_k \rightarrow b'_k(t);\ b_k^{+} \rightarrow b'^{+}_k(t);\ (k=1,2)$$
$$\omega_k{}^2 \rightarrow \omega_k{}^2 - R^2/(4\cdot\lambda_k{}^2);\ (k=1,2). \quad (12)$$

The time-depending operators in (12) are determined in the identical fashion as in (6), if L is replaced by e_{ke} (k = 1,

2) as stated in (9). The present considerations can readily be extended to more general cases, and for simplicity R = 0 is assumed. Since the graphic representation of n identical and mutually coupled circuits (n > 2) is intricate, we have restricted ourselves to a schematic model in Figure 2. The use of the procedure (9) to determine the normal modes can formally be generalized to the arbitrary case of n coupled oscillators, and the parameters of the normal modes now become:

$$\lambda_1 = L + (n-1)\cdot M;\ \omega_1{}^2 = \frac{1}{C\cdot\lambda_1}$$

$$\lambda_k = L - M;\ \omega_k{}^2 = \frac{1}{C\cdot\lambda_k};\ (k = 2,...,n). \qquad (13)$$

The quantization procedure by introduction of creation - and annihilation operators as carried out in (10) and (11) may yield different kinds of representations; we prefer to use Lie groups. The case n = 3 leads to a perturbed SU_3 symmetry, but SU_2 is exactly preserved. In similar fashion the case n = 4 provides a perturbed SU_4 with a corresponding preservation of SU_3. The extension to n = 5 is straightforward with a perturbed symmetry SU_5 and an exact preservation of SU_4, etc. If the parameters L, C and M are suitably modified and interpreted, applications to nuclear and particle physics can be made feasible.

II. Transitions to Continua

In the preceding chapter we have partially repeated and completed some previous results [1]. This is particularly valid with regard to the results of (6 - 13), which shows that by interactions the symmetry of the total system is broken, and by additionally interactions the degeneracy may completely be removed. At the present stage, we have not yet been aware of the fact that the charge Q or current I may be characterized by further degrees of freedom such as spin **σ** or the connection between SU_2 in coupled electric circuits and the is-spin **τ** used nuclear physics.

Case of one Single Circuit (Scalar Field)

In many problems of physics and technical applications one single electromagnetic circuit might be a shortcoming, and one way to overcome the situation incorporate coupled systems of oscillators. However, even these systems may be simplified, and the single parameters L and C are sometimes referred to as 'lumped devices'. Thus we may in particular think of molecular devices as studied in biophysics [11, and references therein], which indicate to perform the replacements: total charge Q → charge density ρ(x, y, z) and total current $I = \dot{Q} \to \dot{\rho}(x,y,z)$. The latter replacement by $\dot{\rho}$ stands in a close relationship to the current density vector **j** owing to the continuity equation:

$$div\cdot\vec{j} + \dot{\rho} = 0\ . \quad (14)$$

We maintain the 'lumped parameters' L and C, but we have to replace the Lagrangian $\mathbf{Ł}_0$ or **Ł** by the corresponding densities (for simplicity, we only consider the case R = 0; if needed, the general case with R ≠ 0 is accounted for via **Ł** according to (1)):

$$Ł_0 = \int d^3x\cdot\tilde{Ł}_0(x,y,z,t)$$

$$\tilde{Ł}_0(x,y,z,t) = \tfrac{L}{2}\cdot\dot{\rho}^2 - \tfrac{1}{2\cdot C}\cdot\rho^2. \qquad (15)$$

The related Lagrange equation provides:

$$\tfrac{d}{dt}\cdot\delta\tilde{Ł}_0/\delta\dot{\rho} - \delta\tilde{Ł}_0/\delta\rho = 0 \to$$

$$L\cdot\ddot{\rho} + \frac{1}{C}\cdot\rho = 0\ . \qquad (15a)$$

The canonical momentum P now has to be replaced by Π(x, y, z, t). By that, the Hamiltonian density reads:

$$\tilde{H}_0 = \frac{1}{2L}\cdot\Pi^2 + \frac{L\cdot\omega_0{}^2}{2}\cdot\rho^2. \qquad (15b)$$

Owing to the introduced substitutions the canonical quantization (5) has to be modified:

$$\Pi(x,y,z,t)\cdot\rho(x',y',z',t) - \rho(x',y',z',t)\cdot\Pi(x,y,z,t)$$

$$= \tfrac{\hbar}{i}\cdot\delta^3(x-x'). \qquad (16)$$

The representation of (15b) and (16) by creation- and annihilation operators follows well-elaborated methods of field theory (see e.g. textbooks of advanced quantum theory). For this purpose we introduce a complete, normalized system of functions and expand (16) according to

$$\int f_k(x,y,z)\cdot f_{k'}(x,y,z)d^3x = 1\ (if\ k=k')\ else\ 0$$

$$\rho = \sum_{k=1}^{\infty} q_k\cdot f_k(x,y,z);\ \Pi = \sum_{k=1}^{\infty} p_k\cdot f^*{}_k(x,y,z)\ . (17)$$

By that, the commutation rule (16) reads:

$$\sum_{k=1}^{\infty} q_k\cdot f_k(x,y,z)\cdot\sum_{k'=1}^{\infty} p_k\cdot f^*{}_{k'}(x',y',z') -$$

$$\sum_{k'=1}^{\infty} p_k\cdot f^*{}_{k'}(x',y',z')\cdot\sum_{k=1}^{\infty} q_k\cdot f_k(x,y,z) = \tfrac{\hbar}{i}\cdot\delta^3(x-x'). \quad (17a)$$

Multiplying both sides of the commutation rule expansion (17) with a function $f^*{}_l(x', y', z')$, and by using the orthogonality relation of the above function system the integration over both sides of (17a) provides:

$$p_k\cdot q_l - q_l\cdot p_k = \frac{\hbar}{i}\cdot\delta_{kl}\ . \qquad (18)$$

This result can readily substituted and generalized with the help of the creation- and annihilation operators according to (10), and the final result now reads:

$$b_k = \alpha_k\cdot q_k + i\beta_k p_k$$

$$b_k^+ = \alpha_k\cdot q_k - i\beta_k p_k$$

$$\alpha_k{}^2 = \tfrac{L\cdot\omega_0}{2\cdot\hbar},\ \beta_k{}^2 = \tfrac{1}{2\cdot\hbar\cdot L\omega_0},\ (k = 1,.....,\infty)$$

$$b_k\cdot b_l^+ - b_l^+\cdot b_k = \delta_{kl}\ (k,l = 1,.....,\infty). \qquad (19)$$

The Hamiltonian (15b) now assumes, in principle, the same shape as previously obtained by (11):

$$\tilde{H}_0 = \frac{\hbar\cdot\omega_0}{2}\cdot\sum_{k=1}^{\infty}(b_k^+\cdot b_k + b_k\cdot b_k^+)\ . (20)$$

This result indicates that by the transition to the continuum the flexibility can be increased. However, since further fundamental properties like *spin* or *iso-spin* are not yet accounted for, we have to consider a more general case, namely four mutually coupled circuits in order to reach further degree of freedom and to describe spin-orbit coupling.

Case of 4 Coupled Circuits - Vector Field with perturbed SU_4

In this section we can use previously elaborated methods. Four mutually coupled circuits (Figure 2) obey the equation

$$L\cdot\ddot{Q}_k + M\cdot\sum_{j\neq k}\ddot{Q}_j + \tfrac{1}{C}Q_k = 0;\quad (k=1,...,4). \qquad (21)$$

The determination of the four normal modes has been previously discussed [1]; see also (9) and (13). Thus we have to replace the inductivity L by the related values of the normal model (the degeneracy is of the order 3, and the quantization yields a perturbed SU_4 with exact conservation of SU_3):

$$\tilde{Q}_4 \to \lambda_4 = L+3M;\quad \tilde{Q}_k \to \lambda_k = L-M\ (k=1,2,3)$$

and

$$\omega_4^2 = \tfrac{1}{\lambda_4\cdot C} = \tfrac{1}{C\cdot(L+3M)};\ \omega_1^2 = \omega_2^2 = \omega_3^2 = \tfrac{1}{C\cdot(L-M)}. \qquad (22)$$

In order to obtain the desired field representation we have to carry out the substitutions:

$$\tilde{Q}_k \to \tilde{\rho}_k \Rightarrow \tilde{Q}_k = \int\tilde{\rho}_k\cdot d^3x$$

$$\dot{\tilde{Q}}_k \to \dot{\tilde{\rho}}_k \Rightarrow \dot{\tilde{Q}}_k = \int\dot{\tilde{\rho}}_k\cdot d^3x\ (k=1,4). \qquad (23)$$

