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On Purely Real Lie Algebras of Skew-Adjoint Operators

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Abstract: **In the given paper we investigate a real von Neumann algebra R on a Hilbert space such that $R \cap iR = \{0\}$ and a real Lie algebra L of skew-adjoint operators on a Hilbert space H such that $R(L) \cap iR(L) = \{0\}$ for the real *-algebra $R(L)$, generated by L in $B(H)$. These algebras are called a purely real von Neumann algebra and a purely real Lie algebra respectively.**

An analog of Gelfand-Naumark theorem for ultraweakly closed purely real Lie algebras of skew-adjoint operators on a Hilbert space is proved. Also, it is proved that the enveloping C* -algebra of such Lie algebra is a von Neumann algebra if this Lie algebra is reversible and it is given a condition in which a purely real Lie algebra of skew-adjoint operators is reversible.

Keywords: **Real Lie Algebra, Real Von Neumann Algebra, Von Neumann Algebra.**

I. Introduction

A weakly closed real *-subalgebra R in $B(H)$ is called a purely real von Neumann algebra if $R \cap iR = \{0\}$, where $iR = \{ix : x \in R\}$. Let $Z_R = \{x \in R : xy = yx \forall y \in R\}$ be the center of R. If Z_R consists of real multiples of 1, i.e. $Z_R = R1$, then R is said to be a purely real factor. Given a purely real von Neumann algebra R it is easy to see that the set $R_k = \{x \in R : x^* = -x\}$ forms a Lie algebra with the Multiplication $[x, y] = xy - yx$. Such Lie algebras and purely real von Neumann algebras were investigated, in particular, in papers [4], [5] by Sh.Ayupov. In these papers it was considered a spacial case of a more general algebraic problem: letting R and S be simple or prime rings with involutions, can every (Lie) isomorphism of $[R_k, R_k]$ onto $[S_k, S_k]$ be lifted to an (associative) isomorphism of R onto S? Here the derived Lie ring $[R_k, R_k]$ is the additive span of all commutators $[x, y]$,where $y\ x, \in R_k$; it is a Lie ideal in Lie ring R_k.

The above problem was solved for simple rings with involution of the first kind were obtained by Martindale [1] and for arbitrary kind by Rosen [2]. But in the case of prime rings it is not possible in general to extend a Lie isomorphism between $[R_k, R_k]$ and $[S_k, S_k]$ to a *-isomorphism between whole R and S. Moreover, there are examples of prime rings with involution such that $[R_k, R_k]$ and $[S_k, S_k]$ are Lie isomorphic but R and S are not *-isomorphic. Concerning purely real von Neumann algebras one can see that every purely real von Neumann algebra is semiprime: it is prime if (but not only if) it is a purely real factor. Examples of simple purely real factors are given by factors of type $\mathrm{I}_n (n < \infty)$, II_1, and a-finite type III [4].

In [5] it was proved that if R and S are real factors not of types I_1 and I_2 then derived Lie algebras $[R_k, R_k]$ and $[S_k, S_k]$ are isomorphic if and only if R and S are *-isomorphic.

Also in the paper [3] there were proved the following statements: let R be a purely real factor except types I_1 and I_2. Then

1. $R_k = [R_k, R_k]$
2. $R = R_k$, where $\mathrm{R_k}$ is the associative sub algebra generated by R_k.

In the given paper analogues of these statements are proved for real von Neumann algebras R, S, satisfying the conditions $R^w(R_k) = R$, $R^w(S_k) = S$, where $R^w(R_k)$, $R^w(S_k)$ are the real von Neumann subalgebras generated by R_k, S_k respectively.

In the given paper it is introduced weakly closed purely real Lie algebras on a complex Hilbert space. These Lie algebras are Banach Lie algebras over the field of real numbers. Banach Lie algebras are investigated in [9], [10]. In this paper we give a representation of all maximal purely real Lie algebras of skew-adjoint elements in the algebra $B(H)$ for a complex Hilbert space H.

Also, it is proved that the enveloping C*-algebra of such Lie algebra is a von Neumann algebra if this Lie algebra is reversible. Also, it is found a condition in which case a purely real Lie algebra of skew-adjoint operators is reversible.

II. Maximal Purely Real Lie Algebras of Skew-Adjoint Operators

In [6] it was proved the following theorem.

