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Dynamics Response of Simply Supported Axial Force Euler-Bernoulli Beam Subjected to Partially Distributed Moving Load

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Abstract: **The dynamic response of simply supported axial force Euler-Bernoulli beam subjected to uniform partially distributed moving loads was examined. The governing fourth order partial differential equation for moving loads was first reduced to second order differential equation by assume a solution in form of series solution and numerically using maple software in order to determine the behaviour of the system under consideration. The effect of mass of the beam on both the moving force and the moving mass was examined. It was observed that as the mass of the beam increases, the transverse displacement (deflection) is equally increases for both the moving force and the moving mass with respect to the tensile force and the compressive force. It was also observed that as the axial force increases the displacement decreases under the action of the moving force and the moving mass for the tensile force while in the case of the compressive force, as the axial force increases the displacement is equally increases.**

Keywords: **Axial Force, Beam, Partially Distributed, Simply Supported**

I. INTRODUCTION

Bridges and viaducts for high speed trains are subject to demanding dynamic loads and much larger and potentially dangerous effects arise from dynamic resonance for speeds above 220 km/h. The basic dynamic response for a moving load on a simply supported beam is known from classical solutions among others by Timoshenko. The response of dynamic effect of moving loads was not known until mid-nineteenth century as a result of the Dee bridge disaster which was a rail accident in England that occurred on 24th May, 1847 precisely in Chester. This terrible incident caused tremendous human loss and created a lot of excitement in the field of civil engineering and this is what makes the study of dynamic structures with moving loads to be an interesting subject of discussion to the engineers and those in the field of physics and applied mathematics.

Akinpelu [1] examined the response of viscously damped Euler-Bernoulli beam to uniform partially distributed moving loads. The author observed that the beam has more than one mode of vibration with each mode having a different natural frequency. It was discovered that as the mass of the load increases the amplitude is also increases and the value of the magnification factor occurs for a value ω less than one for $\varepsilon = 0.1$.

The dynamic analysis of pre-stressed Euler-Bernoulli beam was considered by I.A. Adetunde [2]. The finite difference method was used to solve the pertinent initial boundary value problem numerically and he concluded that the deflection of the moving mass pre-stressed Euller-Bernoulli beam was greater than those of the moving force.

Dynamic response of loads on viscously damped axial force Rayleigh beam was studied by I. A. Adetunde and Baba Seidu [3]. The theory is based on orthogonal functions and the results indicates that the governing differential equation can be transformed into a series of couple ordinary differential equations which is the solution for the corresponding moving distributed force. The resulting governing differential equation is solved numerically by finite central difference method where he concluded that the deflection due to moving mass is greater than that due to moving force.

The response of initially stressed Euler-Bernoulli beam with an attached mass to uniform partially distributed moving loads was carried out by Adetunde, Akinpelu and Gbadeyan [4]. The resulting coupled partial differential equation is solved using finite difference method. It was found that the response amplitude increases

as mass of the load increases under a moving force problem and also that the response amplitude increases with an increase in the mass of the load for various values of time t and ε.

Axial loaded beams on elastic foundation under moving harmonic loads was investigated by Kim Seong-Min [5]. The work was based on the vibration and stability of an infinite Euler-Bernoulli beam on a Winkler foundation by applying a static axial force and a moving load with either constant or harmonic amplitude variations to excite the system and some important results were obtained from the process of changing relative parameters.

Jaiswal and Iyengar [6] examined dynamic response of a beam on elastic foundation of finite depth under a moving force. The dynamics of the infinite beam on a finite elastic foundation base subjected to a moving load was studied. Again, the effects of various parameters such as foundation mass, velocity of the moving load, damping and axial force on the beam were investigated.

Response of beam on visco-elastic foundation to moving distributed load was examined by Roman Bogacz and Wlodzimierz Czyczula [7] where the dynamical problems caused by a distributed load which was acting on a beam on an elastic foundation at a moving velocity were studied. The load was represented by the Heaviside function and by a moving load harmonically distributed in space.