Since the determination of the Lagrange density is analog to (15) and the Lagrange equation to (15a), we restrict ourselves to state the resulting Hamiltonian:

$$\tilde{H}_0 = \sum_{k=1}^{4}\tfrac{1}{2\lambda_k}\cdot\Pi_k{}^2 + \sum_{k=1}^{4}\tfrac{\lambda_k\cdot\omega_k{}^2}{2}\cdot\tilde{\rho}_k{}^2\ . \quad (24)$$

The canonical quantization of vector fields reads:

$$\Pi_k(x,y,z,t)\cdot\tilde{\rho}_l(x',y',z',t) - \tilde{\rho}_l(x',y',z',t)\cdot\Pi_k(x,y,z,t) = \tfrac{\hbar}{i}\cdot\delta_{kl}\cdot\delta^3(x-x')$$
$$(k,l=1,4). \qquad (25)$$

The change from the commutation relation (25) to its algebraic representation by creation - and annihilation operators follows (17 - 19), and the result is:

$$b_{k,k'} = \alpha_{k,k'}\cdot q_{k,k'} + i\beta_{k,k'}p_{k,k'}$$
$$b^{+}_{k,k'} = \alpha_{k,k'}\cdot q_{k,k'} - i\beta_{k,k'}p_{k,k'}$$
$$\alpha_{k,k'}{}^2 = \tfrac{\lambda_{k'}\cdot\omega_{k'}}{2\cdot\hbar},\ \beta_{k,k'}{}^2 = \tfrac{1}{2\cdot\hbar\cdot\lambda_k\cdot\omega_{k'}}$$
$$(k=1,.....,\infty;\ k'=1,4)$$
$$b_{k,k'}\cdot b_{l,l'}{}^{+} - b_{l,l'}{}^{+}\cdot b_{k,k'} = \delta_{kl}\cdot\delta_{k'l'}$$
$$(k,l=1,.....,\infty,\ k',l'=1,4). \quad (26)$$

With the help of (21 - 22) and a suitably extension of (11) the Hamiltonian of the normal modes of four circuits can be stated. However, we cannot establish the spin and a vector potential $\vec{A}$ to describe the interaction of with magnetic fields in our charge quantization formulation, where space coordinates do not be involved. This is the main reason to change to (23) and the related succeeding formulae. The concept of spin enlarges significantly the scope of possible applications.

With regard to (25) we introduce four 2 x 2 σ_k (k = 1,..,4). The Pauli matrices are fixed by σ_1, σ_2, σ_3 and σ_4 is a 2 x 2 unit matrix. In a second step, we apply a generalized quantization method [12, 13] to account for the spin in (24).

$$\sigma^k\cdot\Pi_k(x,y,z,t)\cdot\tilde{\rho}_l(x',y',z',t) - \tilde{\rho}_l(x',y',z',t)\cdot\sigma^k\cdot\Pi_k(x,y,z,t) = \tfrac{\hbar}{i}\cdot\sigma_l\cdot\delta^3(x-x')$$
$$(k,l=1,4). \qquad (27)$$

The commutation rule (27) can be extended to 4 x 4 Dirac matrices [13], and (27) now assumes the shape:

$$\gamma^k\cdot\Pi_k(x,y,z,t)\cdot\tilde{\rho}_l(x',y',z',t) - \tilde{\rho}_l(x',y',z',t)\cdot\gamma^k\cdot\Pi_k(x,y,z,t) = \tfrac{\hbar}{i}\cdot\gamma_l\cdot\delta^3(x-x')$$
$$(k,l=1,4). \qquad (28)$$

The algebra of these γ-matrices can be verified in textbooks of advanced quantum mechanics. A principal difference between (27) and (28) is the incorporation of both types of charges as a new symmetry (charge conjugation). Using (27) the Hamiltonian (24) assumes the shape:

$$\tilde{H}_0 = \sigma^k\cdot\sigma^k\,\tfrac{\Pi_k{}^2}{2\lambda_k} + \sum_{k=1}^{4}\tfrac{\lambda_k\cdot\omega_k{}^2}{2}\cdot\tilde{\rho}_k{}^2\ . \quad (29)$$

When we go to the representation of (29) by creation - and annihilation operators, the state space will be extended by the two-component spinors, but some essential properties resulting from spin are not yet included, e.g. spin-Pauli term, spin-orbit coupling. Therefore an electromagnetic four-vector potential **A** can be introduced via minimal principle to account for these aspects:

$$\tilde{H}_0 = \sigma^k\cdot\tfrac{(\Pi_k - \frac{\tilde{\rho}_k}{c}\cdot A_k)}{\sqrt{2\lambda_k}}\,\sigma^k\cdot\tfrac{(\Pi_k - \frac{\tilde{\rho}_k}{c}\cdot A_k)}{\sqrt{2\lambda_k}}\cdot$$
$$+\sum_{k=1}^{4}\tfrac{\lambda_k\cdot\omega_k{}^2}{2}\cdot\tilde{\rho}_k{}^2\ . \qquad (29a)$$

A constant magnetic field in z-direction can be involved by the properties:

$$\vec{B} = \nabla x\,\vec{A};\ A_x = -B_0\cdot y;\ B_x = B_y = 0$$
$$B_z = B_0 = -\partial A_x/\partial y. \qquad (30)$$

In order to describe the Pauli-spin effect, we only take account of the linear contribution in (29a), which is justified, if the field strength is low:

$$\tilde{H}_0 = \sum_{k=1}^{4}\tfrac{1}{2\lambda_k}\cdot\Pi_k{}^2 + \sum_{k=1}^{4}\tfrac{\lambda_k\cdot\omega_k{}^2}{2}\cdot\tilde{\rho}_k{}^2 + \tfrac{1}{2\cdot\lambda_3\cdot c}\cdot\tilde{\rho}_3\cdot\sigma_3\cdot B_0\ . (31)$$

The component with k = 3 refers to the z - direction, whereas k = 1, 2 to the x- and y - direction (c: velocity of light). Thus the additional term in (31) describes the change of energy by the spin at an external magnetic field In general, the contribution of the vector potential **A** represents the interaction of an external electromagnetic field with coupled circuit system. If **A** is given by plane waves as considered in quantum optics (e.g. transitions induced by external photon fields), we either obtain transitions between different spin states or transitions to excited states of the coupled circuit system of a perturbed SU_4.

With regard to the commutation relations (27) or (28) with should point out that the representations by creation - and annihilation operators (26) are still valid. Now they have to be applied for each spin component. Only by introducing 'second quantization' the related operators are Fermi operators, which have to obey anti-commutation relations.

By taking account of (29a) and (30) without the restriction given by (31) the generalized version of (29a) reads:

$$\tilde{H}_0 = \sum_{k=1}^{4} \frac{1}{2\lambda_k} \cdot \Pi_k^{\,2} + \frac{\tilde{\rho}_1^{\,2}}{2\lambda_1 \cdot c^2} B_0^2 \cdot y^2 + C'$$

$$\sum_{k=1}^{4} \frac{\lambda_k \cdot \omega_k^{\,2}}{2} \cdot \tilde{\rho}_k^{\,2} + \frac{1}{2 \cdot \lambda_3 \cdot c} \cdot \tilde{\rho}_3 \cdot \sigma_3 \cdot B_0. \qquad (32)$$

C' results from some products of the canonical momentum with the related component of **A**; they can be removed by proper substitutions. The quadratic term ~ y^2 in (32) yields a further kind of oscillator well-known in quantum mechanics. It leads to discrete rotation states described by the Landau terms. A resonance effect is obtained, when these rotations are in agreement with spin transitions as known by the Einstein - de Haas effect. In a mechanical sense, we have to introduce a mass M for the coupled circuit system.