Theorem 1.1. Let H be a Hilbert space. Then any maximal purely real *-algebra on H is isomorphic to the algebra $B(\overline{H}_{\mathbb{R}}) \oplus B(\overline{H}_{\mathbb{H}})$, where $\overline{H}_{\mathbb{R}}$and $\overline{H}_{\mathbb{H}}$are Hilbert subspaces of H such that $H_{\mathbb{R}} = \overline{H}_{\mathbb{R}} \oplus \overline{H}_{\mathbb{R}}^{\perp}, H_{\mathbb{H}} = H_{\mathbb{H}} \oplus \overline{H}_{\mathbb{R}}^{\perp}$, the identity elements of the algebras $B(\overline{H}_{\mathbb{R}})$ and $B(\overline{H}_{\mathbb{H}})$ are mutually orthogonal and their sum is equal to the identity element of $B(H)$, and $H_{\mathbb{R}}$ and $H_{\mathbb{H}}$ are Hilbert spaces on R and H respectively such that $B(H) = B(H_F) + iB(H_F)$, where $F = \mathbb{R}\ or\ \mathbb{H}$.

Definition 1.2. Let L be an Lie algebra of skew-adjoint operators on a Hilbert space H. The Lie algebra L is called *a purely real Lie algebra,* if the real *-algebra $R(L)$, generated by L in $B(H)$ satisfies to the condition $R(\mathrm{L}) \cap iR(L) = \{0\}$.

By Zorn's lemma for any purely real Lie algebra of skew-adjoint operators on a Hilbert space there exists a

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maximal purely real Lie algebra of skew-adjoint operators containing the given Lie algebra. Therefore the following theorem takes place.

Theorem 1.3. Let H be a Hilbert space. Then any maximal purely real Lie algebra of skew-adjoint operators on H is isomorphic to the Lie algebra $B(\overline{H}_{\mathbb{R}})_k \oplus B(\overline{H}_{\mathbb{H}})_k$, where $\overline{H}_{\mathbb{R}}$ and $H_{\mathbb{H}}$ are Hilbert subspaces of H such that $H_{\mathbb{R}} = \overline{H}_{\mathbb{R}} \oplus \overline{H}_{\mathbb{R}}^{\perp}, H_{\mathbb{H}} = H_{\mathbb{H}} \oplus \overline{H}_{\mathbb{R}}^{\perp}$, the identity elements of $B(\overline{H}_{\mathbb{R}})$ and $B(\overline{H}_{\mathbb{H}})$ are mutually orthogonal and their sum is equal to the identity element of $B(H)$, and $H_{\mathbb{R}}$, $H_{\mathbb{H}}$ are Hilbert spaces on $\mathbb{R}$ and $\mathbb{H}$ respectively such that $B(H) = B(H_F) + iB(H_F)$, where $F = \mathbb{R}, \mathbb{H}$.

Proof. Let L be a maximal purely real Lie algebra of skew-adjoint operators on a Hilbert space H and $R(L)$ be the real *-algebra, generated by L. By the definition $R(L) \cap iR(L) = \{0\}$. Then there exists a maximal purely real algebra N on H containing the real algebra $R(L)$. Since L is maximal we have $L = R(L)_k = N_k$.

By theorem 1.1 $N \cong B(\overline{H}_{\mathbb{R}})_k \oplus B(\overline{H}_{\mathbb{H}})_k$, where $\overline{H}_{\mathbb{R}}$ and $H_{\mathbb{H}}$ are Hilbert subspaces of H such that $H_{\mathbb{R}} = \overline{H}_{\mathbb{R}} \oplus \overline{H}_{\mathbb{R}}^{\perp}, H_{\mathbb{H}} = H_{\mathbb{H}} \oplus \overline{H}_{\mathbb{R}}^{\perp}$, the identity elements of $B(H_{\mathbb{R}})$ and $B(H_{\mathbb{H}})$ are mutually orthogonal and their sum is equal to the identity element of $B(H)$, and $H_{\mathbb{R}}$ and $H_{\mathbb{H}}$ are Hilbert spaces on $\mathbb{R}$ and $\mathbb{H}$ respectively such that $B(H) = B(H_F) + iB(H_F)$, where $F = \mathbb{R}\, or\, \mathbb{H}$. Hence $L \cong B(\overline{H}_{\mathbb{R}})_k \oplus B(\overline{H}_{\mathbb{H}})_k$.

