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Analyze the Effect of Power Function on Image Sizes

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Abstract **– We apply the simple power function f(x) = x^n for various values of "n", both bigger than 1 and values between 0 and 1, on each pixel value of images. Some examples of the function include x^2, x^3, x^4, x^(1/2), x^(1/3) etc. We analyze the effect of this application both on the size of the images and how it changes the overall quality of the images. We use "Mathematica" code to run the experiment and all the code is provided in the paper for any interested readers to recreate the experiment.**

Keywords **– Zoom, Image Size, Picture, Power Function.**

I. Introduction

Data is something very useful in the present stage. Basically everybody has access to it, some however need it more than others. Some can be threatened by the amount of space needed and their available budget for it. Others are threatened not only by the budget, but by available technology and human patience. With this in mind many will rely on data compression as a way to meet certain requirements and affordable data handling.

Image/Video data compression (Group, n.d.) is a very critical technology for many operations in NASA (Seaman, 2006). The goal of NASA is not only to lower the data amount by compressing it, because this will save money and save space (Cooper & Lorenc, 2006). They also want to improve the time it takes to access that data as some people might need the data to be available right away, or their real time science may demand it.

In NASA, image compression (Prince, Franco, Salva, & Windolf, 2014) is used for three main reasons. First, compression saves space. NASA receives millions of bits of data each day that require a huge storage facility. NASA also has two or more backups for all the information they have. By using compression, NASA saves an enormous amount of hard drive space. Second, image/video compression saves transmission time. For example, the NASA Mars Rovers sends back pictures and data which can take up to years to reach Earth if uncompressed due to the massive distance between the two. Distance also has a direct relationship with transmission rate, for example Mars is 3.74×10^8 km away from Earth, as opposed to the moon that is only 4.05×10^3 km. Lastly, compression saves money by saving hard drive space and time (Rahman Z.).

In this experiment, we deal only a part of the image compression mechanism – power function. This is a very simple experiment to observe some of the effect, both in terms of quantity and quality of the images, when some simple function is applies in each pixel of an image. The experiment is conducted over 20 images. All of these images were taken by a digital camera with uniform setting. We try to consider all the different types of images: images of places, images of human beings, images of dark places, images during the day, images of sky, park etc.

Later we also conduct an observation on the quality of the images where we try to draw some conclusion on the quality related to the type of the images. We use Mathematica (Purdue University, n.d.) software to run our experiment and readers are encouraged to run this experiment in their own. Mathematica is a very powerful computer algebra system and more information can be found in "wolfram" website.

II. Experiment

The experiment was conducted to analyze the effects on an image when ran through a power function. Twenty photographs were taken in the New York City area. The camera took pictures under neutral settings, forgoing flash and other features. The photos were taken at different times of day to allow for variability in the photographs' lighting. Each image was then transported into the program Mathematica. There each images were ran through a program which applied the function X^n to each pixel in the image. The function was ran on each image for n values, 1,2,3, and 4. The experiment was then repeated for $X^{1/n}$ for n values 1,2,3 and 4. The aesthetic and spatial qualities of the images were analyzed.

III. Observation

Two things remained consistent for all images. The first was that the images ran through the function X^n where n ranged from 1 to 4 were considerably darker as n increased. The second was that the images ran through the function $X^{1/n}$ where n ranged from 1 to 4 were considerably lighter as n increased.

Compression ratio is the extent to which an image is compressed in size. Fig. 1 shows the relationship between image size and compression ratio.

$$\text{Compression Ratio} = \frac{\text{Uncompressed Size}}{\text{Compressed Size}}$$

Fig.1

The graphs of all images when X^n and $X^{1/n}$ are applied from n values 1 to 4 can be seen in figures 2 and 3.

Group one comprises of thirteen images: image001, image002, image003, image005, image007, image009, image011, image012, image014, image015, image016, image018, and image020. Group one's trend line increased in compression ratio for both X^n and $X^{1/n}$ from 1 to 4.