Dynamic behavior of non-uniform Bernoulli-Euler beams subjected to concentrated loads travelling at varying velocities was examined by Oni and Omolofe [8]. In this study, Generalized Galerkin's method was used to solve the problem. The method is very versatile and capable of tackling this class of problem for any of the classical boundary conditions often encountered in structural design. The effects of various parameters such as inertia, foundation moduli and axial force on the dynamical system were investigated. It was found that generally, as foundation moduli and axial force were increasing, the response amplitudes of vibrating system decreasing. It was also found that the critical speed for moving mass problem is smaller than that of the moving force problem, hence the resonance is reached earlier in the moving mass problem.

Simply supported beam Response on elastic foundation carrying repeated rolling concentrated loads was examined by Shahin Nayyeri Amiri and Mbakisya Onyango [9]. Fourier sine transformation was adopted to solve the problem. The cases of the response of the beam to loads of different and equal magnitude were studied. Numerical examples were given in order to determine the effects of various parameters on the response of the beam. It was equally observed that within the range of values considered, an increase in speed parameter results in the increase in dynamic deflections; given all other parameters, a decrease in distance parameter also results in the increase in dynamic deflections. Here, it was found that the effect of increasing elastic foundation parameter is an increase to both the fundamental frequencies and the critical speeds of the beam.

Influence of foundation and axial force on the vibration of thin beam under variable harmonic moving load was investigated by Awodola [10]. In this study, the finite Fourier sine transformation was used to reduce the fourth order partial differential equation to a second order partial differential equation. The reduced equation was then solved using the Laplace transformation. Numerical analysis shown that the transverse deflection of the thin beam resting on a uniform foundation under the action of a variable magnitude harmonic load moving with variable velocity decreases as the foundation constant increases. It was also shown that as the axial force increases, the transverse deflection of the beam decreases.

Willis [11] worked on the problem of elastic beam under the action of the moving loads where the assumption that the mass of the beam is greater than that of the load was made and approximate solution of the problem was as well obtained.

Transient Responses of beam with elastic foundation supports under moving wave load excitation was investigated by Yi Wang, Yong Wang, Biaobiao Zhang and Steve Shepard [12]. Newmark integration method for numerical simulation was applied to solve the problem. The results were verified by using commercial ANSYS package through modeling the beam with elastic foundation supports excited by moving point load at a constant speed. The effects of the parameters on the dynamic behavior of the beam or both moving point load and pressure wave load were evaluated. Numerical examples were given in order to determine the effects of various parameters on the response of the beam. Within the range of values considered, an increase in velocity parameter of the moving loads results in the increase in dynamic deflections. This study contains useful contribution to literature on moving loads problems related to transportation system. Couple with this, the technique and the findings can offer a good basis in practical applications such as acoustic wave beam sensor development by means of force reconstruction method.

It is obvious that during flight, aircraft or air plane is always subjected to a wide range of temperature variation which causes tensile or compressive axial force in the beams when they are fixed in the plane direction, that is why the axial beams are usually incorporated and highly essential in the design of aero planes, hence, there is need for the study of moving load which is of great importance in the field of transportation and the examples of the structural element which support the moving masses are roadways, bridges, runways, overhead cranes to mention a few.

To this end, when the distributed loads are moving on a structure, they produce greater deflection and stresses than when such loads are static, thus, the analysis of the behavior of beam under moving loads are of great importance to those in the field of structural engineering, physics and applied mathematics.

This paper examined the dynamic response of simply supported axial force Euler-Bernoulli Beam subjected to partially distributed moving load and the main objectives of the study are stated thus:

i. To reduce the fourth order partial differential equation to second order differential equation

ii. To solve the second order differential equation analytically in series form using maple software.

iii. To examine the effects of mass of the load on the displacement response of simply supported beam under the action of moving force and moving mass.

iv. To determine the effects of axial force on the moving force and the moving mass.

v. To make comparison between the displacement response of simply supported due to moving force and moving mass.

vi. To make comparison between the displacement response of simply supported due to tensile and compressive forces.