With respect to transitions we have not yet accounted for singlet-triplet transitions induced by the remaining spin components. However, this is an internal process. In quantum mechanics, where one regards atoms with a central electrostatic force and an angular momentum **l** besides the spin **s**, the corresponding term is given by:

$$\hat{H}_{ls} = \frac{a_{ls}}{\hbar^2} \cdot \vec{l} \cdot \vec{s}; \quad a_{ls} = \frac{Z \cdot e_0^2 \cdot \mu_0 \cdot \hbar^2}{8 \cdot \pi \cdot m_e^{\,2} \cdot r^3}. \qquad (33)$$

All terms arising in a_{ls} are known from atomic physics, for this reason there is no necessity to explain them. According to [1] we are able to express the coefficient a_{ls} resulting from atomic/molecular physics in terms of so-called Bohr capacitances, C_B inductivity L_B and the radius r_B, which shall be used to set 'g' according to (34) by using (34a). In the more general case, which is of interest here, we consider the interaction of spin with an internal electric field (not necessarily a central field) as described by the charge densities $\tilde{\rho}_k$, and we follow the elaborated methods [12 - 14]. Such a situation occurs in nuclear physics, where interactions of spin with internal fields play a role in the shell model, and in the absence of a central field the notation 'spin-orbit coupling' assumes a generalized meaning. We add the 'spin-orbit' term to the Hamiltonian (32), which now becomes:

$$\tilde{H} = \tilde{H}_0 + g \cdot \vec{\sigma} \cdot (\vec{\Pi} \; x \; \nabla \cdot \phi);$$

$$\phi : scalar\, potential$$

$$g = \frac{Z \cdot e_0}{2 \cdot m_e \cdot c} \cdot \frac{1}{4 \cdot m_e \cdot c^2}$$

$$or: \quad g = \frac{Z \cdot e_0 \cdot \lambda_c^{\,2}}{8 \cdot \hbar^2 \cdot c}. \qquad (34)$$

Z is the nuclear charge, m_e the electron mass, c the velocity of light and λ_c in (34a) the Compton wave length of the electron. The purpose of expressing the coupling constant 'g' in terms of λ_c and electrical parameters is a possible generalization, if we not have strictly to deal with single electrons, e.g. reduced mass of electrons or nucleons. The following relationship between electric and non-electric parameters exists:

$$r_B = \frac{Z \cdot \lambda_c}{2\pi \cdot \alpha} \;\; \text{with} : \alpha = \frac{\hbar \cdot c}{e_0^2}$$

$$L_B = \frac{4 \cdot \pi \cdot \mu_0 \cdot r_B^{\,2}}{\lambda_c}; \; C_B = \frac{4 \cdot \pi \cdot \varepsilon_0 \cdot r_B^{\,2}}{\lambda_c}. \; (34\,a)$$

If the 'spin-orbit' term (34) may be applied to nuclear interactions, then the coupling constant '*g*' would be rather different and some further terms would have to be added. Please note that the vectors of (34) are restricted to the first three components of the canonical momentum and charge densities with SU_3 symmetry; the fourth component is the contribution of the perturbed SU_4, which is only affected by the 2 x 2 unit matrix as previously defined. Thus the latter component is associated with the scalar potential ϕ; with regard to central problems in atomic/molecular physics this scalar potential is only a function of the radius r and leads to (33). We also point out that (34) refers to a Hamiltonian density like all other formulae of this section. In analogy to the language of atomic physics the vector product $(\vec{\Pi} \; x \; \nabla \cdot \phi)$ can be interpreted as an orbit, since the representation of the canonical momentums and charge densities by bilinear products of the operators b_{kl} and b^+_{kl} yields the connection between rotation group (or angular momentum $L_{kk'}$) and generators of SU_3:

$$L_{kk'} = \sum_{l=1}^{4} b_{k,l}^{+} \cdot b_{k,l'} - b_{k,l} \cdot b_{k',l}^{+}. \qquad (34b)$$

It is obvious that usually one does not need (34) in its full generality, and often the principal terms of (24) and (29) are important, while all further terms may be regarded as perturbations. Thus a situation occurs when the vector potential **A** does not contain an external, static magnetic field, but solely electromagnetic plane waves, and the 'spin-orbit' contribution yields transitions between different spin states.

Telegrapher's Equation by a Chain of Coupled Circuits

This chapter follows widely the well-known task in classical mechanics, namely a chain of oscillators, the transition to continuum and quantization. A chain of coupled oscillators with elongation at position a_n (f: force constant, m: mass of each oscillator) is given by:

$$\ddot{q}_n = \frac{f}{m} \cdot (q_n - q_{n+1} + q_n - q_{n-1}). \qquad (35)$$

The transition to the continuum is obtained by:

$$\ddot{q}_n = \frac{f \cdot \varepsilon^2}{m} \cdot (q_n - q_{n+1} + q_n - q_{n-1}) / \varepsilon^2$$

$$\varepsilon = |n' - n| > 0; \; n' = n \pm 1; \; \lim(\varepsilon \to 0, m \to 0, f \to \infty). \quad (35a)$$

This limit procedure provides the well-known wave equation with v: velocity of sound:

$$\frac{\partial^2}{\partial x^2} \cdot q - \frac{1}{v^2} \cdot \frac{\partial^2}{\partial t^2} \cdot q = 0. \qquad (35b)$$

The extension to three elongations q^k (k = 1, 3) is straightforward and yields additionally the transversal contributions.

The above limit procedure (35a) is applied in all disciplines of physics, and the quantization of a Bose fields is achieved by a quantum mechanical treatment of an infinite set of oscillators (phonons, photons, etc.) as stated in (19) or (26).

A chain of coupled electric circuits stands in analogy to (35), if the coupling of the circuits is mediated by a dielectric layer between the capacitances:

$$\ddot{Q}_n = \frac{1}{LC}\cdot(Q_n - Q_{n+1} + Q_n - Q_{n-1}). \qquad (36)$$

With regard to the limit procedure and Figure 3 we slightly modify the meaning of L and C. It is usual to state in the telegrapher's equation L and C per length unit [15 - 20], we rewrite (36) in the form

$$\ddot{Q}_n = \frac{n^2\cdot\varepsilon^2}{L_0 C_0}\cdot(Q_n - Q_{n+1} + Q_n - Q_{n-1})/\varepsilon^2$$

$$L = L_0/l;\ C = C_0/l;\ l = n\cdot\varepsilon. \qquad (36a)$$

The finite length unit l consists of the variable parameter ε multiplied with a factor n. Thus Figure 3 makes still sense, if the distances l goes against 0, i.e. $\varepsilon \to 0$. By that, (36a) assumes the shape:

$$\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}Q - \frac{\partial^2}{\partial x^2}Q = 0. \qquad (37)$$

The velocity c might be that of light, then it would be given by

$$c = n/\sqrt{L_0\cdot C_0}. \qquad (37a)$$

The not yet defined parameter n has to be assigned to specific material properties (e.g. dielectric or magnetic susceptibility). The velocity of light in vacuum is therefore the limit case. A modification of (37, 37a) is obtained by changing from the total electric charge Q to the charge density ρ(x, y, z, t), where the continuity equation (14) has to be fulfilled:

$$\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\rho - \frac{\partial^2}{\partial x^2}\rho = 0. \qquad (38)$$

Both equations, (37) and (38) can be generalized to include Ohm's resistance R:

$$\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}Q + \frac{R_0}{L_0\cdot c^2}\cdot\frac{\partial}{\partial t}Q - \frac{\partial^2}{\partial x^2}Q = 0$$

$$\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\rho + \frac{R_0}{L_0\cdot c^2}\cdot\frac{\partial}{\partial t}\rho - \frac{\partial^2}{\partial x^2}\rho = 0$$

$$R = R_0/l. \qquad (38a)$$

The quantization procedure of (38a) is rather complicated as to be verified from the corresponding cases of single oscillators (6). The application of charges densities ρ indicates a transition of charge clouds instead of discrete charges Q and might be adequate in problems with lacking homogeneity.

There exists an additional possibility to consider a continuum case, namely via magnetic coupling M according to Figure 4. A periodic system of these coupled circuits are described by:

$$\ddot{Q}_n + \frac{M}{L}\cdot(\ddot{Q}_{n+1} + \ddot{Q}_{n-1}) + \frac{1}{LC}\cdot Q_n = 0$$

or:

$$\ddot{Q}_n + \frac{M}{L+2M}\cdot(\ddot{Q}_{n+1} + \ddot{Q}_{n-1} - 2\cdot\ddot{Q}_n) + \frac{1}{(L+2M)\cdot C}\cdot Q_n = 0. \qquad (39)$$

By performing the transition to the continuum the following results are obtained:

$$\frac{\partial^2}{\partial t^2}\cdot[1 - \lambda\cdot\frac{\partial^2}{\partial x^2}]\cdot Q + \omega^2\cdot Q = 0$$

$$\lambda = \frac{v^2}{\omega'^2};\ \omega^2 = \frac{1}{C\cdot(L+2M)};\ \omega'^2 = \frac{1}{C\cdot M}. \qquad (40)$$

or:

$$\frac{\partial^2}{\partial t^2}\cdot[1 - \lambda\cdot\frac{\partial^2}{\partial x^2}]\cdot\rho + \omega^2\cdot\rho = 0. \qquad (40a)$$

