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Mathematical Models of "Mental Diseases" of Robots

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***Abstract* – On the basis of the mathematical concept of robot's emotions introduced in this paper we give a definition of robot mental disease. The paper offers a harmonic function of robot's emotion and describes the mathematical way of diagnosing diseases of robots, presents one of the ways of treatment of robot diseases on the basis of harmonic function of robot's emotion. Also we prove that any mental disease of robots can be treated and is curable.**

***Keywords* – Robot, Emotions, Education, Treatment, Mental Disease, Mathematical Modeling**

I. Introduction

The theory of human psychology says that emotions are responses of an organism to a certain stimulus [1]. For robots, we define this stimulus as a plot in the following definition.

Assume *t* is the time.

Definition 1. The function *S(t)* is a *plot* if it possesses the following properties:

1) the domain of *S(t)* is $t \in [0, t^*]$, $t^* > 0$, $t^* < \infty$;

2) *S(t)*>0 for any $t \in [0, t^*]$;

3) *S(t)* is the one-to-one function;

4) *S(t)* is the limited function.

In [2] we prove the theorem, saying that it is possible to model human and animal emotions for a computer (robot). However, psychological properties of emotions of eutherians and human beings are so complex, intricate and ambiguous that we decided to introduce a special mathematical definition for a robot emotion, abstracting from real human emotions and, at the same time, accumulating in this definition main properties of emotions of human and animals. We also abstract from the content of those emotions.

Definition 2. The function *f(t)*, satisfying the relation $f(t) = a(S(t),t)S(t)$ where *a(s(t),t)* is the arbitrary function, is a function of internal experience of the robot [3].

Assume that the plot *S(t)* generates internal experience of the robot.

Definition 3. The function of robot internal experience *f(t)* is the emotion *M(t)* if it satisfies the following conditions:

1) the domain of *M(t)* is $t \in [0, t^0]$, $t^0 > 0$;

2) $|M(t)|$ is the function differentiable at $(0, t^0)$;

3) $M(t)$ is the one-valued and continuous function at $[0, t^0]$;

4) $M(0) = 0$;

5) $M(t^0) = 0$;

6) there is the only point z in the domain such that $z \neq 0$, $z \neq t^0$ and $\left.\frac{d|M(t)|}{dt}\right|_{t=z} = 0$.

It is easy to see that, for instance, the function

$$M(t) = P\sin\left(\frac{\pi}{t^0}t\right) \qquad (1)$$

for $t \in [0, t^0]$, $P = const$ is an emotion.

A function of the form (1) is a harmonic emotion.

II. Mathematical Models

Let us introduce the following definitions.

Definition 4. The robot is healthy if its function of internal experience is an emotion.

Definition 5. The emotional robot is ill (unhealthy) if its function of internal experience does not satisfy at least one of the conditions defining an emotion.

This definition allows us to introduce a concept of robot's disease severity.

As the definition of emotion includes 6 conditions, the severity of the robot's disease can be characterized by *H* possessing the integral values from 1 to 6 and indicating the number of unsatisfied conditions deviating the internal experience function from the emotion. Assume that that the more severe the disease, the greater is *H*.

Definition 6. The disease symptom vector *X* is a vector with the numbers of unsatisfied emotion conditions according to the numbers of conditions in the definition of emotion.

Definition 7. The robot's disease with the symptom vector is a special case of the robot's disease with the symptom vector if all the elements of the symptom vector X_2 are among the elements of the symptom vector X_1.

Let us look at the examples of robots' diseases.

1. Let us select the function of robot's internal experience *f(t)* satisfying all the conditions of belonging to emotions, except for Condition 5, that is, there is a difference from an emotion which is described by the relation $M(t^0) \neq 0$. It is obvious that in this case the disease severity is equal to unit. Assume that the robot with this function of internal experience suffers from neurasthenia. It is obvious that for neurasthenia the disease symptom vector has the form *X*=(5).

2. Let us select the function of robot's internal experience *f(t)* satisfying all conditions of belonging to emotions, except for Conditions 5, 6. The function $f(t) = t$ can be an example of it. It is obvious that in this case the disease severity is equal to two. If the robot's function of internal experience does not satisfy the conditions of emotion at points 5, 6, then this robot suffers from psychasthenia. Here the psychasthenia symptom vector has the form *X*=(5, 6).

On the basis of values of the symptom vectors of neurasthenia and psychasthenia, we can draw a conclusion that they have a common feature which is unsatisfied

Condition 5, and, according to Definition 7, psychasthenia is a special case of neurasthenia.

Now let us consider the function of robot's internal experience which has the form

$$f(t) = P\sin\left(\frac{\pi}{t^0}t\right) - \frac{1}{2}P, \quad P = const, \quad P > 0, \quad t \in [0, t^0] \tag{2}$$

It is easy to notice that the robot's disease severity is equal to three, and the symptom vector satisfies to *X* = (3, 4, 5).

Let us discuss an issue of treatment of robots' diseases.

Assume $f(t)$ is a function of robot's internal experience which is not an emotion.

The task consists in effecting the internal experience function by the complementary function $g(t)$ knowing the emotion function $M(t)$ so that the resulting function becomes the emotion $M(t)$.

The aforesaid can be written down as the formula

$$M(t) = f(t) + g(t).$$

It is easy to see that this relation is equivalent to the equality

$$g(t) = M(t) - f(t). \tag{3}$$

Let us introduce two more definitions.

