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Computational Algorithm for Rational Least-Squares

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Abstract **– In the present paper arecursive algorithm for rational least-squares(RLS)of observational data is developed. The error analysis of the algorithm is also included.**

Keywords **– Rational Least-Squares, Optimization Techniques, Statistical Approximations.**

I. INTRODUCTION

The least-squares method is one of the excellent techniques that used all times in problems of observational data analysis. On the other hand, the rational least squares (RLS) is rarely found in literatures. The reason is, perhaps the properties of rational functions families are not well known to investigators, as are those of polynomial families [4, 5]. As far as the computational developments are concerned, the models constructed by a rational function are better than that raised by the polynomial ones. This is because, such models are smoother and have minimum oscillations than that evaluated by regular polynomials. Moreover, the models of rational functions are quite easy to handle computationally and easy to fit [1, 3].If the observations is represented by linear combination of independent functions, then the least squares is termed linear least squares[2].On the other hand if the observations are represented by rational functions it is termed as rational least-squares.

In fact, there are countless number of papers and reports on the rational least squares method, as could be recognized in the Internet sits. To the best of our knowledge there is no complete computational algorithm devoted for the RLS

Due to the lack of such algorithm, the present paper is devoted fordevelopingrecursive algorithm for RLS of observational data with error analysis. The mathematical formulations of the algorithm are developed by Sharafin 2003[6]

II. RECURSIVE ALGORITHM FOR RLS OF OBSERVATIONAL DATA

● *Purpose*

To develop algorithm for rational approximation $A(x)/B(x)$ to a discrete function f(x), in the least – squares sense., where

$$A(x) = a_1 + a_2x + a_3x^2 + \cdots a_{n_a} x^{n_a-1},$$

$$Q(x) = b_1 + b_2x + b_3x^2 + \cdots b_{n_b} x^{n_b-1},$$

such that

$$F = \sum_{i=1}^{N}\left[f_i - \frac{A(x_i)}{B(x_i)}\right]^2 = \mathit{minimum}. \tag{1}$$

● *Input*

Positive integers n_a, n_b and N, arguments x_i, and function values $f_i = f(x_i); i = 1,2,\cdots,N$.

● *Output*

The coefficients of the polynomials A(x) & B(x)

● *Computational Sequence*

1-For all $q = 1,2,\cdots,N$

$B_0(x_q) = 1;\ A_0(x_q) = 0$

2-For all $q = 1,2,\cdots,N$

$$f^*(x_q) = f(x_q)\,B_0(x_q) - A_0(x_q)$$

$$\omega(x_q) = \frac{1}{B_0^2(x_q)}$$

$$g_k(x) = -\frac{A_0(x)}{B_0(x)}x^k;\ k = 1,2,\ldots., n_b - 1$$

$$g_k(x) = x^{k-n_b}$$

3- $$S_0 = \sum_{i=1}^{N}\left[f(x_i) - \frac{A_0(x_i)}{B_0(x_i)}\right]^2$$

4-Solve the linear system

$\mathbf{Hc} = \mathbf{R}$,

For the c's coefficients ,where

$$h_{ij} = \sum_{k=1}^{N}\omega(x_k)\,g_i(x_k)\,g_j(x_k)$$

$$r_i = \sum_{k=1}^{N}\omega(x_k)\,g_i(x_k)\,f^*(x_k)$$

5-Compute the increments

$$\Delta B_0(x) = \sum_{i=1}^{n_q-1} c_i x^i, \tag{2}$$

$$\Delta A_0(x) = \sum_{k=n_q}^{n} c_k x^k, \tag{3}$$

$$n = n_a + n_b - 1. \tag{4}$$

6-Form a new approximation :

$$A_1 = A_0 + \Delta A_0$$

$$B_1 = B_0 + \Delta B_0$$

7-Form successive approximations

$$A_{k+1} = A_k + \Delta A_k, \quad (5)$$

$$B_{k+1} = B_k + \Delta B_k. \quad (6)$$

This process is stopped when there is indication of convergence according to certain criteria (Equation (7) or Equation (8)).

8. Refinement of the Calculation Procedure

The above approach has two defects:

1-There is no guarantee for improvement of the rational fit obtained with successive iterations if the initial approximation is not close enough to a true solution.

2- Zeros may come into the denominator, causing a complete breakdown of the procedure. Toover come these defects Breass's[7]purposes the following approach:

Let

$$S_k = \sum_{i=1}^{N} \left[f_i - \frac{A_k(x_i)}{B_k(x_i)} \right]^2 .$$

Then, instead of using the relations (5) and (6) we use

$$A_{k+1} = A_k + \nu\, \Delta A_k,$$

$$B_{k+1} = B_k + \nu\, \Delta B_k,$$

where the relaxation factor ν is chosen so that S_{k+1} is close to a minimum when regarded as a function of ν and $S_{k+1} \le S_k$.Choosing ν sufficiently small also prevents zeros from entering the denominator.

9. Convergence Criteria

Let T_k denote that the sum of the absolute values of the coefficients of $A_k(x)$ and $B_k(x)$.Then ,the process is stopped if either

$$\frac{S_k - S_{k+1}}{S_{k+1}} \le \nu \in, \quad (7)$$

$$\frac{|T_k - T_{k+1}|}{T_{k+1}} \le \nu \in, \quad (8)$$

or

$$S_{k+1} \le \eta T_{k+1}, \quad (9)$$

Suitable values of the tolerances $\in \& \eta$ are $\in = 10^{-5}$ and $\eta = 10^{-11}$,if the diagonal elements of the coefficient matrix **H** were increased **,**only the last test is applied. If none of these condition is met, the process is repeated until the maximum number of iterations is reached about 15 to 20 iterations.

10-End

In concluding the present paper we stress that,arecursive algorithm for rational least-squares(RLS) of observational data was developed in a step by step fashion.The error analysis of the algorithm is also included .

REFERENCES

[1] Casciolaa, G., Romanib, L.: 2009, A Newton-type method for constrained least-squares data-fitting with easy-to-control rational curves, *Journal of Computational and Applied Mathematics, 223, no. 2, 672-692.*

[2] Kopal, Z. and Sharaf, M. A.:1980, Linear Analysis of the Light Curves of Eclipsing Variables, Ap&SS 70, 77.

[3] Novak, J., Miks, A.: 2004, Least-squares fitting of wavefront using rational function, *Optics and Lasers in Engineering*, 43, no.7, 776–787.

[4] Pomentale, T.: 1968, On discrete rational Least-Squares Approximation, NumerischeMathematik 12, 40-46.

[5] Van Barel, M., Bultheel, A.: 1994, Discrete linearized least-squares rational approximation on theunit circle, *Journal of Computational and Applied Mathematics* 50, 545-563.

[6] Sharaf,M.A.:2003,Relationj between the apparent magnitude and the parallax for Hyades stars

[7] Breass,D.: 1966,Ueber,Daempfung,bei,Minimal-isierungsverfahren.Computing,vvol.l,Edition3,264