With regard to (37), (38), (38a), (40) and (40a) we have gained various types of telegrapher's equations, which have already played an important role during the last century, but they are still of interest in modern technologies of networks and even brain research [11, 15 - 21]. In order to be complete we show the quantization procedure of (38), although it follows known principles:
The Lagrange density of (38) is given by:

$$\tilde{L} = \tfrac{1}{2}(\tfrac{1}{c^2}\rho_{/t}{}^2 - \rho_{/x}{}^2) \to \partial\tilde{L}/\partial\rho_{/t} + \partial\tilde{L}/\partial\rho_{/x} = 0. \qquad (41)$$

Equation (41) is equivalent to (38). With the help of the canonical momentum Π the commutation relation reads:

$$\Pi(x)\cdot\rho(x') - \rho(x')\cdot\Pi(x) = \frac{\hbar}{i}\cdot\delta(x - x'). \qquad (42)$$

Using a Fourier expansion of the operators (42) we again obtain an infinite set of harmonic oscillators and related creation/annihilation operators:

$$\rho(x) = \frac{1}{\sqrt{NL}}\sum_{k=1}^{\infty}[\alpha\cdot(b_k\cdot\exp(ikx) + b_k^+\cdot\exp(-ikx)]$$

$$\Pi(x) = \frac{i}{\sqrt{NL}}\sum_{k=1}^{\infty}[-\beta\cdot(b_k\cdot\exp(ikx) + b_k^+\cdot\exp(-ikx)]. \qquad (43)$$

NL refers to the normalization length and the coefficients α and β are defined by:

$$\alpha = \sqrt{\frac{\hbar}{2\cdot\omega_k\cdot L_0}};\ \beta = \sqrt{\frac{\hbar\cdot L_0\cdot\omega_k}{2}};\ \omega_k = c\cdot k = \frac{n}{\sqrt{L_0\cdot C_0}}\cdot k. \qquad (43a)$$

The Hamilton operator finally becomes [22]:

$$H = \sum_{k=1}^{\infty}\hbar\cdot\omega_k\cdot(b_k^+\cdot b_k + \tfrac{1}{2}). \qquad (44)$$

The question arises how to quantize the wave equation (40a), which refers to a chain of circuits with mutual magnetic coupling M. The obstacle is the impossibility to define the canonical momentum due to the product term T of second time and space derivatives:

$$T = \lambda\cdot\frac{\partial^2}{\partial t^2}\cdot\frac{\partial^2}{\partial x^2}\cdot\rho. \qquad (45)$$

On the other hand, equation (40a) can be solved by plane waves; standing waves are of particular interest:

$$\rho(x,t) = A_n\cdot\sin(k\cdot x)\cdot\cos(\omega_n\cdot t). \qquad (46)$$

The boundary conditions are ρ(0, t) = 0 and ρ(l, t) = 0, which imply:

$$k\cdot\frac{\lambda_0}{2} = n\cdot\pi;\quad (n = 0,1,2,....). \qquad (46a)$$

With the help of the fixations of ω^2 and ω'^2 (λ is the wave length of the ground wave) the frequency spectrum ω_n^2 yields:

$$\omega_n^2 = \frac{\cdot\omega'^2\cdot\omega^2\cdot\lambda_0{}^2}{(\omega'^2\cdot\lambda_0{}^2 + 4\cdot n^2\cdot\pi^2\cdot v^2)}\quad (n = 1, 2, ...). \qquad (46b)$$

Due to the linearity of (40a) every superposition of (46) with rather different values of ω_n can also satisfy (40a). This result plays an interesting role with regard to 'beat frequencies' in biophysics [11].

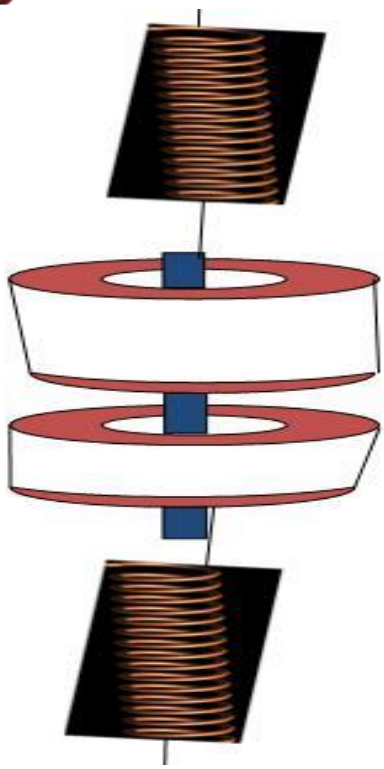

Fig.3. Series of coupled circuits with electric coupling via dielectrics between the capacitances.

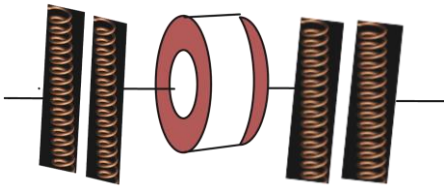

Fig.4. Series of coupled circuits with magnetic coupling M between the circuits and transition to the continuum case. The capacitance is assumed to show cylinder form.

It is evident that Ohm's resistance R has been neglected in Figures 3 and 4. Since this property is more important in Figure 4 due to the magnetic coupling between solenoids, we state the modification with inclusion of R in the case of (40a):

$$\frac{\partial^2}{\partial t^2}\cdot[1-\lambda\cdot\frac{\partial^2}{\partial x^2}]\cdot\rho+R'\cdot\frac{\partial}{\partial t}\cdot\rho+\ \omega^2\cdot\rho=0$$

$$R'=R\cdot C\cdot\omega^2. \qquad (47)$$

Excepted a slight modification the solution of the frequency spectrum follows the 'ansatz' (46):

$$\rho_n=\rho_{0,n}\cdot\exp(i\cdot\omega_n\cdot t)\cdot\sin(k\cdot x)$$

$$k=2\cdot n\cdot\pi/\lambda_0. \qquad (48)$$

The frequency spectrum ω_n is now given by the complex quadratic equation:

$$\beta\cdot\omega_n^2-i\cdot R'\cdot\omega_n-\omega^2=0$$

$$\beta_n=\frac{\omega'^2\cdot\lambda_0^2-4\cdot\pi\cdot n^2\cdot v^2}{\omega'^2\cdot\lambda_0^2}. \qquad (48a)$$

$$\Rightarrow\omega_{n\pm}=\frac{1}{2\cdot\beta_n}[i\cdot R'\pm\sqrt{4\cdot\beta_n\cdot\omega^2-R'^2}]. \qquad (48b)$$

The general solution function now becomes:

$$\rho_{n\pm}(x,\ t)=\rho_{0,n}\cdot\sin(\frac{2\cdot n\cdot\pi\cdot x}{\lambda_0})\cdot$$

$$\cdot\exp(\frac{-R'\cdot t}{2\cdot\beta_n}\pm\frac{i\cdot t}{2\cdot\beta_n}\cdot\sqrt{4\cdot\beta_n\cdot\omega^2-R'^2}). \qquad (49)$$

Thus (49) includes the possibility of superposition of solutions with different n-values and an exponential decreasing damping function resulting from R ≠ 0. The special cases R = 0 (without damping) and/or M = 0 (without mutual magnetic coupling) are accounted for in the family of solutions (49).

Connection to the wave-function representation in the position space

From the basic principles of quantum mechanics it is well-known that $|\psi|^2$ represents a probability function of the particle density. Assume N electrons (as bosons) with charge e_0. For the scalar field (16) this implies a modification of the probability density function yielding the charge density function ρ of this study:

$$\rho=e_0\cdot|\psi(x_1,...,x_N,t)|^2\ and$$

$$\Pi=L\cdot\dot{\rho}=L\cdot e_0\cdot\frac{\partial}{\partial t}|\psi(x_1,...,x_N,t)|^2. \qquad (50)$$