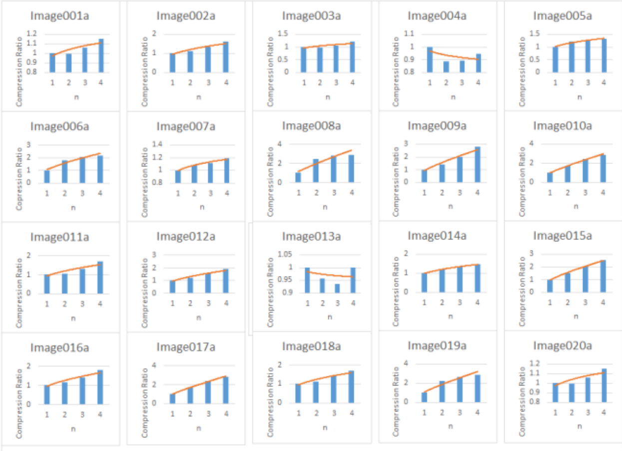

Fig.2. Trend lines for images 001 to 020 for X^n from n values 1 to 4.

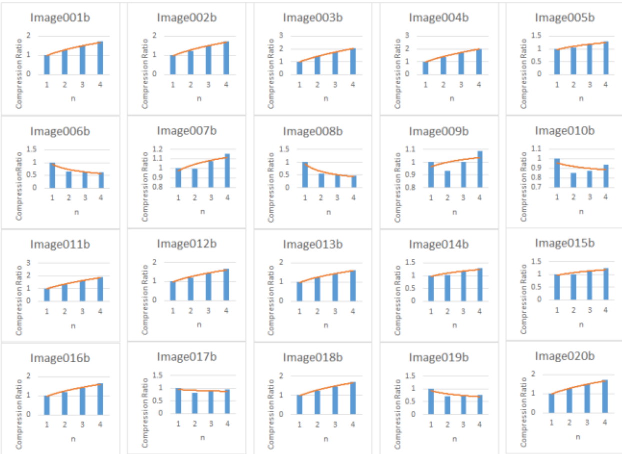

Fig.3. Trend lines for images 001 to 020 for $X^{1/n}$ from n values 1 to 4.

Group two comprises of five images: image006, image008, image010, image017, and image019. Group two's trend line increased in compression ratio for X^n from 1 to 4 and decreased in compression ratio from $X^{1/n}$ from 1 to 4.

The three comprises of two images: image004 and image013. Group three's trend line increased in compression ratio for X^n from 1 to 4 and decrease in compression ratio from $X^{1/n}$ from 1 to 4.

IV. Analysis

For the images' RGB (Image Compression, 2011) values, $X^{1/n}$ increases their RGB value because taking the root of a fraction produces a larger number. So as the RGB values approach 1, the images become lighter because 1 represents an RGB value that is completely white on the RGB scale.

For the images' RGB values, X^n decreases their RGB value because taking the power of a fraction produces a smaller number. So as the RGB values approach 0, the images become lighter because 0 represents an RGB value that is completely black on the RGB scale.

While there are differences between groups' one, two, and three's compression ratios, they do share similarities as well. If we look at their compression ratios for both X^n and $X^{1/n}$ most samples see an increase in compression ratio. When the image in transported into Mathematica and have X^1 applied to its pixel, the resulting image size is consistently larger than when X^n runs from 2 to 4. It is possible that some error is ran into when the image in transported into Mathematica, skewing the data.

V. Conclusion

There is a trend of image compression ratios increasing for both X^n and $X^{1/n}$ from 2 to 4 for most samples. But overall the experiment is inconclusive in a relationship between power functions and image size. The experiment did prove a relationship between power functions and image intensity. Roots of the pixels in an image cause the image to become lighter and powers of the pixels in an image cause them to become darker. Further research into image the effect on an image size when transported into a computer program is suggested.

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Dr. Tanvir Prince has a Ph.D. in Mathematics and is currently working as an Assistant Professor of Mathematics at Hostos Community College. He is interested in such areas as Topological Quantum Field Theory and Recreational Mathematics. He has recently become very interested in Mathematics Education. He has presented at numerous international and national conferences. He is regularly publishing articles focusing both on pure mathematics and mathematics education in various peer reviewed journals. Dr. Prince is also currently collaborating on projects that aim to expose community college students to mathematical research early in their academic careers. His other interests include traveling and cooking.

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