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Solar Radiation Data Analysis in Baku by using Daubechies Wavelets

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Abstract – **Wavelets widely applied in filtering the data and help to forecast in many separate fields. In this study, Daubechies Wavelet was used to study hourly solar radiation in Baku city, Azerbaijan. The results demonstrated that timing of global radiation changes have a periodical and non-linear nature. Apparently, the level 5 is sufficient enough, despite the fact that wavelet was extended to level 12 originally. Analysis presented that original series interval is 4 and series stayed unchanged. Also, it was indicated that the original signal is restored on the basis of wavelet factor d1-d5, trends should first be removed from the original series**

Keywords – **Solar Radiation In Baku, Wavelet Analysis, Time Series, Fourier Series, Wavelet Expansion in Azerbaijan, Daubechies Function, Orthonormalized Daubechies.**

I. Introduction

Wavelet analysis is one of the most advanced data analysis technologies and its tools find use in various fields of intellectual activity. Wavelets are widely used in image recognition, processing and synthesis of various signals, for investigation of properties of turbulent fields [1], image analysis [2], convolution (packaging) of significant amounts of information [3], and many other occasions [4]. The term of wavelet (literal translation - a small wave) first appeared in the mid-80s. It was introduced by Grossman and Morlet in association with analysis of properties of seismic and acoustic signals [1].

The most prominent works are by I. Daubechies [5, 6], K.Chui [7], W. Sweldens [8, 9], A. Louis et al. [10]. All of these works provide a detailed insight on wavelet analysis.

Many works have focused on using wavelets for filtering of compared prognostic and actual fields for the purposes of removing noises in the process of verification of prediction results. It has been noted that the use of wavelets allows to clear the compared fields of noises and improve the quality of forecasts. However, a more efficient application of wavelets demands for development of new quality criteria, which are different to standard metric and quadratic similarity measures, such as standard deviation or coefficient of anomaly correlation.

II. Setting Objectives

Wavelet analysis is a special type of linear conversion of information signals and signal-reflected physical data about processes and physical properties of natural environments and objects. It allows to not only identify the typical frequencies of a signal but obtain information about certain local coordinates in which these frequencies occur [1].

The classic fast-Fourier-transform algorithm does not provide an opportunity to analyze frequency-response data at an arbitrary point of time. Unlike the Fourier-transform algorithm where basic functions are harmonic functions, wavelet transformation is based on expansion in functions of varying frequency and time-constrained.

It should be reminded that Fourier-transform algorithm only allows processing signals in either time or frequency domain.

When analyzing non-stationary signals, Fourier-transform does not allow to analyze the signal features as these are smeared in the frequency domain through the entire frequency range of the spectrum (discontinuities, steps, peaks, etc). New high-frequency components emerge although these are absent in the original signal which already bears steps and discontinuities. This happens because "harmonic basic expansion functions is not capable of reflecting the signal differential with infinite slopes like square pulses because this requires for an infinitude of the terms of the series. When the number of the terms of the Fourier series is infinite in the neighborhood of steps, discontinuities, etc., the regenerated signal displays a special conduct known as Gibbs phenomenon.

The classic Fourier-transform algorithm cannot be used for non-stationary signals to get a frequency-time image due to the lack of information on which frequencies are present in the signal at a given moment of time. For stationary signals this information is not required because frequency components of signal and their amplitude are identical at any time interval.

Fourier-transform reflects general information on frequencies of the investigated signal and does not provide details on signal's local properties during rapid stopgap amendments in its frequency content. For instance, Fourier-transform does not make a difference between signals with a sum of two sinusoids (stationary signals) and signals with two consecutive sinusoids of identical frequencies (non-stationary signal) because spectrum factor are computed by means of integration of the entire signal set point interval.

Fourier-transform gives a general idea of which frequencies are present within the signal range but does not provide information on lifetime of spectral components of a signal. Time-domain localization of spectral components demands for designing frequency-time image of the signal. This task can be resolved by means of the so-called Fourier's window transform although wavelet transform is a better solution. Wavelets appear to be an acceptable tool for processing signals and conduct local properties of non-stationary signals.