Inserting (50) to (16) implies a commutation relation between the square of the wave-function and the time derivative of the square (**j** is the quantum mechanical expression of the current density vector):

$$div\cdot\bar{j}=-\frac{\partial}{\partial t}\cdot\rho$$

$$|\psi(x',y',z',t)|^2\cdot div\cdot\bar{j}(x,y,z)-div\cdot\bar{j}(x,y,z)\cdot|\psi(x',y',z',t)|^2$$

$$=\ \frac{\hbar}{L\cdot e_0\cdot i}\cdot\delta^3(x-x'). \qquad (50a)$$

Since we have started our considerations by a canonical quantization of the charge and the related current, the extension of (16) to (50a) represents rather a novel result: The representation of (50a) by creation/annihilation operators leads to a current algebra, which is realized by (19). If we restrict ourselves (16), (50) and (50a) to charged Boson fields, the commutation rules are in agreement with the connection between spin and statistics, and the Cooper pairs in superconductivity may represent suitable examples of this way of field quantization. However, we should have to deal with Fermions, and therefore we have to turn to their properties. The commutation rule (27) or (28) need a more detailed analysis with regard to properties of Fermions of the wave-function ψ. According to [23 - 26] the commutation rules (27) and (28) can be classified as para-Fermions. This behavior becomes feasible by the anti-commutation rule for creation/annihilation operators of Fermions:

$$a_k\cdot a_l^+ + a_l^+\cdot a_k\ =\ \delta_{kl}. \qquad (51)$$

The indices k and l refer to a set of quantum numbers; bilinear products of the operators (51) satisfy commutation rules to form a basis set of Lie algebras. Since in (27, 28) we have to deal with operator functions of the densities and their time-derivatives, the transcription to Fermi operators has to be based on the mentioned bilinear products. Thus the properties of para-Fermions can be verified by (28), which can be brought to the form:

$$\gamma^l\gamma^k\cdot\Pi_k(x,y,z,t)\cdot\tilde{\rho}_l(x',y',z',t)-$$
$$\gamma^l\tilde{\rho}(x',y',z',t)\cdot\gamma^k\cdot\Pi_k(x,y,z,t)=\tfrac{\hbar}{i}\cdot\gamma^l\gamma_l\cdot\delta^3(x-x')$$
$$(k,l=1,4). \qquad (52)$$

We use the anti-commutation rule of the γ-matrices (I_u is the 4 x 4 unit matrix):

$$\gamma^k\cdot\gamma^l+\gamma^l\cdot\gamma^k=2\cdot g^{kl}\cdot I_u. \qquad (52a)$$

By that, we obtain:

$$\gamma^k\cdot\Pi_k(x,y,z,t)\cdot\gamma^i\cdot\tilde{\rho}_l(x',y',z',t)+$$
$$\gamma^l\cdot\tilde{\rho}(x',y',z',t)\cdot\gamma^k\cdot\Pi_k(x,y,z,t)=$$
$$\tfrac{\hbar}{i}\cdot\gamma^l\cdot\gamma_l\cdot\delta^3(x-x')\ +T.\quad (k,l=1,4). \qquad (52b)$$

Thus at the right hand-side of (52b) an additional term appears, which is a characteristic feature of para-Fermions:

$$T=2\cdot g^{kl}\cdot\Pi_k\cdot\tilde{\rho}_l. \qquad (52c)$$

A rigorous procedure to come to a pure Fermion field is given by considering (28) and (52b) as a result of a continuum representation on the first quantization level. Then by treating the eigen-vectors ψ_k and ψ^+_k of the Hilbert space as anti-commuting operators the whole problem is formulated in a second quantization theory. In this way, one also proceeds with regard to the field quantization of the Dirac or Pauli equation. For practical problems, it is easier to represent Π_k *and* $\tilde{\rho}_k$ by infinite sequences of bilinear products of a_k and a^+_l, where the indices k and l refer to sets of quantum numbers (inclusive spin and perturbed SU_4). This way is regarded in (50a). The Hamiltonian related to the para-Fermion system can be brought to the form:

$$H=\hbar\cdot\omega'\cdot$$
$$\cdot\sum_{k,l,m,n}[C_{klmn}\cdot a_k^+\cdot a_l^+\cdot a_m\cdot a_n+D_{kl}\cdot a_k^+\cdot a_l\,]. \qquad (52d)$$

With C_{klmn}, D_{kl} and ω' we express differences in the corresponding oscillator modes and terms resulting from (52b) and (52c); (52d) is a typical example of an interaction Hamiltonian.

III. Klein-Gordon (KG) Equation and Dirac Equation in the Charge Space

We consider the KG equation in a version such that in the non-relativistic limit we obtain the related Schrödinger equation:

$$\sum_{k=1}^{3}\tfrac{1}{2L}(\Pi_k^{\,2}+\tfrac{L\cdot\omega^2}{2}\cdot Q_k^2)\varphi=i\cdot\hbar\cdot\tfrac{\partial}{\partial t}\varphi$$
$$\Pi_k\rightarrow\tfrac{\hbar}{i}\tfrac{\partial}{\partial Q_k}. \qquad (53)$$

For simplification, we restrict ourselves to three oscillator circuits. The KG equation in charge space without capacitance has to obey the generalized energy-momentum relation, but the rest energy $E_0 = mc^2$ must still be maintained:

$$E^2=E_0^{\,2}+\lambda^2\cdot\vec{\Pi}^2$$
$$E\rightarrow i\hbar\tfrac{\partial}{\partial t}\ \ and\ \ \vec{\Pi}\rightarrow\tfrac{\hbar}{i}\cdot\nabla. \qquad (53a)$$

According to the next section (application to nuclear physics) the requirement to maintain E_0 is consistent, since the total electromagnetic energy of a circuit can be associated with the relativistic expression of the energy. We have to include the potential energy related to charges in a way to yield the corresponding expression in (53a). By that, we write the KG-equation by the 'ansatz' (λ is a parameter to account for a possible difference to the velocity of light c with regard to an energy-momentum relation in the charge space):

$$E^2=\lambda^2\cdot\vec{\Pi}^2+E_0^2+E_0\cdot L\cdot\omega^2\cdot\vec{Q}^2\rightarrow$$
$$-\hbar^2\cdot\tfrac{\partial^2}{\partial t^2}\psi=-\hbar^2\cdot\lambda^2\cdot\sum_{k=1}^{3}\tfrac{\partial^2}{\partial Q_k^{\,2}}\psi+$$
$$E_0\cdot L\cdot\omega^2\cdot\sum_{k=1}^{3}Q_k^2\psi+E_0^2\psi. \qquad (54)$$

With the help of the 'ansatz' $\psi=\exp(-i\cdot E_0\cdot t/\hbar)\cdot\varphi$ and by neglecting the second time derivative $\ddot{\varphi}$ we obtain the above non-relativistic limit:

$$i\cdot\hbar\cdot\dot{\phi}=-\tfrac{\hbar^2\cdot\lambda^2}{2E_0}\cdot\sum_{k=1}^{3}\tfrac{\partial^2}{\partial Q_k^{\,2}}\phi+\tfrac{L}{2}\cdot\omega^2\cdot\sum_{k=1}^{3}\tfrac{\partial^2}{\partial Q_k^{\,2}}\phi. \qquad (54a)$$

A comparison with (53) determines the parameter λ:

$$\lambda^2=E_0/L=mc^2/L. \qquad (54b)$$

In the sense of quantum mechanics of position space we have to replace L → m which provides λ = c. The transition to the Dirac equation based on (54) is trivial, if the term resulting from the charges (potential energy) is neglected. However, the inclusion of the charge terms (Q_k, k = 1, 3) can only provide quadratic terms of the charges via iterated Dirac operator, if the first-order Dirac equation in charge-time is restricted to a linear combination of the charges without any quadratic contributions. Therefore we proceed by the following fashion:

$$i\cdot\hbar\tfrac{\partial}{\partial t}\psi=\lambda\cdot\vec{\alpha}\cdot\vec{\Pi}\cdot\psi+\beta\cdot E_0\cdot\psi+\lambda'\cdot\vec{\alpha}\cdot\vec{Q}$$
$$\lambda'=\sqrt{L\cdot E_0\cdot\omega^2}. \qquad (55)$$

The algebra of the matrices **α** and β is known from relativistic quantum theory, it obeys:

$$\alpha_k\alpha_l+\alpha_l\alpha_k=2\cdot\delta_{kl};\ \beta\alpha_k+\alpha_k\beta=0$$
$$(k,l=1,3). \qquad (55a)$$

However, the iterated bispinor equation yields further terms resulting from spin, if λ' ≠ 0 which can be realized via ω = 0. This can readily be seen by setting the operator formula $A\cdot\psi=B\cdot\psi$, from which the operator iteration results

$$A\cdot\psi=B\cdot\psi\ \rightarrow A^2\cdot\psi=B^2\cdot\psi+[A\cdot B-B\cdot A]\ \psi$$
$$\rightarrow-\hbar^2\cdot\tfrac{\partial^2}{\partial t^2}\psi=\lambda^2\cdot\vec{\Pi}^2\psi+E_0^2\cdot\psi+\lambda'^2\cdot\vec{Q}^2\cdot\psi$$
$$+i\cdot\hbar\cdot\lambda'\cdot(\tfrac{\partial}{\partial t}\vec{\alpha}\cdot\vec{Q})\cdot\psi+\lambda\cdot\lambda'\cdot(\vec{\Pi}\cdot\vec{Q})\cdot\psi. \qquad (56)$$

In the iterated Dirac case with non-vanishing potential (i.e. λ' ≠ 0) the term A·B - B·A in (56) is non-vanishing, too.

