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Some Properties of the Strong Chain Equivalent Set and Strong Chain Recurrent Set

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Abstract: **In this paper, we present some properties of strong chain equivalent set and strong chain recurrent set with the continuous maps of a metric space into itself. Further we show the connection between chain recurrent point and strong chain recurrent point. Finally, we give an example to show this result.**

Keywords: **Chain Equivalent Set, Strong Chain Equivalent Set, Chain Recurrent Set, Strong Chain Recurrent Set.**

I. Definitions and Notations

Let (X,d) be a compact metric space. Denoted by $C^0(X)$ the set of all continuous maps from X to itself. Let $f\in C^0(X)$. Let Z_+ be the set of all positive integers and $N=\{0\}\bigcup Z_+$. Denoted by $B(x,\varepsilon)=\{y\in X:d(x,y)<\varepsilon\}$ the ball field of x, for some $x\in X$, $\varepsilon>0$.

Definition 1.1[1] A point $y\in X$ is said to be a limit point of f if for some sequence of integers $n_i\to\infty$, $f^{n_i}(y)\to x$ holds. We define the limit set of a point x to be the set of all limit points of x.Recorded as $\omega(x,f)$

Definition 1.2[1] For any $\varepsilon>0$, if the sequence $\{x_0,x_1,\cdots,x_n\}$ such that $d(f(x_i),x_{i+1})<\varepsilon$, the sequence is called a ε chain from x_0 to x_n.

Definition 1.3[1] $CR(x,\varepsilon,f)=\{\,y\in X:$ there exist a ε chain from x to y under f $\}$. It is evident by definition that $CR(x,\varepsilon_1,f)\subset CR(x,\varepsilon_2,f)$, when $0<\varepsilon_1<\varepsilon_2$.

Definition 1.4[1] $S(x,f)=\{y\in X,\forall\varepsilon>0,y\in CR(x,\varepsilon,f)\}=\bigcap_{\varepsilon>0}CR(x,\varepsilon,f)$;

$$S(x,f)=\bigcap_{k=1}^{\infty}CR(x,\frac{1}{k},f)=\lim_{k\to\infty}CR(x,\frac{1}{k},f)$$

Definition 1.5[1] $CE(x,f)=\{\,y\in X$, there exist a ε chain from x to y under f. In addition to this, there exist a ε chain from y to x under f $\}=\{y\in X, y\in S(x,f)$ and $x\in S(y,f)\,\}$; $CE(x,f)$ may be empty.

Definition 1.6[1] For any $\varepsilon>0$,if there exist a ε chain from x to itself under f, x is called a chain recurrent point of f. We define the chain recurrent set of f to be the set of all chain recurrent points of f. Recorded as $CR(f)$.

Definition 1.7[2] For any $\varepsilon>0$, if the sequence $\{x_0,x_1,\cdots,x_n\}(n\in Z^+)$ such that $\sum_{i=0}^{n-1}d(f(x_i),x_{i+1})<\varepsilon$, the sequence is called a strong ε chain from x_0 to x_n.

Definition 1.8 $SCR(x,\varepsilon,f)=\{\,y\in X:$ there exist a strong ε chain from x to y under f $\}$. It is evident by definition that $SCR(x,\varepsilon_1,f)\subset SCR(x,\varepsilon_2,f)$, when $0<\varepsilon_1<\varepsilon_2$.

Definition1.9 $SS(x,f)=\{y\in X,\forall\varepsilon>0,y\in SCR(x,\varepsilon,f)\}=\bigcap_{\varepsilon>0}SCR(x,\varepsilon,f)$.

Definition 1.10[2] For any $\varepsilon>0$,if there exist a strong ε chain from x to itself under f, x is called a strong chain recurrent point of f. We define the strong chain recurrent set of f to be the set of all strong chain recurrent points of f. Recorded as $SCR(f)$.

Definition 1.11 $SCE(x,f)=\{\,y\in X$, there exist a strong ε chain from x to y under f. In addition to this, there exist a strong ε chain from y to x under f $\}=\{\,y\in X, y\in SS(x,f)$ and $x\in SS(y,f)\,\}$.