Thus, Fourier expansion and wavelet expansion shall be considered as supplementary to each other.

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III. Theoretical Analysis

The use of wavelet expansion in simulation of solar radiation transport is caused by the fact that the graph of global radiation behavior has a periodic and non-linear nature. Wavelet expansion enables for representation of the local features of actinometrical data, such as steps and discontinuities. On the other hand, well-designed tools of harmonic analysis are optimal for the use with the determined portion of time series of solar radiation which are close to stationary. Thus, wavelet analysis is not very useful for strongly determined components. We shall now take a closer look at application of wavelet analysis for investigation of alterations in solar radiation.

Any non-stationary signal s(t) can be represented as a sum of basic functions (mother wavelets), multiplied by the factors $C_к$ [3]:

$$s(t) = \sum_k C_k \psi_k(t) \quad (1)$$

Basic functions of wavelets should have zero average value throughout the entire interval and subside at infinity. This property sometimes results in names like "short waves". As wavelet activities are limited they should be capable of shifting through the entire space. It also has a scaling feature which is similar to alteration of harmonic frequency of Fourier-transform. These two features provide for the key advantage over sinusoids which is about more exact representation of local peculiarities of signals with potential steps and discontinuities.

A typical example of the basic wavelet family is provided in the formula $\psi(t)$

$$\psi(t) = 2^{-j/2}\psi(2^{-j}t - k) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \qquad j,k \in Z \quad (2)$$

Where

ψ - is some a certain decreasing function (for instance $|\psi(t) < C(1+|t|)^{-(1+s)}$) with a certain degree of smoothness and providing for $\int \psi(t) = 0$.

As the analysis performed on the basis of (2) is invariant in respect to integral shifts along the time axis and in respect to expansions that are multiple to 2. Solar radiation is represented as a function of a single argument (time). If so, then a definable function and wavelet functions will provide for performance of the algorithm of quick wavelet transform. The family of basic functions determined with the use of shifts and dimensional scaling will look as follows:

$$\psi(2^j t - k) = \sum_m h_\psi(m-2k)\sqrt{2}\psi(2^{j+1} - m)$$

$$\varphi(2^j t - k) = \sum_m h_\varphi(m-2k)\sqrt{2}\varphi(2^{j+1} - m)$$

Here,

h_ψ and h_φ are scale factor coefficients of the refinable and wavelet functions.

The key results in construction of basis were achieved by Ingrid Daubechies. The methods of construction and analysis of properties of several families of orthonormalized Daubechies wavelets $\psi_{j,k}$ are provided in [5]. Haar wavelet (db1) was proposed earlier in time and is based on the step-function:

$$\psi(t) = \begin{cases} 1 & \text{if } 0 \le t < \frac{1}{2} \\ -1 & \text{if } \frac{1}{2} \le t < 1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

This function does not comply with the above-mentioned smoothness requirements but offers a number of conveniences. Much smoother constructions were found only in the 1980s, particularly by Meyer (dmey wavelets) who associated outbursts with the theory of approximation. Daubechies proposed orthogonal wavelets which are focused on the finite interval of time. These wavelets have a well localized spectrum in the frequency domain. The domain of definition of orthonormalized Daubechies wavelets is wider than in Haar wavelets and they provide for a greater amount of expansion coefficients, yet maintaining the information value of input data.

It should be noted that just like Fourier series expansion, expansion in wavelet series makes a sequence of coefficients compliant with the continuous function. In our case, the expansion is determined by the following transforms:

$$W_\varphi(j,k) = \frac{1}{\sqrt{M}}\sum_t f(t)\varphi_{j,k}(t) \quad (4)$$

$$W_\psi(j,k) = \frac{1}{\sqrt{M}}\sum_t f(t)\psi_{j,k}(t) \quad (5)$$

Here t=0,1,2,…,M-1; j=0,1,2,…,J-1;k=0,1,2,….,2^{j-1}.