In order to obtain the above Dirac equation in the charge space in the usual version by γ-matrices we carry out the substitutions $\gamma^1 = \beta$, $\gamma^{k'} = \beta \cdot \alpha_{k'}$ (k' = 2,..,4) and divide the resulting equation by c:

$$i \cdot \hbar \cdot \gamma^k \cdot \frac{\partial}{\partial \xi_k} \psi = m \cdot c \cdot \psi + \frac{\lambda'}{c} \cdot \sum_{k'=2}^{4} \gamma^{k'} \cdot Q_{k'} \cdot \psi$$

$$\xi_1 = c \cdot t\,;\ \xi_{k'} = \frac{\lambda}{\hbar \cdot c} \cdot Q_{k'}\ (k' = 2,..,4). \qquad (57)$$

Since $Q_k = \frac{\hbar \cdot c}{\lambda} \cdot \xi_k$ we are able to write the Dirac equation (A8a) and its solution function (58) in the form:

$$i \cdot \hbar \cdot \gamma^k \cdot \frac{\partial}{\partial \xi_k} \psi = m \cdot c \cdot \psi + \frac{\lambda' \cdot \hbar}{\lambda} \cdot \sum_{k=2}^{4} \gamma^{k'} \cdot \xi_{k'} \cdot \psi\,. \qquad (58)$$

$$\psi_{pqr} = N(p,q,r) \cdot \exp(-i \cdot k_\nu \cdot \xi_\nu) \cdot$$

$$\cdot \exp(-\frac{i \cdot \eta^2}{2} \cdot \sum_{j=2}^{4} \xi_j^2) \cdot H_p(\frac{\xi_1}{\eta}) \cdot H_q(\frac{\xi_2}{\eta}) \cdot H_r(\frac{\xi_3}{\eta}). \quad (58a)$$

$N(p,q,r)$ refers to the normalization of the wave-function. H_p, H_q, and H_r represent Hermite polynomials. The γ-matrices obey the relation (52a) and from (58) and (58a) follows:

a. ground state (p = q = r = 0):

$$E = mc^2 + \gamma^{k'} \cdot c \cdot \hbar \cdot k_{pw,k'} + \frac{1}{2} \hbar \cdot \omega \cdot \sum_{k'=2}^{4} \gamma^{k'}. \qquad (59)$$

In (59) the wave vector of plane waves is defined by $\mathbf{k_{pw}}$ in order to avoid confusion with the index parameter k.

b. general solution:

$$E = mc^2 + \gamma^{k'} \cdot c \cdot \hbar \cdot k_{pw,k'} +$$

$$+ \hbar \cdot \omega \cdot [(p + \tfrac{1}{2}) \cdot \gamma_2 + (q + \tfrac{1}{2}) \cdot \gamma_3 + (r + \tfrac{1}{2}) \cdot \gamma_4]. \quad (59a)$$

Thus it is nice to see that the case of absence of the potential energy with λ' = 0 leads to the Dirac equation of pure currents. The general solution with λ' ≠ 0 still provides the SU_3-symmetry in the space of four-component spinors.

A principal note refers to the determination of N(p, q, r) of (58a) as well as to the normalization of the previously regarded non-relativistic cases. In position space the normalization is given by the integration over the complete interval $[-\infty \le x \le +\infty]$ and in Euclidean geometry this assumption makes sense, whereas in charge space the same principle would imply charges Q running from $-\infty$ *to* $+\infty$, which is apparently not imaginable. Therefore we have restricted all conclusions to those predictions independent of the normalization, e.g. the eigen-values or modes of the related equations. ´

IV. Applications

In order to show a rather wide field of possible applications of systems of coupled circuits, we consider now topics of molecular physics/biophysics and nuclear interactions. The principal difference exists in the interpretation of the currents and the mutual coupling between them. However, the methodology is in agreement with usual principles of quantum mechanics. Thus molecular physics is based on electromagnetic interactions, whereas in nuclear physics the interaction concept of quantum theory is extended to other kinds of interactions.

Singlet-triplet Transitions in Molecular Physics

Due to the importance in molecular - and biophysics we should like to return to the previously studied case of show an increased importance with regard to singlet - triplet transitions compared to '*p - electrons*'. There are - besides measurements in solvents - different ways to consider the electronic structure of H_3PO_4 by theoretical means [1, 22, 27 - 29].

Recently even *'Ab-initio calculations'* of H_3PO_4 have been made available [28 and references therein]. However, these calculations suffer from realistic conditions with regard to interactions with a solvent (e.g. H_2O), and only one definite conformation is possible to be accounted for. Due to the progress in computer facilities calculations of the electronic structure have been made possible in the past decade stated in the references of [28], whereas semi-empirical methods governed the field of quantum chemistry in earlier decades (see e.g. [27]).

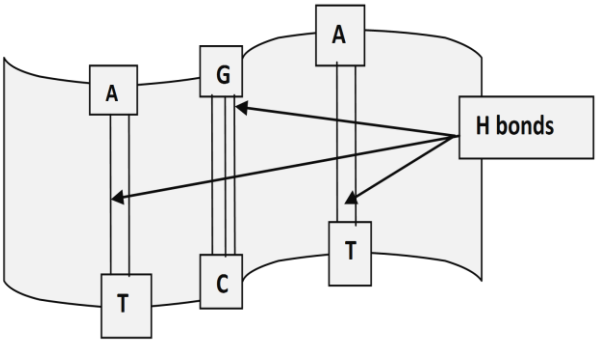

Fig.5. Model of a DNA-chain with the H bonds between the related base pairs A - T and G - C.

Figure 5 shows the proton current between the complementary base pairs. Due to the magnetic coupling of the currents the resonance frequencies cannot be described by one eigen-frequency denoted by the corresponding energy e_0V_0 (see Table 1). Figure 6 has been taken from a previous paper [11] referring to the temperature $T = 300^0$ K. Thus the proton motions between the related base pairs incorporates a typical case of the quantum mechanical tunneling effect, which indicates the increasing importance of this theory in problems of molecular biology.

According to previous results [1] the absorptions of phosphoric acids in solvents lie between 4.76 eV and 4.84 eV for the P = O double bond with essential influence of 3d-electrons of P, whereas the P - OH bonds lead to absorption bands between 5.52 eV and 5.77 eV. We should also recall that the range of 3d-electrons is above 3 Å; by that, a suitable interaction with π-electrons of nucleic bases A, C, T, G (and U in the case of RNA) is realized. With regard to triplet excitations of phosphoric acid according to (34) we have to account for the charge densities of all constituents as given in [27 - 29].

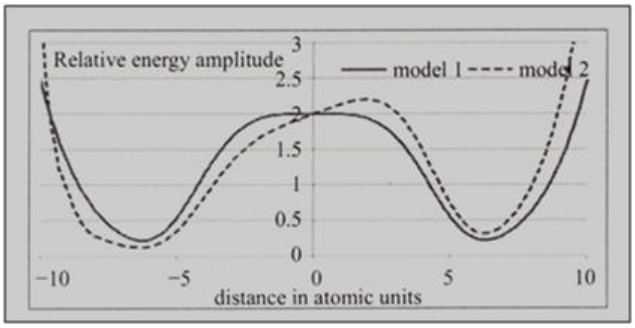

Fig.6. Double minimum potential between the pairs A - T. Model 2: Dashed line with left-side minimum → keto-tautomer of A; at right side minimum → enol - tautomer of T. Model 1: Symmetric approximation between the base pair A - T.

By the use of the elaborated methods in this study we obtain a triplet excitation energy of the P = O double bond at 3.37 eV, but due the weak interaction with solvents (water) and the influence of the P - OH (single bounds) these energy levels are split up to yield 4 very narrow levels between 3.32 eV and 3.41 eV. The triplet energy levels of the single bonds lie between 4.1 eV and 4.5 eV. If we have to deal with chain molecules like DNA these energy levels rather may be identified with band structures than single energy levels. It should be pointed out that the triplet bases of DNA/RNA lie between 3.17eV and 3.45 eV [27, 29], and the energy levels are less affected by keto-enol-tautomery of the base pairs. This tautomery results from the H-bonds between A...T and G...C.

Table 1: Resonance frequencies of the protons (H bonds of DNA) without ($\omega_0 \rightarrow M = 0$) and with couplings ($M \neq 0$) between the base pairs A - T and G - C. Note: the frequency is given by $\upsilon = 1/\tau$ in sec^{-1}; $E_{Hydrogen}$ energy in eV = $h \cdot \upsilon$ - ground state energy: $E_{Hydrogen} = 0.5 \cdot h \cdot \upsilon$.