The orbit of x under f, the set of fixed points under f,the set of periodic points under will be denoted by $O(x,f)$, $F(f)$, $P(f)$, respectively.

II. Some Properties of Strong Chain Equivalent Sets

Proposition 2.1 If $f\in c^{\circ}(X)$, then the following properties are established:

(1) $SCE(x,f)$ is invariant and closed;

(2) $SCE(x,f)\neq\phi\Leftrightarrow x\in SCR(f)$;

(3)If $x\in SCR(f)$,

$SCE(x,f\,|_{SCE(x,f)})\subset SCE(x,f)$;

(4) If $SCE(x,f)\neq\phi$, $\overline{Orb(x,f)}\subset SCE(x,f)$;

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(5) For $\forall x$, if $y, z \in \omega(x, f)$, then $y \in SCE(z, f)$.

Proof: The proof of (1): Firstly, we prove $SCE(x, f)$ which is under f is an invariant set. It means we must get $f(SCE(x, f)) \subset SCE(x, f)$.

For any $y \in SCE(x, f)$, there exists a strong ε chain from x to y denoted $x_0 = x, x_1, x_2, \cdots x_{m-1}, x_m = y$ such that $\sum_{i=0}^{m-1} d(f(x_i), x_{i+1}) < \varepsilon$ for any $\varepsilon > 0$. Then the sequence $x_0 = x, x_1, x_2, \cdots x_{m-1}, x_m = y, x_{m+1} = f(y)$ can made $\sum_{i=0}^{m} d(f(x_i), x_{i+1}) = \sum_{i=0}^{m-1} d(f(x_i), x_{i+1}) + d(f(y), f(y)) = \sum_{i=0}^{m-1} d(f(x_i), x_{i+1}) < \varepsilon$ hold.

So $x_0 = x, x_1, x_2, \cdots x_{m-1}, x_m = y, x_{m+1} = f(y)$ is a strong ε chain from x to $f(y)$. Furthermore, we want to get a is the strong ε chain from $f(y)$ to x.

Because of $f \in c^{\circ}(X)$, for any $\varepsilon > 0$, we can choose a $0 < \delta < \frac{\varepsilon}{2}$ so that $d(u, v) < \delta$ implies $d(f(u), f(v)) < \frac{\varepsilon}{2}$.

Let $y_0 = y, y_1, y_2, \cdots y_n = x$ be a strong δ ($\delta < \frac{\varepsilon}{2}$) chain from y to x. It means $\sum_{i=0}^{n-1} d(f(y_i), y_{i+1}) < \delta$,

We wish to show that $z_0 = f(y), z_1 = y_2, \cdots, z_{n-1} = y_n = x$ is a strong ε chain.

$$\sum_{i=0}^{n-2} d(f(z_i), z_{i+1})$$
$$< d(f^2(y), y_2) + \sum_{i=2}^{n-1} d(f(y_i), y_{i+1})$$
$$< d(f^2(y), f(y_1)) + d(f(y_1), y_2) + \sum_{i=2}^{n-1} d(f(y_i), y_{i+1})$$
$$< d(f(y_1), y_2) + \sum_{i=1}^{n-1} d(f(y_i), y_{i+1}) < \frac{\varepsilon}{2} + \delta < \varepsilon$$

Therefore, $z_0 = f(y), z_1 = y_2, \cdots, z_{n-1} = y_n = x$ is the strong ε chain from $f(y)$ to x. It means that $f(y) \in SCE(x, f)$, if $y \in SCE(x, f)$. so that $SCE(x, f)$ under the f is invariant.

Next, we will prove the $SCE(x, f)$ is closed set as follow.

We prove $\overline{SCR(x, \varepsilon_1, f)} \subset SCR(x, \varepsilon_2, f)$ for any $0 < \varepsilon_1 < \varepsilon_2$. Let $\varepsilon = \varepsilon_2 - \varepsilon_1$, for any $y \in \overline{SCR(x, \varepsilon_1, f)}$, there exists $y' \in SCR(x, \varepsilon_1, f)$ which make $d(y, y') < \varepsilon$.