Definition 8. A pill is the function $g(t)$ satisfying to Eq. (3) where $M(t)$ is the emotion and $f(t)$ is the robot's internal experience function.

Definition 9. Treatment is the effect of a pill on the robot's internal experience function.

From (3) we can conclude that the pill exists always, i.e. any disease of the robot connected with robot's emotions is curable.

Let us give $M(t)$ in the form of the function (1) to set the following goal: estimate *P* in such a way that the internal experience function $f(t)$ would differ from $M(t)$ as little as possible.

Assume that there are integrals $\int_0^{t^0} \sin\left(\frac{\pi}{t^0}\tau\right) f(\tau)d\tau$ and $\int_0^{t^0} f^2(\tau)d\tau$.

It is obvious that in this case for finding *P* it is necessary to solve the optimizing problem:

$$\min_P J(P) = \min_P \int_0^{t^0} \left[P\sin\left(\frac{\pi}{t^0}\tau\right) - f(\tau)\right]^2 d\tau.$$

Finding the unconditional extremum of function $J(P)$ and expressing *P* from the obtained formulas on the basis of (1) we obtain the relation defining $P = P_{\min}$ which performs a minimum deviation of *M(t)* from the internal experience function:

$$P_{\min} = \frac{2\int_0^{t^0} \sin\left(\frac{\pi}{t^0}\tau\right) f(\tau)d\tau}{t^0} \tag{4}$$

Let us introduce the following definition.

Definition 10. The residual disease Ω after treatment is

$$\Omega = \int_0^{t^0} \left[P_{\min}\sin\left(\frac{\pi}{t^0}\tau\right) - f(\tau)\right]^2 d\tau.$$

Assume that there is the integral $\int_0^{t^0} f(\tau)d\tau$.

To estimate the value of *P* allowing us to treat the robot so that the elementary education obtained from the emotion is minimally different from the elementary education obtained from its experience function, we need to solve the following optimizing problem:

$$\min_P I(P) = \min_P \left[\int_0^{t^0} P\sin\left(\frac{\pi}{t^0}\tau\right)d\tau - \int_0^{t_0} f(\tau)d\tau\right]^2.$$

It is easy to see that $I(P)$ possesses the minimum value when the value of $P = P^*$ satisfies the relation

$$P^* = \frac{\pi\int_0^{t^0} f(\tau)d\tau}{2t^0}. \tag{5}$$

It is easy to notice that the value of $P = P^*$ given by (5) makes the minimum of $I(P)$ vanish, which indicates that it is possible to cure the robot in such a way that its elementary education from the internal experience function $f(t)$ which is not an emotion, becomes equal to the elementary education from the corresponding harmonic emotion.

Thus, if the robot's internal experience function the can be described by mathematical formulas, then any robot's mental diseases can be treated by means of harmonic emotion in terms of equality of elementary educations implied by the experience function and harmonic emotion. In this case the pill is estimated by the formula

$$g(t) = P^*\sin\left(\frac{\pi}{t^0}t\right) - f(t).$$

It is obvious that depending on a form of the function $f(t)$ the pill $g(t)$ can be an alternating function.

Let us introduce Definition 11.

Definition 11. A disease severity coefficient is the value α, satisfying the expression

$$\alpha = \left|\frac{\int_0^{t_0} f(t)dt}{\int_0^{t_0} P_{\min}\sin\left(\frac{\pi}{t_0}t\right)dt}\right| = \frac{\pi\left|\int_0^{t_0} f(t)dt\right|}{2t_0 \mid P_{\min} \mid}. \tag{6}$$

It is easy to see that α is a dimensionless value.

Assume that the more is $|\alpha - 1|$, the more severe is the robot's disease.

The definitions and formulas given above were applied in the experiments with practicing doctors working on estimation of disease severity of psychasthenic patients. 24 patients took part in those experiments.

The essence of the experiments is as follows. First of all, we set up the time of emotion duration t_0 = 5 sec. Then the patient was given some information (in terms of the robot theory, they receive the plot) able to initiate insignificant sparing internal experience (in terms of the robot theory, the internal experience function $f(t)$ is formed). Then we fixed the variation of the patient's internal experience within the given time interval $[0, t_0]$. $P_{\min}$ was estimated by (4), and then α was estimated by (6).

The analysis of the experiment results showed that the greatest value of $|\alpha - 1|$ corresponds to the most severe disease state within 87,5% of the patients. At that, $\alpha \in [1{,}18; 1{,}52]$.

The experiments demonstrated that the theory of "mental diseases" of robots offered in the present paper can be applied to a first approximation in description of the psychasthenia of human. It indicates adequacy and reasonability of mathematical models for "mental diseases" of robots offered in the paper.

III. Conclusion

So, the present paper makes another attempt to introduce the formalized mathematical concept of "robot's mental disease" where this concept is based on the degree of deviation of internal experience function from emotion.

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Author's Profile

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Doctor of Technical sciences, Professor of Department of Control Flows and Information Security, Perm State University (Russia), Honored worker of Higher Vocational School of Russian Federation. Prof. Pensky was born in April, 6, 1959. His scientific interests include internal ballistics of building ordnance and math theory of robots' emotion and mental diseases. He is an author of 110 papers including 7 books. He has more than 35 patents for inventions and applicable models, certificates of Rospatent and Branch collection of algorithms and software (Russian Federation).