A - T: $E_{Hydrogen}$			
e_0V_0	e_0V_{10}	e_0V_{20}	
0.0120	0.01603	0.0104	
G - C: $E_{Hydrogen}$			
e_0V_0	e_0V_{10}	e_0V_{20}	e_0V_{30}
0.0105	0.0153	0.0093	0.0117

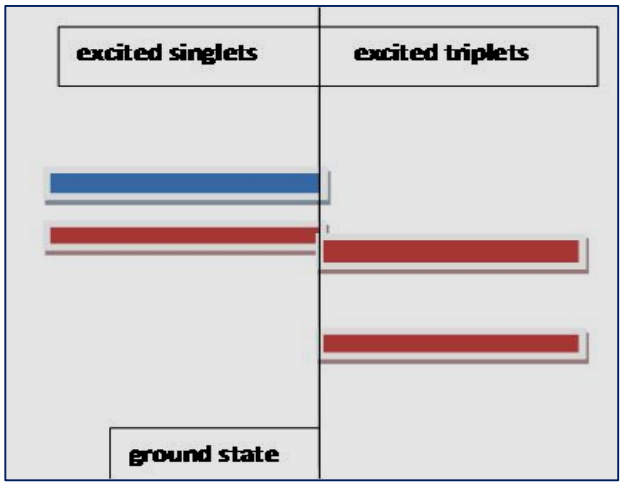

Fig.7. Band structure of DNA with excited singlet and triplet states.

The study of DNA band structures have a long history [30 - 34]. In particular, triplet DNA excitations either can be reached via singlet-triplet transitions with an exorbitant role of the triplets of phosphoric acid due to the long range of 3d-electrons and spin-orbit coupling or by direct excitation of the triplet conduction band by weak X-rays [32] via scattering processes. For the reason of couplings between neighboring molecules in a chain, the triplet excitations lead to charge transfers along the chain, which is referred to as soliton excitation of DNA [33]. This mechanism is in contrast to singlet excitations in molecular chains, which is usually regarded as an exciton mechanism; these excitons are damped by emission of light.

The energy levels of the ground state and excited states have been determined by results [11], which are based on a nonlinear/nonlocal Schrödinger equation and applied to formulas of the preceding section. In particular, we point out the role of (27 - 34a) with reference to triplets. Thus the 3d-electrons of phosphoric acids yield couplings to the π-electrons of the base pairs, but also to the 3d-electrons of the neighboring phosphoric acid of the complementary strand; this is also a consequence of their long range. This fact implies that all energy levels of the isolated molecules suffer further splitting up. The H-bonds described above contribute to this splitting up in an additional fashion. However, the complete band structure still consists of a set of discrete levels with very small energy differences; a continuum description represents a useful approximation. The transitions to the continuum description previously developed must be collated to this approach (Figure 7), since the DNA strands do not show periodicity in a rigorous sense; the nucleic bases differ somewhat in their charge distributions.

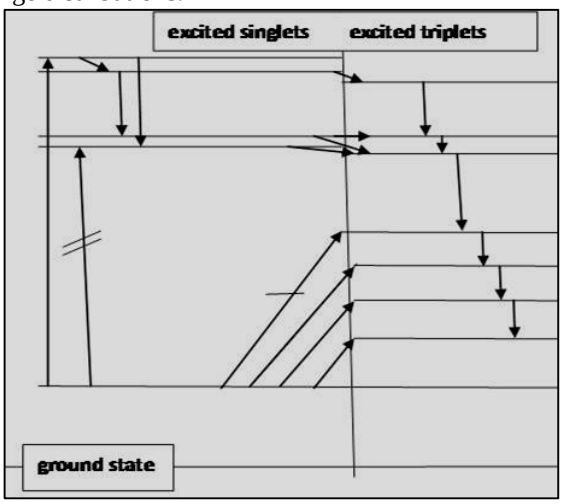

Fig.7a. Term scheme of possible singlet and triplet excitations. The arrows show excitations from the singlet ground state. Transitions to triplet states are very probable, if the triplets are quasi-degenerate to the related excited singlet states (double resonances) or if the energy differences to the ground state are low. The arrow with one dash indicates a rather small transition probability, whereas two dashes represent an extremely small probability, which is usually referred to as 'forbidden transition'.

A particular feature with regard to biochemical excitations of triplet states can be verified in Figure 7a, which shows many excited states with comparably small energy differences. Due to the interaction of DNA with tryptophan (this amino acid exhibits a similar term scheme as the nucleic acids and represents an essential constituent of non-Histon proteins), the triplet levels are split up again. This fact results from the interaction at small distances and lowers triplet levels again. By that, the release of the ATP/GTP-decay via triplet solitons (free energy: 0.5 eV) provides cascade excitations leading to a pumping mechanism [11, 33], and owing to this pathway the DNA receives the necessary energy for processes like cell division or transcription.

Model of the Deuterium Nucleus based on Circuits

While all preceding studies [1] and the present one so far refer to problems of molecular [35] and radiation physics, the extension to other fields appears to be adequate, where the currents are mediated by mesons. According to the connection between rest mass and rest energy (M_p, E_p: proton and M_n, E_n: neutron) we define:

$$E_p = \frac{\hbar\cdot\omega_p}{2} = \frac{\hbar}{2\cdot\sqrt{C\cdot(L+M_s)}} = M_p\cdot c^2 = 938.257\ MeV$$

$$E_n = \frac{\hbar\cdot\omega_n}{2} = \frac{\hbar}{2\cdot\sqrt{C\cdot(L-M_s)}} = M_n\cdot c^2 = 939.565\,MeV. \quad (60)$$

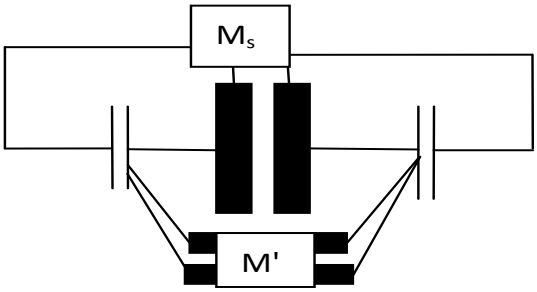

Fig.8. Two identical resonator circuits with mutual coupling M_s (proton (p) - neutron (n)) mediating the exchange of π- and K-mesons incorporate the corresponding rest energies. The 'capacitances' are connected by a further current with weak coupling M' (electromagnetic interaction between p and n) induced by the proton - neutron motion. The kinetic energy per nucleon amounts to 24 MeV.

In (60) we have made use of the possibility to formulate deuteron via iso-spin concept by two coupled oscillators, the coupling current is defined by M_s (M_s: coupling of strong interaction), which is now mediated by the exchange of charged and uncharged mesons. We define the following ratio $\varepsilon = E_p/E_n$ yielding $\varepsilon^2 = 0.9972176708$. By that, we obtain a correspondence between L and M_s:

$$\varepsilon^2 = \frac{4\hbar^2\cdot(L-M_s)\cdot C}{4\hbar^2\cdot(L+M_s)\cdot C} \rightarrow$$

$$M_s = \frac{L\cdot(1-\varepsilon^2)}{1+\varepsilon^2} = L\cdot 0.0027746093. \quad (60a)$$

In the following, we reconsider Figures 1A and 1B to provide the starting-point of the contents of Figure 8. For this purpose, we, at first, restrict ourselves to exclusively strong interaction and neglect the electromagnetic contribution of *p* and *n*. Then *p* and *n* are identical nucleons with different iso-spin **τ** (this is similar to electrons with different spin); the rest energy E_{rest} is given by:

$$E_{rest} = \frac{\hbar\cdot\omega_{nucleon}}{2} = \frac{\hbar}{2\cdot\sqrt{L\cdot C}}. \quad (61)$$

In the framework of the quark model we have to consider 3 coupled oscillators, where SU_2 is exactly preserved: Thus *p* is characterized by the quarks 2 x up-quark $(+2\cdot e_0/3)$ + 1 x down-quark $(-e_0/3)$, and *n* by 2 x down-quark $(-e_0/3, -e_0/3)$ + 1 x up-quark $(+2\cdot e_0/3)$. In a second step, we regard Figure 1B as a one nucleon system, and the coupling M_s to the second nucleonic circuit is considered as a virtual process. Thus both coupled circuits (Figure 1B) simultaneously consist of a *real* and *virtual* circuit. The coupling constant M_s due to meson exchange now expresses the property that the proton is a charged nucleon, whereas the neutron is not. If we turn now to a system with two nucleons and both circuits are subjected to an exchange interaction, since via meson exchange the nucleon with $\tau = +1/2$ can be converted to $\tau = -1/2$ and reverse. E_{rest} according to (61) is determined by simple manipulations via either addition or subtraction to yield:

$$C\cdot L = \frac{1}{8}\cdot\left(\frac{1}{E_p^2}+\frac{1}{E_n^2}\right)\cdot\hbar^2$$

$$C\cdot M_s = \frac{1}{8}\cdot\left(\frac{1}{E_p^2}-\frac{1}{E_n^2}\right)\cdot\hbar^2. \quad (62)$$