Because of $y' \in SCR(x, \varepsilon_1, f)$, there exists $x_0 = x, x_1, x_2, \cdots x_m = y'$ such that $\sum_{i=0}^{m-1} d(f(x_i), x_{i+1}) < \varepsilon_1$.

We consider the sequence $y_0 = x_0 = x, y_1 = x_1, \cdots, y_{m-1} = x_{m-1}, y_m = y$. As

$$\sum_{i=0}^{m-1} d(f(y_i), y_{i+1})$$
$$= \sum_{i=0}^{m-2} d(f(x_i), x_{i+1}) + d(f(x_{m-1}), y)$$
$$\leq \sum_{i=0}^{m-2} d(f(x_i), x_{i+1}) + d(f(x_{m-1}), x_m) + d(y, y')$$
$$\leq \sum_{i=0}^{m-1} d(f(x_i), x_{i+1}) + d(y, y') \leq \varepsilon_1 + \varepsilon = \varepsilon_2,$$

$y \in SCR(x, \varepsilon_2, f)$ holds. and

$SS(x, f) \subset \bigcap_{n=1}^{\infty} \overline{SCR(x, \frac{1}{n}, f)} = \bigcap_{n=2}^{\infty} \overline{SCR(x, \frac{1}{n}, f)} \subset \bigcap_{n=1}^{\infty} SCR(x, \frac{1}{n}, f) = SS(x, f)$ holds, so that $SS(x, f)$ is a closed set. Coupled with $SCE(x, f) = SS(x, f) \bigcap SS(x, f^{-1})$, then $SCE(x, f)$ is a closed set. Now (1) has been proved.

The proof of (2): Because $SCE(x, f) \neq \phi$, there exists a $y \in SCE(x, f)$. By definition 1.11, $y \in SS(x, f) \bigcap SS(x, f^{-1})$ holds. And then, for any $\varepsilon > 0$, there exists one strong $\frac{\varepsilon}{2}$ chain from x to y and another strong $\frac{\varepsilon}{2}$ chain from y to x. Combine two chains together, we get a strong ε chain from x to x, then $x \in SCR(f)$. Then if $x \in SCR(f)$, it exists the strong ε chain of x to x, so that $x \in SCE(x, f)$, then $SCE(x, f) \neq \phi$. The proof of (2) is end.

(3) is evident by definition.

The proof of (4): Because $SCE(x, f) \neq \phi$, there exists $y \in SCE(x, f)$. By definition 1.11, $y \in SS(x, f) \bigcap SS(x, f^{-1})$ holds. And then, for any $\varepsilon > 0$, there exists one strong $\frac{\varepsilon}{2}$ chain from x to y and another strong $\frac{\varepsilon}{2}$

chain from y to x. Combine two chains together, we get a strong ε chain from x to x, then $x \in SCR(f)$. Hence $x \in SCE(x, f)$. And $SCE(x, f)$ is invariant, $f(x) \in SCE(x, f)$ establish.

Further, $Orb(x, f) \subset SCE(x, f)$.In addition to $SCE(x, f)$ is closed, $\overline{Orb(x, f)} \subset SCE(x, f)$. Now, (4) has been proved.

The proof of (5): We just need to prove that $y \in SS(z, f)$. Because the position of y is equal to z, $y \in SS(z, f)$ implies $z \in SS(y, f)$. We can get $y \in SCE(z, f)$ further. Then (5) can be proof.

Because of $f \in c^{\circ}(X)$, for any $\varepsilon > 0$,we can choose a $0 < \delta < \frac{\varepsilon}{2}$, so that $d(u, v) < \delta$ implies $d(f(u),.$

$f(v)) < \frac{\varepsilon}{2}$. By definition of $z \in \omega(x, f)$, there exists $k \in Z^+$ such that $f^k(x) \in B(z, \delta)$. Further $d(f(z),.\ f^{k+1}(z)) < \frac{\varepsilon}{2}$. By definition of $y \in \omega(x, f)$, there exists $l \in Z^+$ make $f^{k+l}(x) \in B(y, \delta)$ hold. Consider $z_0 = z, z_1 = f^{k+1}(x), \cdots, z_{l-1} = f^{k+l-1}(x),\ z_l = y$.