Definition 1.4. Let L be an Lie algebra of skew-adjoint operators on a Hilbert space H. The Lie algebra L is called *reversible*, if for any elements $a_1, a_2, \dots a_{2n} \in L$ the following conditions hold

$$a_1 a_2, \dots a_{2n} - a_{2n} a_{2n-2} \dots a_1 \in L$$

$$a_1 a_2, \dots a_{2n-1} - a_{2n-1} a_{2n-2} \dots a_1 \in L.$$

Theorem 1.5. Let L be an ultra weakly closed purely real reversible Lie algebra of skew-adjoint operators on a Hilbert space H, $R^w(L)$ be a real von Neumann algebra, generated by L in $B(H)$. Then $R^w(L)_k = L$.

Proof. It is evident that $L \subseteq R^w(L)_k$.

Let $a \in R^w(L)_k$. Then a* = —a and there exists a sequence $(b_m) \subset R^w(L)_k$ suchthat (b_m) weakly converges to a and $b_m = \sum_{i=1}^{k_m} a_{i1} a_{i2} \dots a_{in_i}$, where a_{i1}, a_{i2}, ..., $a_{in_i} \in L$ for all i. Since $b_m^* = -b_m$ we have that $-b_m = \sum_{i=1}^{k_m} (-1)^{n_i} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}$ and $b_m = \sum_{i=1}^{k_m} (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}$. Then

$$b_m = (1/2)\left(\sum_{i=1}^{k_m} a_{i1} \; a_{i2} \; \dots a_{in_i} + \sum_{i=1}^{k_m} (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right) = \left(\tfrac{1}{2}\right) \sum_{i=1}^{k_m} (a_{i1} \; a_{i2} \; \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}).$$

Note that in the last sum

$$\left(a_{i1} a_{i2} \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)^* = -\left(a_{i1} a_{i2} \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right) =$$
$$= \left(a_{i1} a_{i2} \dots a_{in_i} - a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)^* = -\left(a_{i1} a_{i2} \dots a_{in_i} - a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)$$

if n_i is an even natural number for any index i, and

$$\left(a_{i1} a_{i2} \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)^* = -\left(a_{i1} a_{i2} \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right) =$$
$$= \left(a_{i1} a_{i2} \dots a_{in_i} - a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)^* = = -\left(a_{i1} a_{i2} \dots a_{in_i} + a_{in_i} \; a_{in_{i-1}} \dots a_{i1}\right)$$

if f n_i is an add natural number. Hence, for any index i

$$a_{i1} a_{i2} \dots a_{in_i} + (-1)^{n_i+1} a_{in_i} \; a_{in_{i-1}} \dots a_{i1} \in L$$

by reversibility of the Lie algebra L. Therefore $b_m \subseteq L$. Since L is ultraweakly closed in $B(H)$ we have $a \in L$.

Theorem 1.6 .Let L be an ultraweakly closed reversible Lie algebra of skew- adjoint operators on a Hilbert space H, $R^w(L)$ be a real von Neumann algebra, generated by L in $B(H)$. Then the following conditions are equivalent

(1) $R^w(L) \cap iR^w(L) = \{0\}$,

(2)there exist Lie sub algebras $L_1 L_2$ in L such that $L_1 \subseteq B(H_{\mathbb{R}})_k$, $L_2 \subseteq B(H_{\mathbb{H}})_k$, and $L = L_1 \oplus L_2$ for some Hilbert spaces $H_{\mathbb{R}}$, $H_{\mathbb{H}}$ on $\mathbb{R}$ and $\mathbb{H}$, respectively, such that $B(H) = B(H_F) + iB(H_F)$, where $F = \mathbb{R}, \mathbb{H}$.