The factors described by formulas (4) and (5) are the approximation and detailing factors correspondingly. Approximation factors of the largest scale are input data for subsequent computations,

$$W_\varphi(J,n) = f(t)$$

where J is the largest scale.

IV. Computer Experiment

We will now move further to formulation of the method of discrete wavelet expansion using the example of global hourly solar output in the Baku city in 2012.

The wavelet function which used was Daubechies function of the 12th degree. The function's degree is dictated by the value of a vector of approximation factors.

Picture 2 shows schedules of factors produced with the use of Daubechies wavelet (dbwavelet) up to 12 levels - a two dimensional image has values of measurements for the analyzed time series horizontally, which can be taken for hours (from zero to 8760), and the so-called scale expressed in global radiation and showing which interval of time has certain features, vertically (in our case periodicity). The scale for each wavelet is clearly associated with the period of series under analysis. This is the key distinction and advantage of wavelet analysis compared to Fourier transform. As a result, we get a confirmation that the analyzed series has a spectrum in the shape of a single frequency and information about the moments of time (along x axis) where function

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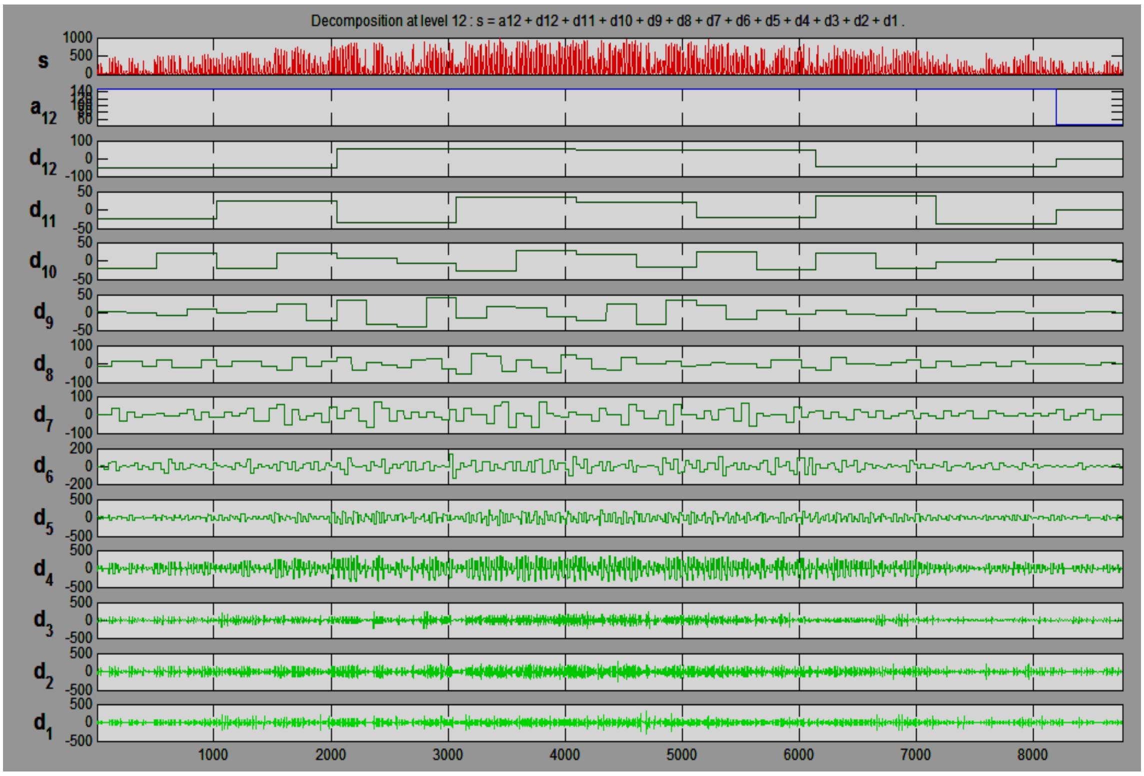