The additional information given in (60a) eliminates M_s and reduces the system of coupled circuits to one free parameter, e.g. the capacitance *C*, which can be obtained via internal charge distributions of the corresponding nucleon and based on the relativistic quark-oscillator theory [36]. With the help of these determinations we finally consider Figure 8, which represents a modification of Figure 1B. Figure 8 implies the following equation of the system of coupled circuits:

$$\begin{aligned}
&L\cdot\ddot{Q}_1 + M_s\cdot\ddot{Q}_2 + M'\cdot(\ddot{Q}_3+\ddot{Q}_4) + \tfrac{1}{C}\cdot Q_1 = 0\\
&L\cdot\ddot{Q}_2 + M_s\cdot\ddot{Q}_1 + M'\cdot(\ddot{Q}_3+\ddot{Q}_4) + \tfrac{1}{C}\cdot Q_2 = 0\\
&L'\cdot\ddot{Q}_3 + M'\cdot\ddot{Q}_4 + \tfrac{1}{C}\cdot Q_1 = 0\\
&L'\cdot\ddot{Q}_4 + M'\cdot\ddot{Q}_3 + \tfrac{1}{C}\cdot Q_2 = 0. \quad (63)
\end{aligned}$$

Thus by writing $q_1 = Q_1+Q_2$, $q_2 = Q_1 - Q_2$, $q_3 = Q_3 + Q_4$, $q_4 = Q_3 - Q_4$, $\lambda_1 = L-M_s$, $\lambda_2 = L+M_s$, $\lambda_3 = L'+M'$ and $\lambda_4 = L'-M'$ we obtain the normal mode equations:

$$\lambda_1\cdot\ddot{q}_1 + \tfrac{1}{C}\cdot q_1 = 0. \quad (63a)$$

$$\lambda_2\cdot\ddot{q}_2 - 2\cdot M'\cdot\tfrac{1}{C\cdot\lambda_3}\cdot q_2 + \tfrac{1}{C}\cdot q_2 = 0. \quad (63b)$$

$$\lambda_3\cdot\ddot{q}_3 + \tfrac{1}{C}\cdot q_2 = 0. \quad (63c)$$

$$\lambda_4\cdot\ddot{q}_4 + \tfrac{1}{C}\cdot q_1 = 0. \quad (63d)$$

The solutions inclusive quantization procedure of (63a) - (63b) are clear, they lead to (60) and some corrections due to the coupling of the proton - neutron motion expressed by M' in (63), whereas (63c) and (63d) lead to 'potential energies' depending on q_1 and q_2:

$$-\frac{\hbar^2}{2\cdot\lambda_1}\frac{\partial^2}{\partial q_1^2}\psi+\frac{1}{\lambda_1\cdot C}\cdot q_1^2\psi=E_1\cdot\psi$$

$$-\frac{\hbar^2}{2\cdot\lambda_2}\frac{\partial^2}{\partial q_2^2}\psi+\frac{1}{\lambda_2\cdot C}\cdot q_1^2\psi-\frac{2\cdot M'}{C\cdot\lambda_2\cdot\lambda_3}\cdot q_2^2=E_2\cdot\psi$$

$$-\frac{\hbar^2}{2\cdot\lambda_3}\frac{\partial^2}{\partial q_3^2}\psi+\frac{1}{\lambda_3\cdot C}\cdot q_3^2\psi=E_3\cdot\psi$$

$$-\frac{\hbar^2}{2\cdot\lambda_4}\frac{\partial^2}{\partial q_4^2}\psi+\frac{1}{\lambda_4\cdot C}\cdot q_4^2\psi=E_4\cdot\psi. \qquad (64)$$

The charges at the capacitances remain always identical for Q_1 and Q_3, Q_2 and Q_4; therefore we have used $q_1 = q_3$ and $q_2 = q_4$. By that, all equations of (64) are solved by the well-known methods of harmonic oscillators. These properties give reason to interpret the motion of the nucleons *p* and *n* according to Figure 8 as oscillators in the potential given by deuterium nucleus D_1^2. As already indicated the kinetic energy related to the 'velocities' $\dot{q}_3$ *and* $\dot{q}_4$ amounts to 24 MeV per nucleon associated to the ground state. The determination of the Lagrangian and Hamiltonian follows elaborated principles, which lead to (64). The numerical value of 24 MeV may be taken from a previous investigation based on a generalized nuclear shell theory [12], while the present consideration cannot derive this value and has to be a given magnitude. Since the mass of D_1^2 is somewhat less than the mass of the isolated nucleons *p* and *n*, the related energy difference implies the corresponding binding energy, standing in the direct relationship to the terms to L and M, i.e. $L'=e^2\cdot\sqrt{3}\cdot L$ and $M'=e^2\cdot\sqrt{\frac{3}{4^3}}\cdot M$. This delivers the binding energy $E_{binding}$ = 1.7044 MeV (measurement: 1.71355157 MeV). The advantage of the present model resulting from an identification of the current by a meson exchange is that the numerical effort of the calculation of complex nuclei inclusive reactions with protons or light nuclei can be kept rather bearable. This fact should be appreciated, since radiotherapy with protons and light nuclei has reached a feasible modality.

V. Discussion and Conclusion

The present study provides a significant extension of a previous paper [1] to account for at a first glance rather different fields of modern physics: molecular physics including the band structure of DNA and nuclear physics.

With regard to molecular physics the change from isolated molecules to chains incorporates the most outstanding extension. The quantization of chains of coupled circuits and singlet-triplet transitions resulting from spin-orbit couplings bears new facilities in biophysical chemistry. However, the application to realistic problems like physico-chemical processes in DNA double strands makes apparent the difficulty that we have strictly speaking not to deal with rigorously periodic systems and for a good measure with a finite number of constituents. These constraints have to be accounted for and show the restricted applicability of the approach. A similar situation exists at the quantization procedure of chains of mechanical oscillators (35, 35a) of rather idealistic systems, whereas the reality consists of dislocations and other perturbations of a lattice.

With regard to soliton excitations in DNA we have to differ between solitons travelling along the strands, where the long-range coupling between base pairs is mediated via 3d-electrons of phosphoric acids and the mutual coupling of H-bonds between environmental base pairs. The latter magnetic coupling is extremely weak, and therefore the additional beat frequencies can be connected to the very slow biorhythms considered in chronobiology [11]. Since the protons have spin 1/2, the overall mechanism may be interpreted as spin waves.

The quantization of coupled circuits reveals rather outstanding features, since the procedure can be applied to a field with neither similarity to tasks of molecular physics/biophysics, namely to nuclear interactions and internal symmetries such as SU_3 [36]. There are two possible applications to nuclear physics:

1. We are able to consider a proton and a neutron by one circuit with different iso-spin, respectively, and the different mass is expressed by splitting up the resonance frequency. The handling of the corresponding charge density of the two particles with distinguished iso-spin is evident, but the internal current between the nucleons now has to be interpreted by a meson exchange. The coupling of this current between the two particles leads to deuterium D_1^2. Some test calculations revealed that a assembly of several nucleons is principally possible, and the numerical effort is much easier to handle than by performing a many-body problem in conventional quantum field theory.
2. We should like to have again a look to the Dirac harmonic oscillator in charge space (57 - 59) with SU_3 as an internal symmetry. By that, we can verify a quite natural access to the relativistic quark-oscillator theory developed in [36] and applied to various problems in particle physics by numerous authors (the interested reader should have a look into the web; therefore we do not cite some further papers here). It appears rather exciting that an extension of the presented generalization of the Dirac equation (55 - 59) to a system of coupled internal circuits/resonators provides a nice access to modern theories in particle physics:

$$V(\vec{Q})=\lambda'\cdot\vec{\alpha}\cdot\vec{Q}\rightarrow\lambda'\cdot\vec{\alpha}\cdot(\vec{Q}_n-\vec{Q}_{n+1}+\vec{Q}_n-\vec{Q}_{n-1})$$

$$\lambda'=\sqrt{L\cdot\omega^2\cdot m\cdot c^2}. \qquad (65)$$

Thus by performing the analogous transition to the charge density $\vec{Q}_n\rightarrow\vec{\rho}_n$ as in (36 - 38), then a nonlinear Dirac equation will emerge for the description of particle fields with internal symmetries. As an example we mention the string theory, which results from a system of coupled relativistic oscillators in the quark theory according to [36] in the position space. However, these aspects go beyond the scope of the present study.

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