Because $\sum_{i=0}^{l-1} d(f(z_i), z_{i+1}) = d(f(z), f^{k+1}(z)) + d(f^{k+l}(z), y)$ $< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} < \varepsilon$. It means that $y \in SS(z, f)$. Similar to the proof of $y \in SS(z, f)$, the proof of $z \in SS(y, f)$ is trivial. Then $y \in SCE(z, f)$.

Proposition 2.2 Let $f \in c^{\circ}(X)$, $x \in SCR(f^n)$, $n \in N_+$, then

(1) $SCE(f^n(x), f^n) = SCE(x, f^n)$;

(2) $\bigcup_{i=0}^{n-1} SCE(f^i(x), f^n) \subset SCE(x, f)$;

(3) $f(SCE(f^i(x), f^n)) = SCE(f^{i+1}(x), f^n), (i = 0, 1, \cdots n-1)$.

Proof: Firstly, we should prove $x \in SCR(f)$. Because $x \in SCR(f^n), n \in N_+$, for any $\varepsilon > 0$, there exists a strong ε chain from x to x under the f^n, marked as $x_{0n} = x, x_{1n}, \cdots x_{mn} = x$, which let $\sum_{i=0}^{m-1} d(f^n(x_{in})$ $x_{i+1n}) < \varepsilon$ hold.

For any $j = nk + i\ (i, k \in N, 0 \le i < n, 0 \le k < m)$, let $x_j = f^i(x_{kn})$, so that for the chain of $x_{0n} = x_0 = x$, $x_1, \cdots x_{n-1}, x_{1n}, \cdots x_{mn} = x$, we can obtain that $\sum_{i=0}^{mn-1} d(f(x_i), x_{i+1}) = \sum_{i=0}^{m-1} d(f^n(x_{in}), x_{i+1n}) < \varepsilon$. This is a strong ε chain from x to x under the f.

Hence, $x \in SCR(f)$, we can know $SCE(f^n(x), f^n)$ and $SCE(x, f^n)$ are not empty.

The proof of (1): let $f^n = g$, so we should prove $SCE(g(x), g) = SCE(x, g)$.In order to get that, we should prove $SCE(g(x), g) \subset SCE(x, g)$ and $SCE(x, g) \subset SCE(g(x), g)$.

We prove $SCE(g(x), g) \subset SCE(x, g)$ firstly. For any $y \in SCE(g(x), g)$, according to the definition of strong chain equivalent set, we can know that for any $\varepsilon > 0$, there exists a strong $\frac{\varepsilon}{2}$ chain $x_0 = g(x), x_1, \cdots, x_m = y$ from $g(x)$ to y, and a strong $\frac{\varepsilon}{2}$ chain $y_0 = y, y_1, \cdots, y_l = g(x)$ from y to $g(x)$. Due to $x \in SCR(g)$, then $x \in SCE(x, g)$. Because $SCE(x, g)$ is invariant, $g(x) \in SCE(x, g)$.

For the above ε, there exists a strong $\frac{\varepsilon}{2}$ chain from $g(x)$ to x, and we sign it as $z_0 = g(x), z_1, \cdots,.$ $z_m = x$. It can be easily proved that $x, x_0 = g(x), x_1, x_m = y$ is the strong ε chain of x to y, and $y_0 = y, y_1, \cdots, y_l = g(x) = z_0, z_1, \cdots, z_m = x$ is the strong ε chain of y to x. Hence $y \in SCE(x, g)$. Then we can get $SCE(g(x), g) \subset SCE(x, g)$.