Image Daubechies wavelet expansion to the 12th level

features (time series) such as maximums and minimums are observed. The analyzed sequence of data is shown in the top part of the image (colored red). The maximum radiation value has been registered along the horizontal lines between points 4000 and 6000 (which corresponds to the period between mid-Julys to September). Subsequent schedules (colored in yellow) show factor expansion by levels. Factor alteration domain $d_1 \div d_5$ is [-500,+500],d_6:[-200,+200],$d_7 \div d_8$ and factor d_{12}: [-100,+100], the remaining factors are distributed in the interval of [-50,+50]. The wider the range of changes, the greater is the factor's contribution to expansion. As seen on the picture, the first 5 factors have high frequency changes. However, d_1 and d_3 are more regular (with almost permanent amplitudes) than factors d_4 and d_5. All of these factors have smaller amplitudes in the beginning, compared to the middle section. We can see that the structure of $d_1 \div d_5$ factor schedules clearly demonstrates a sinusoidal component of the signal even in the area of very insignificant factor values and allows for identification of the oscillation period. The image of skeleton lines shows the consistency of signal up to its actual disappearance and initial increase with subsequent decrease of the amplitude with time. Statistical features of the

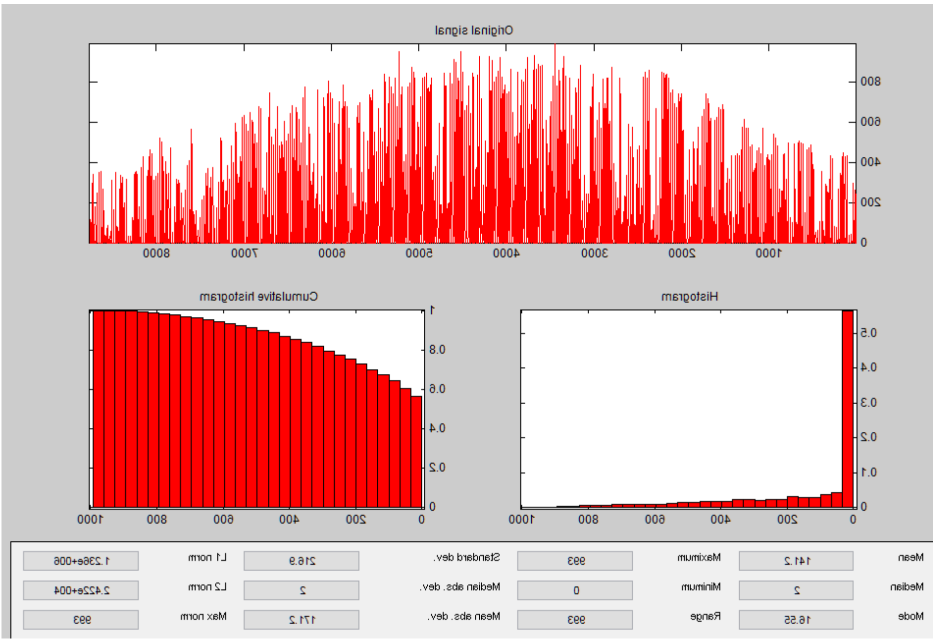

Image2 Statistical features of wavelet expansion factor in the initial series of solar radiation in Baku in 2012.

signal expansion (image 2) shows that zero (almost 50%) and non-zero signals make an almost identical contribution in generation of the original signal. Histograms show that signals with the amplitude of up to 400 has higher weight (approximately 40%) than signals with the amplitude of 400 to 800 (approximately 10%), while signals with the amplitude of over 800 could be ignored. This means that zero radiation during the investigated period totaled approximately 4380 hours, and radiation with the amplitude ranging from 400 ÷to 800 mJ/hour totaled 3504 hours. Average hourly radiation per annum (including zero values) totaled 141.2 mJ/hour.