II. THE GOVERNING EQUATION

Considering the dynamics response of simply supported axial force Euler-Bernoulli beam subjected to partially distributed moving load equation which is given by:

$$\frac{\partial^2}{\partial^2 x}\left(EI(x)\frac{\partial^2 y}{\partial x^2}-Sy\right)+m(x)\frac{\partial^2 y}{\partial t^2}=F(x,t) \quad (2.1)$$

Subjected to the following boundary and initial conditions:

$$y(0,t)=y^{''}(0,t)=0 \quad (2.2)$$

$$y(L,t)=y^{''}(L,t)=0, \quad (2.3)$$

$$y(x,0)=y^{'}(x,0)=0 \quad (2.4)$$

Where; E is the young modulus, I is the moment of inertial, EI is the flexural rigidity, y is the deflection/transverse displacement, x is the Coordinate, t is the time, M is the mass of the load or beam, m is the mass per unit length, g is the acceleration due to gravity, H is the Heaviside unit function, Ɛ is the length of the beam, $\xi = vt + \frac{\varepsilon}{2}$ is the fixed length of the load, V is the constant velocity of the load, S is the axial force, and F (x, t) is the applied force. The applied force per unit length $F(x,t)$ is the uniform partially distributed moving load defined as

$$F(x,t)=$$
$$\left(Mg-M(\frac{\partial^2 y}{\partial t^2}+2v\frac{\partial^2 y}{\partial x\partial t}+v^2\frac{\partial^2 y}{\partial x^2}\right)\left(H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})\right) \quad (2.5)$$

III. Method of Solution

Substitute equation (2.5) into equation (2.1) to have

$$\frac{\partial^2}{\partial^2 x}\left(EI(x)\frac{\partial^2 y}{\partial x^2}-Sy\right)+m(x)\frac{\partial^2 y}{\partial t^2}=$$
$$\left(Mg-M\frac{\partial^2 y}{\partial t^2}+2v\frac{\partial^2 y}{\partial x\partial t}+v^2\frac{\partial^2 y}{\partial x^2}\right)\left(H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})\right) \quad (3.1)$$

To reduce the fourth order partial differential equation in (3.1) to second order ordinary differential equation, the solution is then assumed as

$$\text{Y (x, t)} = \sum_{i=1}^{n} Y_i(x)T_i(t) \quad . \quad (3.2)$$

Substitute equation (3.2) into equation (3.1) to get;

$$EI(x)\sum_{i=1}^{n}Y_i^{iv}(x)T_i(t)-S\sum_{i=1}^{n}Y_i^{''}(x)T_i(t)+m(x)\sum_{i=1}^{n}Y_i(x)\ddot{T}_i(t)$$
$$\left(Mg-M\sum_{i=1}^{n}Y_i(x)\ddot{T}_i(t)+2MV\dot{Y}_i(x)\dot{T}_i(t)-mV^2Y_i^{''}(x)T_i(t)\right)\left(H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})\right) \quad (3.3)$$

Multiply equation (3.3) by Y_j (x) and integrate along the length of the beam to have

$$\int_0^L [EI(x)\sum_{i=1}^{n}y_i^{iv}(x)T_i(t)-S\sum^{n}y_i^{''}(x)T_i(t)+m(x)\sum_{i=1}^{n}y_i(x)\ddot{T}_i(t)]y_j dx=$$
$$\text{Mg}\int_0^L y_i(x)\ [H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]dx$$
$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_0^L y_i(x)y_j(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]dx$$
$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_\circ^L y_i^{'}(x)y_j(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]dx$$
$$-MV^2\sum_{i=1}^{n}T_i(t)\int_0^L y_i^{''}(x)y_j(x)[H(x-\xi+\frac{\varepsilon}{2})-H(x-\xi-\frac{\varepsilon}{2})]dx \quad (3.4)$$

Using the orthogonality condition $\int_{x_1}^{x_2} x_i(x)\delta(x-x_i)dx = x-x_i = x_j(x)$ provided that $x_0 < x_1 < x_2$ Then, equation (3.4) can be written as

$$EI\sum_{i=1}^{n}\int_0^L y_i^{iv}(x)T_i(t)y_j(x)dx + s\sum_{i=1}^{n}\int_0^L y_i^{ii}(x)T_i(t)y_j(x)dx + m(x)\sum_{i=0}^{n}\int_{\circ}^{L} y_i(x)\ddot{T}_i(t)y_j(x)dx$$

$$= Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_j(x)dx - M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i(x)y_j dx - 2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i^{'}(x)y_j dx - MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i^{''}(x)y_j dx \qquad (3.5)$$