Next, we will prove $SCE(x, g) \subset SCE(g(x), g)$ .Because f is uniformly continuous, we can know that g is uniformly continuous. Hence, for any $\varepsilon > 0$,there exists $0 < \delta < \frac{\varepsilon}{2}$ such that $d(f(u), f(v)) < \frac{\varepsilon}{2}$, if $d(u, v) < \delta$. As $y \in SCE(x, g)$, for this δ, we can choose a strong δ chain from x to y denoted as $x_0 = g(x), x_1, \cdots, x_m = y$,and a strong δ chain from x to y denoted as $y_0 = y, y_1, \cdots, y_l = x$. It is evident that $y_0 = y,$

$y_1,\cdots,y_l=x,y_{l+1}=g(x)$ is the strong δ chain from y to $g(x)$. We will consider the $z_0=g(x),z_1=x_2,z_2=x_3,\cdots,z_{m-1}=x_m=y$ as follow. We get

$$\sum_{i=0}^{m-2}d(g(z_i),z_{i+1})=d(g^2(x),x_2)+\sum_{i=2}^{m-1}d(g(x_i),x_{i+1})$$
$$<d(g^2(x),g(x_1))+d(g(x_1),x_2)+\sum_{i=2}^{m-1}d(g(x_i),x_{i+1})$$
$$<d(g^2(x),g(x_1))+\sum_{i=2}^{m-1}d(g(x_i),x_{i+1}).$$

Since $d(g(x),x_1)<\sum_{i=0}^{m-1}d(g(x_i),x_{i+1})<\delta$, and g is uniformly continuous, that $d(g^2(x),g(x_1))<\frac{\varepsilon}{2}$ holds.

Hence $\sum_{i=0}^{m-2}d(g(z_i),z_{i+1})<\frac{\varepsilon}{2}+\delta<\varepsilon$. Then $z_0=g(x),z_1=x_2,z_2=x_3,\cdots,z_{m-1}=x_m=y$ is a strong ε chain from $g(x)$ to y, so $y\in SCE(g(x),g)$, then $SCE(x,g)\subset SCE(g(x),g)$.

In conclusion, $SCE(g(x),g)=SCE(x,g)$. (1) has been proved.

The proof of (2): For any $y\in SCE(f^i(x),f^n)$, it is evident that $y\in SS(f^i(x),f^n)$. According to the definition we can know that, for any $\varepsilon>0$, there exists a strong $\frac{\varepsilon}{2}$ chain from $f^i(x)$ to y under the f^n, denoted as $x_{0n}=f^i(x),x_{1n},\cdots x_{mn}=y$.

For any $j=nk+i\ (i,k\in N,0\le i<n,0\le k<m)$, let $x_j=f^i(x_{kn})$. Consider the chain $x_{0n}=x_0=f^i(x),x_1,\cdots,x_{n-1},x_{1n},\cdots,x_{mn}=y$ and we can obtain that $\sum_{i=0}^{mn-1}d(f(x_i),x_{i+1})=\sum_{i=0}^{m-1}d(f^n(x_{in}),x_{i+1n})<\varepsilon$.

Hence $y\in SS(f^i(x),f)$. According to the definition we can know $f^i(x)\in SS(x,f)$. Thus, $y_0=x,y_1=f(x),\cdots,y_i=x_0=f^i(x),y_{i+1}=x_1,\cdots,y_{mn+i}=x_{mn}=y$ is a strong ε chain from x to y, so that $y\in SS(f^i(x),f)$.

Since $y\in SCE(f^i(x),f^n)$, we can know that $f^i(x)\in SS(x,f)$. It means that there exists a strong $\frac{\varepsilon}{2}$ chain from y to $f^i(x)$ under f^n, recorded as $y_{0n}=y,y_{1n},\cdots,y_{mn}=f^i(x)$, which satisfying the condition $\sum_{i=0}^{m-1}d(f^n(x_{in}),x_{i+1n})<\frac{\varepsilon}{2}$. Similarly, for any $j=nk+i\ (i,k\in N,0\le i<n,0\le k<m)$, we let $y_j=f^i(y_{kn})$, Consider the chain $y_{0n}=y_0=y,y_1,\cdots,y_{n-1},y_{1n},\cdots y_{mn}=f^i(x)$ we can obtain that,

$$\sum_{i=0}^{mn-1}d(f(y_i),y_{i+1})=\sum_{i=0}^{m-1}d(f^n(y_{in}),y_{i+1n})<\frac{\varepsilon}{2}.$$