Based on the analysis results, further research is performed using 5-level wavelet expansion. (image 3) As seen on image 3, d_1–d_4 factors have high alternating frequencies. If certain constants are deducted (the value of these constants are shown in dotted lines), we arrive at even higher frequency changes, which allows for detailed research of in-row peculiarities. On the left side we can see spectral expansion of restored and original factors. As we can see, they are almost identical. To get a clearer image for researching the properties of wavelet transform factors, we need to identify various intervals of factor consistency (image 4.) As seen from image 4, the consistency interval is wide between the values of 2800 to 7100 which corresponds to

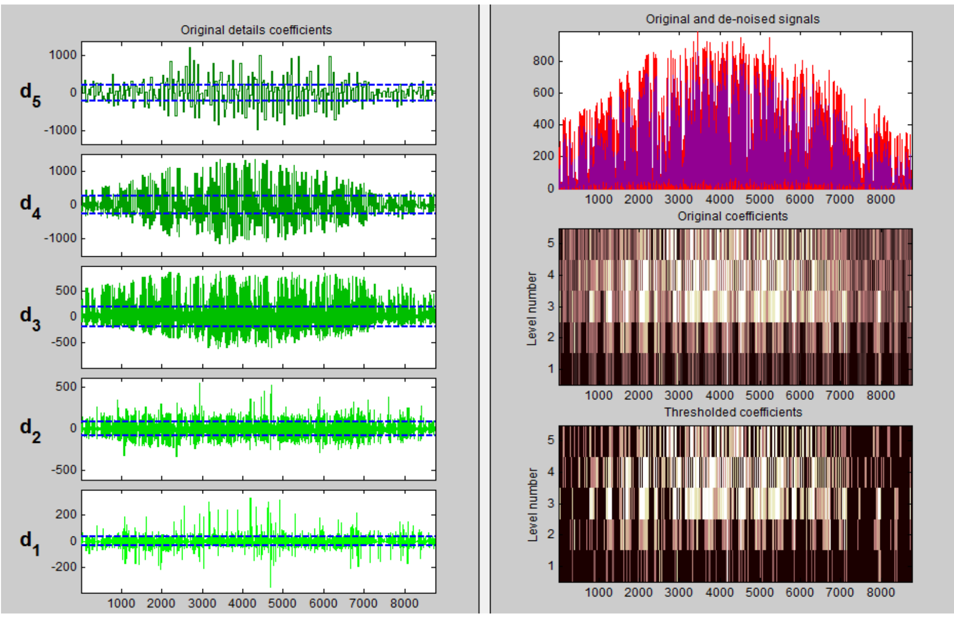

Image 3. Daubechies wavelet expansion of 5 level with the restored series (top left side with red lines)

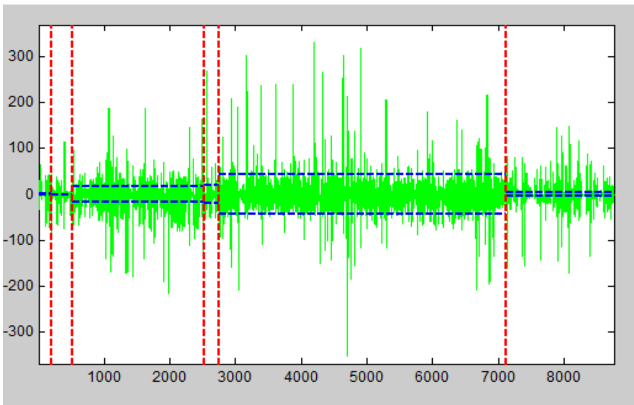
Image 4. Expansion of Daubechies factors in consistency interval.

April - September. Indeed, Baku constantly has high temperature and clear skies during this period.
A multi resolution analysis of function expansion by orthonormalized basis of outbursts is provided on image 5.