For free vibration, $EIy^{iv} = m\omega^2 y$ so that equation (3.5) becomes

$$m\omega^2 T_i(t) + ST_i(t) + m(x)\ddot{T}_i(t) =$$

$$Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_j(x)dx - M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i(x)y_j dx - 2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i^{'}(x)y_j dx - MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i^{''}(x)y_j dx \qquad (3.6)$$

For simply supported axial displacement;

$$Y_i(x) = \sqrt{\frac{2}{L}}\sin\left(1 + \frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi x}{L}\right) \qquad (3.7)$$

Differentiate equation (3.7) and substitute in (3.6) accordingly to get

$$m(x)\ddot{T}_i(t) + (m\omega^2 + S)T_i(t) = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{j\pi}{L}\right)dx$$

$$- M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{j\pi}{L}\right)\right)dx$$

$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi}{L}\right)\cos\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{j\pi x}{L}\right)\right)dx$$

$$+MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left(\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi}{L}\right)\right)^2\sin\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{\frac{1}{4}}\left(\frac{j\pi x}{L}\right)\right)dx$$

(3.8)

For convenience, let P = $\left(1+\frac{S}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{\frac{1}{4}}$ and Q = $\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{\frac{1}{4}}$ then, (3.8) becomes

$$\ddot{T}_i(t) + (\omega^2 + \frac{S}{m})T_i(t) = \frac{Mg}{m}\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin Q\frac{j\pi}{L}xdx$$

$$-\frac{M}{m}\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin P\frac{i\pi x}{L}\left(\sqrt{\frac{2}{L}}\sin Q\frac{j\pi x}{L}\right)dx$$

$$-\frac{2MV}{m}\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\left(\sqrt{\frac{2}{L}}P\frac{i\pi x}{L}\cos P\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin Q\frac{j\pi x}{L}\right)dx$$

$$+\frac{MV^2}{m}\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\left(\sqrt{\frac{2}{L}}\left(P\frac{i\pi x}{L}\right)^2\sin P\frac{i\pi x}{L}\right)\left(\sqrt{\frac{2}{L}}\sin Q\frac{j\pi x}{L}\right)dx$$

(3.9)

Applying trigonometry identity on equation (3.9) so that

$$\ddot{T}_i(t) + (\omega^2 + \frac{S}{m})T_i(t) = \frac{Mg}{m}\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\ Q\ \frac{j\pi}{L}\ xdx$$
$$-\frac{2M}{mL}\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} -\frac{1}{2}\left(\cos\ P\ \frac{\pi x}{L}(i+j) - \cos\ Q\ \frac{\pi x}{L}(i-j)\right)dx$$
$$-\frac{4MV}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} P\frac{i\pi}{L}\left(\frac{1}{2}(\sin P\frac{\pi}{L}x(i+j) - \sin Q\frac{\pi x}{L}(i-j)\right)dx$$
$$+\frac{2MV^2}{mL}\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}(P\frac{i\pi}{L})^2\left(-\frac{1}{2}[\cos P\frac{\pi x}{L}(i+j) - \cos Q\frac{\pi x}{L}(i-j)\right)dx \quad (3.10)$$

Then equation (3.10) becomes

$$\ddot{T}_i(t)+(\omega^2+\frac{S}{m})T_i(t) = -\frac{Mg}{m}\sqrt{\frac{2}{L}}(\frac{L}{Qj\pi})\left[\cos Qj\frac{\pi x}{L}\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} - \frac{2M}{2mL}\sum_{i=1}^{n}\ddot{T}_i(t)\left[(\frac{L}{P\pi(i+j)})\sin P\frac{\pi}{L}(i+j)x - \frac{L}{Q\pi(i-j)}\sin Q\frac{\pi}{L}(i-j)x\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}$$
$$-\frac{4MV}{2mL}(P\frac{i\pi}{L})\sum_{i=1}^{n}\dot{T}_i(t)\left[\frac{-L}{P\pi(i+j)}\cos P\frac{\pi}{L}(i+j)x + \frac{L}{Q\pi(i-j)}\cos Q\frac{\pi}{L}(i-j)x\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}$$
$$-\frac{2MV^2}{2mL}(\frac{pi\pi}{L})^2\sum_{i=1}^{n}T_i(t)\left[\frac{L}{p\pi(i+j)}\sin P\frac{\pi}{L}(i+j)x - \frac{L}{Q\pi(i-j)}\sin Q\frac{\pi}{L}(i-j)x\right]_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} \quad (3.11)$$