Hence, $f^i(x)\in SS(y,f)$. As $x\in SCR(f)$ and $SCR(f)$ is invariant, we know $f^i(x)\in SCR(f)$, then $x\in SS(f^i(x),f)$. It means that there exists a strong $\frac{\varepsilon}{2}$ chain from $f^i(x)$ to x under the f, signed as $z_0=f^i(x),z_1,\cdots,z_i=x$. Therefore, $a_0=y_{0n}=y_0=y,a_1=y_1,\cdots a_{n-1}=y_{n-1},a_{1n}=y_{1n},\cdots,a_{mn}=y_{mn}=f^i(x)=z_0,\cdots,a_{mn+i-1}=z_i=x$ is a strong ε chain from y to x under f. It means $x\in SS(y,f)$. Further, $y\in SCE(x,f)$.

Hence $\bigcup_{i=0}^{n-1}SCE(f^i(x),f^n)\subset SCE(x,f)$. (2) has been proved.

III. Example

The example of $SCR(f)\neq CR(f)$ on the circle had been given by reference [2]. Now, we will give an example, which on the interval as follow:

Example 3.1: Let $I=[0,4]$ and $f:I\to I$ is a separable continuous map, which been defined as

$$f=\begin{cases}\sqrt{x}, & x\in[0,1)\\ x, & x\in[1,2)\\ 2x-2, & x\in[2,3)\\ 16-4x, & x\in[3,4)\end{cases}.$$

Proposition 3.2: Let $f:I\to I$ as defined by example 4.1. Then $x\in CR(f)$ for each $x\in(0,1)$.

Proof: Because $f(x)=\sqrt{x}$ on the $(0,1)$, so that $f^n(x)=x^{\frac{1}{2^n}}$. Because for any $x>0$, $\lim_{n\to\infty}x^{\frac{1}{2^n}}=1$. It means that for any $\varepsilon>0$, there exists $N\in N_+$, $N>0$ such that $|x^{\frac{1}{2^n}}-1|<\varepsilon$, if $n>N$. Then

$x_0 = x, x_1 = f(x), x_2 = f^2(x), \cdots, x_N = f^N(x)$ $, x_{N+1} = f^{N+1}(x), x_{N+2} = 1$ is a ε chain from x to 1.

Let $M = [\frac{1}{\varepsilon}] + 1$, then $y_0 = 1, y_1 = 1 + \frac{\varepsilon}{2}, \cdots,$ $y_{2M} = 1 + 2([\frac{1}{\varepsilon}] + 1) * \frac{\varepsilon}{2}, y_{2M+1} = 2$ is a ε chain from 1 to 2.

Because $\{\frac{1}{2^n}\}$ is a series which converges to 0. It means that for any $\varepsilon > 0$, there exists $N_1 \in N_+$, such that $\frac{1}{2^n} < \varepsilon$, if $n > N_1$. Then $z_0 = 2, z_1 = 2 + \frac{1}{2^n}$, $z_2 = f(z_1), \cdots, z_n = f(z_{n-1}) = \frac{5}{2}, z_{n+1} = 3, z_{n+2} = 4,$ $z_{n+3} = 0$ is a ε chain from 2 to 0.

When $x \in (0,1)$, $\{x^{2^n}\}$ converges 0. It means that for any $\varepsilon > 0$, there exists $N_2 \in N_+$, such that $x^{2^n} < \varepsilon$, if $n > N_2$.Then $h_0 = 0, h_1 = x^{2^n}$, $h_2 = f(h_1), \cdots, h_n = f(h_{n-1}) = x^2$, $h_{n+1} = x$ is a ε chain from 0 to x.

Overall, if we put these together, we can obtain a ε chain from x to x, and according to the arbitrarily of ε, we can know that $x \in CR(f)$.

Proposition 3.3: Let $f : I \to I$ as defined by example 4.1, hence for any $x \in (0,1)$, we can obtain $x \notin SCR(f)$.

Proof: Let $\varepsilon_0 = \frac{1}{4}(\sqrt{x} - x)$, assume $x_0 = x, x_1, x_2, \cdots x_n$ is a strong ε_0 chain.