The division of the original series of solar radiation $s(t)$ with the use of dyadic expansion by wavelets is representation of s(t) signal as a sum of low-frequency components $\bar{s}(t)$ and the sum of scaled oscillating components w_j:

$$s(t) = \bar{s}(t) + \sum_{j\ge 0} w_j(2^j t) \qquad (5)$$

or as a more generalized representation

$$s(t) = \bar{s}_J(t) + \sum_{j\ge J} w_j(2^j t),$$

where

$$\bar{s}_J(t) = \bar{s}(t) + \sum_{0\le j\le J} w_j(2^j t).$$

The multitude of $\mathbf{V}_J$, built along the slow components $\bar{s}_J(t)$ is known as the resolution space. The multitude of $\mathbf{W}_J$, built along all w_J, is known as detail space and is considered as s function expansion to consistent layers of adjusting scale, each of which is more detailed than a previous one. This is why it is more convenient to represent the sum as a binary along time shifts (k) and the scales (j). As we move towards lower levels of detail, image representations reduce but opportunities for wavelet filtration of signal, noise reduction and efficient compression of signals emerge. Signal(s) in wavelet analysis is divided to approximate (A1) and detailed (D1). The approximate signal (A1) in its turn could be divided into two levels -

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approximate (A2) and detailed (D2). The same process is then repeated for the approximate portion (A3 and D3):
s = A1+D1 = A2+D2+D1 = A3+D3+D2+D1.

Output values of AJ (J=1,2…n) of a previous iteration are used at each new step of iteration as input data. DJ branches are remaining unused.

As one can see from image 4, approximate factors (a_1-a_5) contain zero amplitude up to level 5, which provides for harmonious changes of the wavelet factor d_1-d_5. Thus, if an original signal is restored on the basis of wavelet factor d_1-d_5, we shall begin with removing trends from the original series.

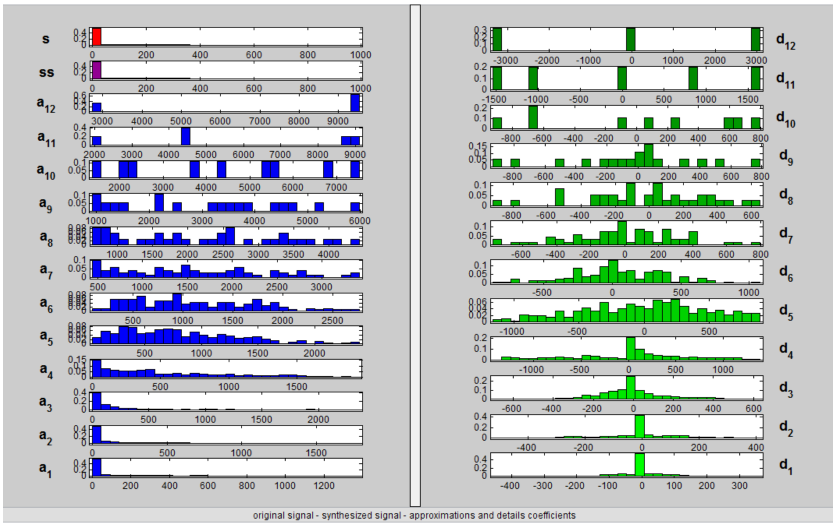

Image 5. Histograms of changes, original - s synthesized, ss approximated - a and wavelet transformed - d.

V. Conclusion

To sum up the research above, the following conclusions can be made:

Discrete wavelet expansion has been researched using the example of global hourly solar radiation in Baku in 2012. The research has revealed that schedules of global radiation changes have a periodical and non-linear nature. As a result, we first used Daubechies wavelet expansion to level 12 and then the research showed that expansion to level 5 was sufficient.

Analysis of expansion factor changes (approximated and detailed) has shown that the original series has interval 4 where the series is remaining unchanged. These conclusions comply with air temperature changes in Baku. Besides, it was demonstrated that where the original signal is restored on the basis of wavelet factor d_1-d_5, trends should first be removed from the original series.

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