$$\ddot{T}_i(t)+(\omega^2+\frac{S}{m})T_i(t) = \left[\frac{-Mg\sqrt{2L}}{mL}\left((\frac{L}{Qj\pi})\cos Qj\frac{\pi}{L}(\xi+\frac{\varepsilon}{2}) - \cos Qj\frac{\pi}{L}(\xi-\frac{\varepsilon}{2})\right)\right]$$
$$+(\frac{M}{mL})(\frac{L}{P\pi})\sum_{i=1}^{n}\ddot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\sin P\frac{\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\sin Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\\ -\left(\frac{1}{(i+j)}\sin P\frac{\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\sin Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right]$$
$$+\frac{2MV}{mL}(P\frac{i\pi}{L})(\frac{L}{P\pi})\sum_{i=1}^{n}\dot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\cos\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\\ -\left(\frac{1}{(i+j)}\cos\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right]$$
$$-\frac{MV^2}{mL}(P\frac{i\pi}{L})^2(\frac{L}{P\pi})\sum_{i=1}^{n}T_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\sin\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\sin Q\frac{\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})\right)\\ -\left(\frac{1}{(i+j)}\sin\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2}) - \frac{1}{(i-j)}\sin Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right] \quad (3.12)$$

Further simplification gives

$$\ddot{T}_i(t)+(\omega^2+\frac{s}{m})T_i(t)=-\frac{Mg\sqrt{2L}}{Qmj\pi}\left(-2\sin Q\frac{j\pi}{L}\xi\sin Qj\frac{\pi\varepsilon}{2L}\right)$$
$$+\frac{M}{Pm\pi}\sum_{i=1}^{n}\ddot{T}_i(t)\left[\begin{array}{l}\left((\frac{1}{(i+j)}\sin P\frac{\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i+j)}\sin P\frac{\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)\\-\left((\frac{1}{(i-j)}\sin\frac{Q\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i-j)}\sin\frac{Q\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right]$$
$$+\frac{2MVi}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\cos\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i+j)}\cos\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)\\-\left(\frac{1}{(i-j)}\cos\frac{Q\pi}{L}(i-j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i-j)}\cos Q\frac{\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right]$$
$$-\frac{MV^2Pi^2\pi}{mL^2}\sum_{i=1}^{n}T_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}\sin\frac{p\pi}{L}(i+j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i+j)}\sin\frac{p\pi}{L}(i+j)(\xi-\frac{\varepsilon}{2})\right)\\-\left(\frac{1}{(i-j)}\sin Q(i-j)(\xi+\frac{\varepsilon}{2})-\frac{1}{(i-j)}\sin\frac{Q\pi}{L}(i-j)(\xi-\frac{\varepsilon}{2})\right)\end{array}\right] \quad (3.13)$$

$$\ddot{T}_i(t)+(\omega^2+\frac{S}{m})T_i(t)=\frac{2Mg\sqrt{2L}}{Qmj\pi}\left(\sin Qj\frac{\pi}{L}\xi\sin Qj\pi\frac{\varepsilon}{2L}\right)$$
$$+\frac{M}{Pm\pi}\sum_{i=1}^{n}\ddot{Ti}(t)\left[\begin{array}{l}\frac{1}{(i+j)}(2\cos\frac{p\pi}{L}(i+j)\xi\sin\frac{P\pi}{2L}(i+j)\varepsilon)\\-\frac{1}{(i-j)}(2\cos Q\frac{\pi}{L}(i-j)\xi\sin Q\frac{\pi}{2L}(i-j)\varepsilon)\end{array}\right]$$
$$+\frac{2MVi}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}(-2\sin\frac{p\pi}{L}(i+j)\xi\sin\frac{P\pi}{2L}(i+j)\varepsilon\right)\\-\left(\frac{1}{(i-j)}(-2\sin Q\frac{\pi}{L}(i-j)\xi\sin\frac{Q\pi}{2L}(i-j)\varepsilon)\right)\end{array}\right]$$
$$-\frac{MV^2Pi^{2\pi}}{mL^2}\sum_{i=1}^{n}T_i(t)\left[\begin{array}{l}\left(\frac{1}{(i+j)}(2\cos\frac{p\pi}{L}(i+j)\xi\sin\frac{P\pi}{2L}(i+j)\varepsilon\right)\\-\left(\frac{1}{(i-j)}(2\cos\frac{Q\pi}{L}(i-j)\xi\sin\frac{Q\pi}{2L}(i-j)\varepsilon)\right)\end{array}\right] \quad (3.14)$$