Proof. $1 \Rightarrow 2$: By theorem 1.3 $R^w(L)_k = L$ and by theorem 4 in [6] there exist central projections Z_1 and Z_2 in $R^w(L)$ such that $Z_1 R^w(L) \subseteq B(H_{\mathbb{R}})$, $Z_2 R^w(L) \subseteq B(H_{\mathbb{H}})$ and $R^w(L) = Z_1 R^w(L) \oplus Z_2 R^w(L)$. At the same time $R^w(L)_k = L$. Hence $L = Z_1 R^w(L)_k \oplus Z_2 R^w(L)_k$ and

$L_1 \subseteq B(H_{\mathbb{R}})_k, L_2 \subseteq B(H_{\mathbb{H}})_k$ where

$L_1 = Z_1 R^w(L)_k, L_2 = Z_2 R^w(L)_k.$

$2 \Rightarrow 1$: The converse statement of the theorem is obvious.

III. Reversibility of Purely Real Lie Algebras of Skew-Adjoint Operators

Theorem 2.1. Let L be a purely real Lie algebra of skew-adjoint operators on a Hilbert space $H, R^w(L)$ be a real von Neumann algebra, generated by L in $B(H)$. Suppose that $M_3(\mathbb{R})_K$ can be embedded in L and in this embedding $M_3(\mathbb{R}) \subseteq R^w(L)$ and the unit of $M_3(\mathbb{R})$ coincides with the unit of $R^w(L)$. Then L is reversible.

Proof. By the conditions of the theorem there exists a complete system $\{e_{ij}\}_{i,j=1,2,3}$ of matrix units in $R^w(L)$, i.e. $e_{11}+e_{22}+e_{33}=1, e_{ii}^2=e_{ii}, e_{ij}e_{ij}^*=e_{ii}, e_{ij}^*e_{ij}=e_{ii}$, where $i \neq j$ and $i,j = 1,2,3$, such that $e_{ij} - e_{ji} \in L$ for all $i = j$.

Then by [8, lemma 2.8.2] the set $A_0 = \{a \in R^w(L) + iR^w(L) : ae_{ij} = e_{ij}a \forall i = 1,2,3\}$ is a *-subalgebra of $R^w(L) + iR^w(L)$ and the map $(a_{ij}) \longrightarrow \sum_{ij=1,2,3} a_{ij}e_{ij}$ is a *-isomorphism of $M_3(A_o)$ onto $R^w(L) + iR^w(L)$.

Let

$$R_{ij} = \{a \in A_0 ae_{ij} - a^*e_{ji} \in L\}, i \neq j, i,j = 1,2,3.$$

Note that $R_{ij} = R_{ji}$, where $i \neq j$ and $i,j = 1,2,3$. We prove that $R_{12} = R_{13} = R_{23}$.

Let i,j,k be pairwise distinct indexes in $\{1,2,3\}, a \in R_{ji}$. Since $e_{jk} - e_{kj} - \in L$ and

$$[e_{jk} - e_{kj}, ae_{ij} - a^*e_{ij}] = a^*e_{ki} - a \quad e_{ik}$$

then $a^*e_{ki} - a \quad e_{ik} \in L$. Hence $a \in R_{ki}$. Therefore $R_{12} = R_{13} = R_{23}$. Also it is easy to see $a^* \in R_{ij}$.

Let $R = R_{12} = R_{13} = R_{23}$ and $a,b \in R$. Then the elements $ae_{12} - a^*e_{21}, ae_{23} - a^*e_{32}$ belong to L and $[ae_{12} - a^*e_{21}, ae_{23} - a^*e_{32}] = abe_{13} - b^*a^*e_{31} \in L$ Hence $ab \in R_{13} = R$. Therefore R is a real *-algebra.

Let

$$x = \sum_{i<j}(a_{ij}e_{ij} - a_{ji}e_{ji}) \in L, a_{ij} \in A_0, i, j = 1,2,3.$$

Since $x^* - x$ then $a_{ij}^* = a_{ji}$. It is clear that

$$[e_{ij} - e_{ji}, a_{ij}e_{ij} - a_{ij}^*e_{ji}] = 0, \forall i \neq j.$$

Let $p_{ij} = e_{ij} - e_{ji}$ for all $i \neq j$. Then x consequently multiplying by p_{12} and p_{23} we get $[p_{23}, [p_{12}, x]] = a_{13}e_{13} - a_{13}^*e_{31}$.

Hence $a_{13}e_{13} - a_{13}^*e_{31} \in L$. Analogously we have $a_{12}e_{12} - a_{12}^*e_{21}$, $a_{23}e_{23} - a_{23}^*e_{32} \in L$. Hence $a_{ij} \in R$ for all $i \neq j$.