Claim: For each $1 \le k \le n$, we have $x < x_k < 1 + \varepsilon_0 < 2$.

Proof of the claim: If $k = 1$, we have

$$x_1 > f(x) - \varepsilon_0 = \sqrt{x} - \frac{1}{4}(\sqrt{x} - x)$$

$$= \frac{3}{4}\sqrt{x} + \frac{1}{4}x > x$$

On the other hand. $x_1 < f(x) + \varepsilon_0 = 1 + \varepsilon_0 < 2$. Thus we have $x < x_1 < 1 + \varepsilon_0 < 2$.

If for each $2 \le i < k \le n$, we have $x < x_i < 1 + \varepsilon_0 < 2$.

Hypothesis that $x_k \ge 1 + \varepsilon_0$.Since $x_0 = x, x_1, x_2,$ $\cdots, x_n$ is a strong ε_0 chain, we have

$\sum_{n=0}^{k-1} d(f(x_i), x_{i+1}) < \varepsilon_0$. If $x < x_{k-1} \le 1$, then

$$\sum_{n=0}^{k-1} d(f(x_i), x_{i+1}) \ge d(f(x_{k-1}), x_k) \ge x_k - 1 > \varepsilon_0.$$

That is a contradiction. Thus $x_{k-1} > 1$. Let $t = \max\{i : 1 \le i \le k, x < x_{i-1} < 1\}$. Then $x < x_{t-1} < 1$, $1 \le x_i < 1 + \varepsilon_0$ for $t \le i < k$. Thus,

$$\sum_{i=0}^{k-1} d(f(x_i), x_{i+1}) \ge \sum_{i=t-1}^{k-1} d(f(x_i), x_{i+1})$$

$$\ge \sum_{i=t}^{k-1} |x_{i+1} - x_i| + x_t - 1$$

$$\ge \left| \sum_{i=t}^{k-1} (x_{i+1} - x_i) \right| + x_t - 1$$

$$= x_k - 1 > \varepsilon_0.$$

This is a contradiction. Thus $x_k < 1 + \varepsilon_0$.

On the other hand, we have.

$$x_k \ge f(x_{k-1}) - \varepsilon_0 > f(x) - \varepsilon_0 > x$$

The proof of claim is completed.

If $x \in SCR(f)$, then there exists a strong ε_0-chain $x_0 = x, x_1, x_2, \cdots x_n = x$. It follows by the Claim that $x < x$. This is a contradiction. Thus, we have $x \notin SCR(f)$.

IV. Conclusion

We study continuous maps of a metric space into itself. We find some properties of strong chain equivalent set with the maps. Further we give an example in interval to show $SCR(f) \ne CR(f)$.

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References

[1] Gongfu Liao, Lidong Wang, Qinjie Fan "Iterate and chaos dynamic system", Science publication, 1999.
[2] Junling Zhao, "Strong chain recurrent set and shadowing properties", Mathematics Study, 37(3), 2004, 286-291.
[3] Block L S, Coppel W A, "Dynamics in one dimension", New York: Spring-er-verlag, 1992.
[4] Block L, Franke J E, "The chain recurrent set for maps of the interval", Proceedings of the American Mathematical Society, 87(4), 1983, 723-727.
[5] Taixiang Song, "Dynamics of tree map", Guangxi: Guangxi Science and Tech-nology Press, 2011.

[6] Walters P,"On the pseudo orbit tracing property and its relationship to stability. In: The structure of attractors in dynamical systems", Berlin: Spring-Verlag, 1978, 231-244.
[7] Shimomura T. China recurrence and P.O.T.P, "In: Dynamical systems and singular phenomena", Singapore: World Sic. Publishing, 1987, 224-241.
[8] Coven E M, Kan I, Yorke J A, "Pseudo-orbit shadowing in the family of tent maps. Trans", Amer .Math. Soc, 308, 1988, 227-241.
[9] Pilyugin S Y, "Shadowing in dynamical systems", Berlin: Spring-Verlag, 1999.
[10] Aoki N, "Topological dynamics", In: Topics in General Topology. Amsterdam: North Holland, 1989, 624-741.

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