Substitute for P and Q into equation (3.15) and rearranged to have;

$$\ddot{T}_i(t)+(\omega^2+\frac{s}{m})T_i(t)=\frac{\sqrt{8L}}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}j\pi}Mg\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{j\pi}{L}\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{j\pi}{2L}\varepsilon$$
$$-\frac{2M}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\pi}\sum_{i=1}^{n}\ddot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\-\frac{1}{(i+j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$+\frac{4MVi}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$+\frac{2MV^2\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}i^2\pi}{mL^2}\sum_{i=1}^{n}T_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right) \quad (3.15)$$

IV. Numerical Analysis and Discussion of Result

In this paper, two cases were considered

Case 1: The Moving Force

In the case of moving force, the inertia effect of the moving load is neglected from equation (3.15) and only the first term on the right hand side of the equation is retained, hence equation (4.1) is obtained

$$\ddot{T}_i(t)+(\omega^2+\frac{s}{m})T_i(t)=\frac{\sqrt{8L}}{m(1+\frac{S}{EI}(\frac{L}{j\pi})^2)^{1/4}j\pi}Mg\sin(1+\frac{S}{EI}(\frac{L}{j\pi})^2)^{1/4}\frac{j\pi}{L}\xi\sin(1+\frac{S}{EI}(\frac{L}{j\pi})^2)^{1/4}\frac{j\pi}{2L}\varepsilon \quad (4.1)$$

Case 2: The Moving Mass

In this case, both the inertia effect of the moving load and the force effect were taken into consideration resulted into equation (4.2) as follows;

$$\left[1+\frac{2M}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\pi}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)\right]\ddot{T}_i(t)$$

$$+\frac{4MVi}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$\left[(\omega^2+\frac{s}{m})-\frac{2MV^2\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}i^2\pi}{mL^2}\sum_{i=1}^{\infty}\frac{1}{(i-j)}\left(\begin{array}{l}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i-j)}\cos\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)\right]T_i(t)$$

$$=\frac{\sqrt{8L}}{m\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}}Mg\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}j\frac{\pi}{L}\xi\sin\left(1+\frac{S}{EI}\left(\frac{L}{j\pi}\right)^2\right)^{1/4}\frac{j\pi}{L}\varepsilon \quad (4.2)$$

Equations (4.1) and (4.2) were solved numerically by using the the following numerical values: ε= 0.1, m=70, V=3.3, g= 9.8, π =3.142, $\lambda = \frac{n\pi}{2}$, ε=0.1, $E = 2.07\times10^{11}$, m=70, $I = 12.27\times10^{-6}$, $\omega = \sqrt{\frac{\lambda^4 EI}{m}}$, L=10, α=10, θ=0.1, $\xi = vt + \frac{\varepsilon}{2}$

Figures 4.1 to 4.4 are displacement against time for moving force and moving mass for the tensile and the compressive forces with various values of mass M. It was observed that the displacement increases as M increases for both the tensile and the compressive forces for the moving force and the moving mass.

Figures 4.5 to 4.8 are displacement against time for moving force and moving mass for the tensile and the compressive forces with various values of axial force, S. Figures 4.5 and 4.7 shown that displacement decreases as S increases for the tensile force for the moving force and the moving mass respectively, but in the case of the compressive force, figures 4.6 and 4.8 shown that displacement increases as S increases for the compressive force for the moving force and the moving mass respectively.