Conversely, Let $x = \sum_{i<j}(a_{ij}e_{ij} - a_{ji}e_{ji})$, where $a_{ij} \in R, a_{ij}^* = a_{ji}, i,j = 1,2,3$. Then we have to show $x \in L$. For the last statement to be proved it is sufficient to establish $ae_{ji} - ae_{ij} \in L$ for all $i \neq j$. In turn the last statement is true by the definition of R. Hence $M_3(R)_k = L$. By the last equality L is reversible.

VI. The Enveloping C*-Algebra of a Purely Real Lie Algebra of Skew-Adjoint Operators

Theorem 3.1. Let L be a weakly closed purely real reversible Lie algebra of skew- adjoint operators on a Hilbert space H, $R^w(L)$ be a real von Neumann algebra, generated by L in $B(H)$. Suppose that $R^w(L)$ does not have nonzero direct summands of type I. Then the enveloping C*-algebra $C^*(L)$ of L is a von Neumann algebra.

Proof. By theorem 1.5 we have $L = R^w(L)_k$, where $R^w(L)_k = \{x \in R^w(L) : x^* = -x\}$, and $R^w(L)_{sa}$ is a JW-algebra with no nonzero direct summands of type I.

Let p be a projection in $R^w(L)_{sa}$. Then $pR^w(L)_{sa}p$ is a JW-algebra with no nonzero direct summands of type I. Hence there exist pairwise orthogonal and pairwise equivalent by symmetry projections p_1, p_2, p_3, p_4 in $pR^w(L)_{sa}p$ such that $p = p_1 + p_2 + p_3 + p_4$. Let $\{s_{ij}\}_{i<j,i,j=1,\dots,4}$ be partial symmetries in $pR^w(L)_{sa}p$ such that if $i < j$ then $p_j = s_{ij}p_is_{ij}$ for all $i,j = 1,2,3,4$.

Let $e_{ii} = p_i$ and if $i < j$ then $e_{ij} = p_is_{ij}$ for all $i,j = 1,2,3,4$. Then the set $\{e_{ij}\}_{i,j=1,2,34}$ is a system of matrix units and

$$R^c(\{e_{ij}\}_{i,j=1,\dots,4}) \cong M_4(\mathbb{R}),$$

where $R^c(\{e_{ij}\}_{i,j=1,\dots,4})$ is a real associative algebra, generated by the set $\{e_{ij}\}_{i,j=1,\dots,4}$. At the same time

$$R^c(R^c(\{e_{ij}\}_{i<j,i,j=1,\dots,4})_k) = R^c(\{e_{ij}\}_{i,j=1,\dots,4}),$$

where $R^c(R^c(\{e_{ij}\}_{i<j,i,j=1,\dots,4})_k)$ is a real associative algebra, generated by the Lie algebra $R^c(\{e_{ij}\}_{i,j=1,\dots,4})_k$. Also, since $L = R^w(L)_k$ we have $R^c(\{e_{ij}\}_{i,j=1,\dots,4})_k \subseteq L$. Hence $R^c(R^c(\{e_{ij}\}_{i<j,i,j=1,\dots,4})_k) \subseteq R^c(L)$ and $R^c(\{e_{ij}\}_{i<j,i,j=1,\dots,4}) \subseteq R^c(L)$. Since $p \in R^c(\{e_{ij}\}_{i,j=1,\dots,4})$ we have $p \in R^c(L)$. The projection p was chosen in $R^w(L)_{sa}$ arbitrarily, and a JC-algebra, generated by the set of all projections of the algebra $R^w(L)_{sa}$ coincides with $R^w(L)_{sa}$. Hence $R^w(L)_{sa} = R^c(L)_{sa}$.

Therefore

$$C^*(R^w(L)_{sa}) = C^*(R^c(L)_{sa}) = R^c(L) + iR^c(L) = C^*(L).$$

At the same time by [12] and [7] we have

$C^*(R^w(L)_{sa}) = R^w(L) + iR^w(L)$ and $R^w(L) + iR^w(L)$ is a von Neumann algebra, i.e. $W^*(L) = R^w(L) + iR^w(L)$. Hence $C^*(L) = W^*(L)$.

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