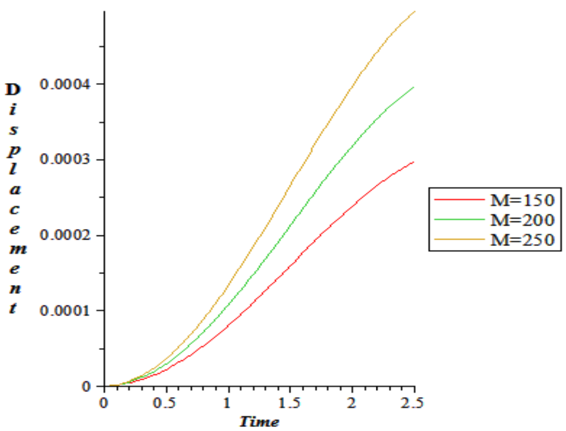

Figure 4.1: Displacement against time for moving force with tensile force for various values of M for Box shape

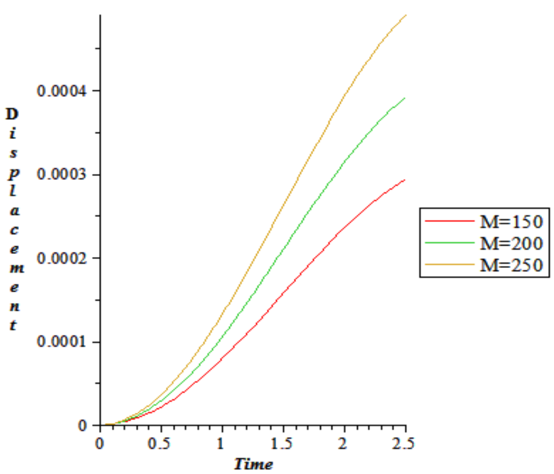

Figure 4.2: Displacement against time for moving force with compressive force for various values of M for box shape

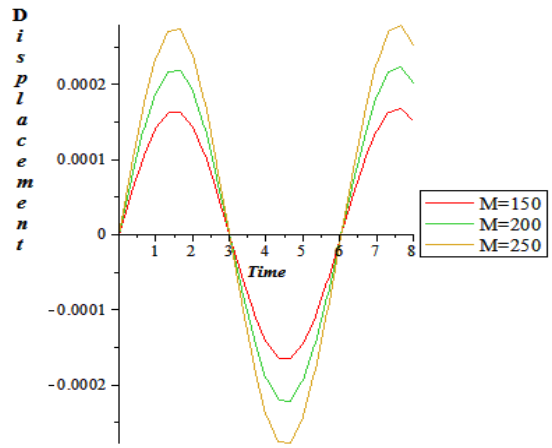

Figure 4.3: Displacement against time for moving mass with tensile force for various values of M for box shape

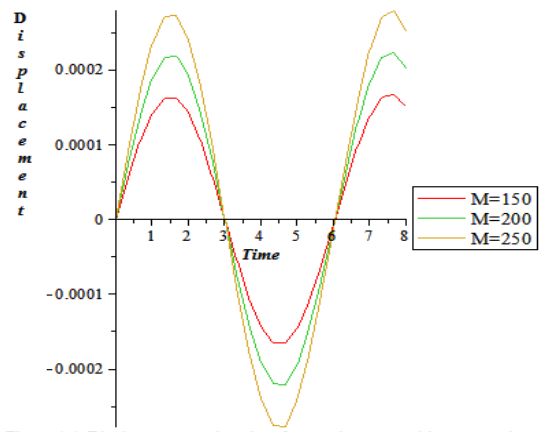

Figure 4.4: Displacement against time for moving mass with compressive force for various values of M for box shape

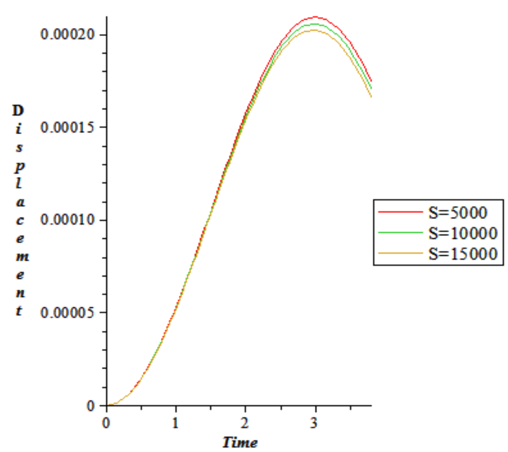

Figure 4.5: Displacement against time for moving force with tensile force for various values of S for box shape

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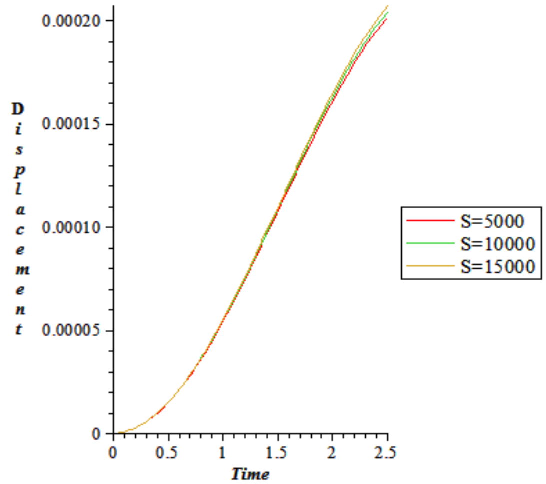

Figure 4.6: Displacement against time for moving force with compressive force for various values of S for box shape

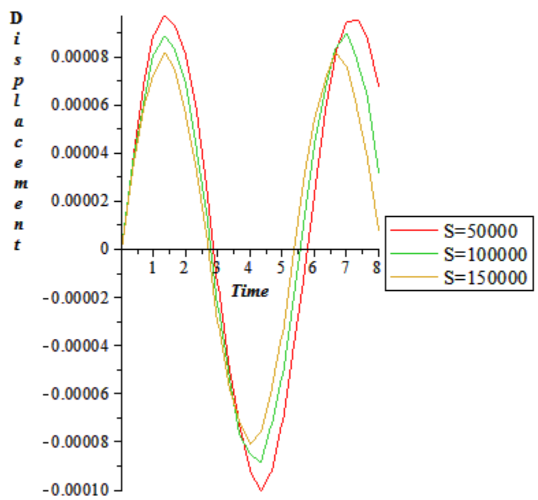

Figure 4.7: Displacement against time for moving mass with tensile force for various values of S for box shape

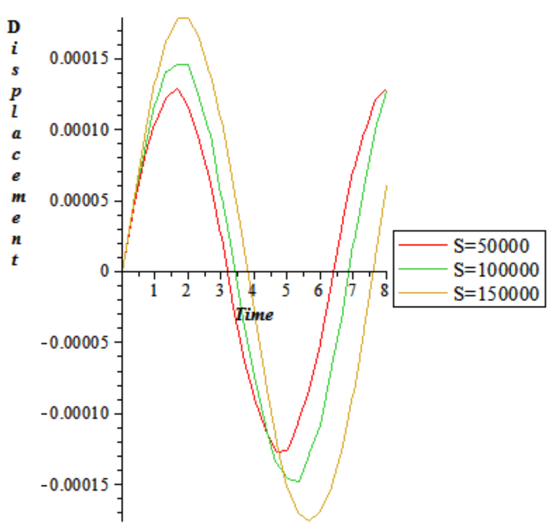

Figure 4.8: Displacement against time for moving mass with compressive force for various values of S for box shape

CONCLUSION

The Dynamic response of simply supported axial force Euler-Bernoulli Beam subjected to partially distributed moving loads was investigated. The governing fourth order partial differential equation for moving loads was first reduced to second order differential equation by assume a solution in form of series solution and numerically using maple software in order to determine the behaviour of the system under consideration.

The effect of axial force and mass of the beam on the moving force were examined accordingly with respect to both the tensile and compressive forces. It was observed that as the mass of the beam increases, the transverse displacement is also increases for both the tensile and the compressive forces under the actions of the moving force and that of the moving mass. It was equally observed that as the axial force is increasing, the displacement is decreasing for the tensile force for the moving force while in the case of compressive force, as the axial force increases, the displacement is also increases for both the moving force and the moving mass.

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